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Reliability Investigation of Steel Cased Columns

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Abstract: Reliability analysis of steel cased columns designed in accordance with BS5950[1] was carried using First Order Reliability Method (FORM). Design variables such as design axial load and moment, concrete strength, yield strength of steel, cross–sectional dimensions of the plate and universal column, as well as thickness of the base plate were considered random and stochastic. It was shown among other findings that, as the material strength P_y was increased, the safety of the designed section considering axially loaded decreased. However, considering combined axial load and bending condition, as the material strength was increased, the safety of the section increased. It was also shown that applied bending moment has more significant effect on concrete strength of the cased column than on steel strength.

Keywords: Reliability, steel cased columns, safety index, structural design, BS5950

INTRODUCTION

The design of any structure should be able to safely meet the requirements of functionality, aesthetics and economy (Clarke, A.B. and Converman, S.H., 1987). When an engineering structure is loaded in some way it will respond in a manner which depends on the type and magnitude of the load, and the strength as well as stiffness of the structure. Whether the response is considered satisfactory depends on the requirements which must be satisfied.

Cased or Composite columns comprise steel section with a concrete encasement or core. Encased column usually consist of standard universal beam (UB) or universal column (UC) sections with a square or rectangular or circular concrete section cased around them to form a solid composite section. Additional reinforcement is placed in the concrete cover around the steel section in order to prevent spalling under axial stress.

In many cases of design, concrete or some other protective material is needed around steel columns for reasons of fire-resistance and durability. It would seem appropriate, therefore to develop composite action between the steel and concrete, thereby taking advantage of the inherent compressive strength of concrete, increasing the capacity of the section and leading to considerable savings in the cost of the steelwork. Even ignoring this composite action, the slenderness of the steel column in lateral buckling is reduced, thereby increasing the compressive stress that can be resisted by the steel section.

Much research has gone into the behaviour of concrete filled tubular sections; architecturally tubular columns have many attractive features; concrete filling has no visual effect on their appeal. The advantages from a structural point of view are, first, the tri-axial confinement of the concrete within the section, and secondly, the fire resistance of the column which largely depends on the residual capacity of the concrete core.

Cased columns could be designed in accordance with BS5950 as either those (McGinley, T.J., and Ang, T.C., 1990) that transmit axial loads or those that transmit both axial compressive loads and bending moments.

Design of axially loaded cased columns is governed by both compression resistance as well as short-strut capacity. In addition, the design of cased columns transmitting both axial compressive loads and bending moments is governed also by moment capacity of designed section, for short struts, as well as the buckling resistance moment for slender members.

They are also used, though less frequently, in multi-storey rigid-frame construction. In each of these cases, it is assumed by definition that no angular rotation takes place, though this is unlikely to be achieved, it is generally accepted that sufficient rigidity can be obtained to justify the assumption.

The resistance of a structural member as well as the loads applied to it is a function of several variables, most of which are random (Melchers, R.E., 1999). Therefore, the use of probabilistic approach in the design

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of structures enables the structural safety to be treated in a more rational manner.

The study of structural reliability is concerned with the calculation and prediction of the probability of limit state violation for engineered structures at any stage during their life. In particular, the study of structural safety is concerned with the violation of the ultimate or safety limit states for the structure (Madsen, H.O., *et al.,* 1986).

The effect of uncertainties in design is included by the use of safety factors that are based on engineering judgment and previous experience with similar structure. Due to the fact that safety involves a consideration of random variables and the realization of the limitations in design by the deterministic method, it is now generally accepted that the rational approach to the analysis of safety is through the use of probabilistic models (Morries, L.J., *et al.,* 1987). Under-estimation of these uncertainties sometimes leads to adverse results of collapse such as those reported by Carino *et al.,* (1983) and Igba (1996). In general, because of uncertainties, the question of safety and performance has arisen (McGinley, T.J., and Ang, T.C., 1990).

Hence it is necessary to evaluate the level of safety implied in the design of cased columns. The BS5950 has been found to be very conservative (Abubakar, I. and Sanusi J., 2006).

The work presented therefore investigates the safety associated with the design criteria of grade 43 steel cased columns subjected to axial loads on one hand, and to combination of axial loads and bending moment on the other.

First Order Reliability Procedure:

Probabilistic design is concerned with the probability that a structure will realize the functions assigned to it. In this work, the reliability method employed is briefly reviewed.

If R is the strength capacity and S the loading effect(s) of a structural system which are random variables, the main objective of reliability analysis of any system or component is to ensure that R is never exceeded by S. In practice, R and S are usually functions of different basic variables. In order to investigate the effect of the variables on the performance of a structural system, a limit state equation in terms of the basic design variable is required. This limit state equation is referred to as the performance or state function and expressed as:

$$g(x_i) = g(x_1, x_2, ..., x_n) = R - S \qquad (1)$$

where $X_{i,}$ for i = 1,2,..., n, represent the basic design variables.

The limit state of the system can then be expressed as

$$g(x_i) = 0. \qquad (2)$$

Graphically, the line $g(x_i) = 0$ represents the failure surface while $g(x_i) > 0$ represents the safe region and $g(x_i) < 0$ corresponds to the failure region. This is shown in Fig. 1. Introducing the set of uncorrelated reduced variates,

$$x'_i = \frac{(X_i - \mu_{x_i})}{\sigma_{x_i}}, i = 1,2,....n \qquad (3)$$

and in terms of these reduced variates the limit state equation becomes:

$$g(s_{xi}X'_1 + \mu_{xi}, s_{x2}X'_2 + \mu_{x2},.., s_{xn}X'_n + \mu_{xn}) = 0, \qquad (4)$$

where μ and s are the means and standard deviations of the design variables. The distance D, from a point $X'_i = (X'_1, X'_2, ..., X'_n)$ on the failure surface $g(x'_i) = 0$ to the origin of X_i space is also given as

$$D = \sqrt{X'^2_1 + X'^2_2 + ... + X'^2_n} \qquad (5)$$

In matrix form:

$$D = (X'_1 X'_2 \cdots X'_n) \begin{vmatrix} X'_1 \\ X'_2 \\ \cdot \\ \cdot \\ X'_n \end{vmatrix} = (X'^t_i X')^{1/2} \qquad (6)$$

The point on the failure surface (X'_1^*, X'_2^*, ..., X'_n^*), having the minimum distance to the origin may be determined by minimizing the function D and subjecting equation (6) to the constraint $g(X_i) = 0$. For this purpose, the method of Langrange's multiplier may be used. Let

$$L = D + \lambda g(X_i) \qquad (7)$$

where D is the minimum distance to the origin of the circle in Fig. 3, l is the value of the Langrange's multiplier and $g(X_i)$ is the limit state function.

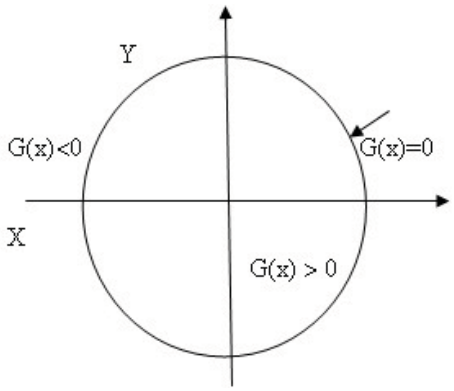

Fig. 3: The most likely failure point[11]

Substituting equation (6) in (7) gives

$$L = (X'^t_i X'_i)^{1/2} + \lambda g(X_i) \qquad (8)$$

In scalar notation,

$$L = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n} + \lambda g(x_1, x_2, \ldots, x_n) \qquad (9)$$

in which $X_i = s_{xi} X'_i + \mu_{xi}$ where μ_{xi} and s_{xi} are the means and standard deviations of the design variables. Minimizing L, we obtain (n+1) equations with (n+1) unknown as

$$\frac{\partial L}{\partial x_i} = \frac{X'_i}{\sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n}} + \ldots + \frac{\lambda \partial g}{\partial X'_i} = 0 \qquad (10)$$

and,

$$\frac{\partial L}{\partial \lambda} = g(X_1, X_2, \ldots, X_n) = 0 \qquad (11)$$

The solution to equations (10) and (11) would yield the most probable failure point (X'_1^*, X'_2^*, ..., X'_n^*). Introducing the gradient vector,

$$G = \frac{\partial g}{\partial X'_1}, \frac{\partial g}{\partial X'_2}, \ldots, \frac{\partial g}{\partial X'_n} \qquad (12)$$

in which

$$\frac{\partial g}{\partial X'_i} = \frac{\partial g}{\partial X_i} \cdot \frac{\partial X_i}{\partial X'_i} = \sigma_{X_i} \frac{\partial g}{\partial X_i} \qquad (13)$$

Therefore, in vector form we have

$$\frac{X'}{(X'^t X')^{1/2}} + \lambda G = 0 \qquad (14)$$

From which

$$X' = \lambda D G \qquad (15)$$

From equation (6)

$$D = [(\lambda D G^t)(\lambda D G)]^{1/2} = \lambda D (G^t G)^{1/2} \qquad (16)$$

and,

$$\lambda = (G^t G)^{-1/2} \qquad (17)$$

Where G^t is the transpose of the gradient vector G. Substituting equation (17) into equation (15) gives

$$X' = \frac{-GD}{(G^t G)^{1/2}} \qquad (18)$$

Multiplying both sides of equation (18) by G^t, the transpose of the gradient vector matrix, we have

$$G^t X' = \frac{-G^t G D}{(G^t G)^{1/2}} = -(G^t G)^{1/2} D \qquad (19)$$

which implies

$$D = \frac{-G^t X'}{(G^t G)^{1/2}} \qquad (20)$$

The minimum distance from the origin describing the variable space to the line representing the failure surface equals b and therefore equation (20) becomes

$$\beta = \frac{-G^* X'^*}{(G^* G^*)^{1/2}} \qquad (21)$$

where G* is the gradient vector at the most probable failure point (X'^*_1, X'^*_2, ..., X'^*_n). It is the value of b which tells us of the safety of any given design under uncertainties in the decision variables.

Performance Functions:

The calculation of the performance functions is performed for discrete combination of basic variables into the following equations considering an axially loaded cased column:

$$G(X) = P_{CS} - F_C, \qquad (22)$$

And,

$$G(X) = P_c - F_c, \qquad (23)$$

and into equations (24) to (25) considering a cased column subjected to both applied axial load and bending moment:

$$G(X) = P_b S_x - M_{cx} \qquad (24)$$

Also,

$$G(X) = P_y S_X - M_X \qquad (25)$$

Where in equations (22) to (24):

$$F_c = 0.45 f_{cu}(B+150)(D+150) + A_s P_y + 0.87 f_y A_r, \qquad (26)$$

$$P_{CS} = (A_g + {}^{0.25 f_{cu} A_C}/_{P_y}) P_y, \qquad (27)$$

$$P_C = (A_g + {}^{0.45 f_{cu} A_C}/_{P_y}), \qquad (28)$$

And,

$$M_{cx} = (057 + 0.33B + 0.10\beta^2) M_x \qquad (29)$$

In equations (22) to (29), P_{cs} and P_c are short strut and axial carrying capacities of the cased section respectively; F_c and M_X are the applied axial load and moment in the major axis respectively; P_y and P_b are material and bending strengths respectively; S_X is plastic modulus of the steel section; B and D are respectively width and depth of the steel section; f_{cu} is the characteristic strength of concrete; M_{cx} is the moment capacity of the cased section; A_g and A_c are gross areas of steel section and concrete respectively; and β is a factor which depends on end restraint of the columns.

Design is said to be satisfactory if conditions set-out in the code of practice is satisfied by estimating

$P_f = P(G(X) £ 0),$ (30)

for varying values of the relevant design variables in the limit state equation.

The procedure of the FORM in the previous section, which was coded in a FORTRAN module (Gollwizer, S., *et al.,* 1988), was employed for the computation of the reliability indices.

Example of Steel Cased Columns:

A grade 43 steel cased column designed to transmit axial load as well as one that transmits both axial load and moment, were in each case, designed in accordance with the provisions set-out in BS5950 (BS5950 1990).

Design of Cased Columns:

Cased columns transmitting an axial compressive factored load, F_c= 1900kN on a clear height of 4.2m on one hand; and another transmitting an axial load, F_c= 1200kN with an applied moment of 85kNm on a clear height of 9m on the other were in each case designed. A 203x203x52 UC was designed in the case of axial load condition and a 203x203x60 UC was designed in the other. In both cases, characteristic strength of concrete was 20N/mm (Clarke, A.B., and Coverman, S.H., 1987).

Results of Reliability Analysis:

Reliability analyses of the cased columns designed in sections 4 above were achieved by the use of FORM by estimating the reliability levels at varying values of material strength, P_y; characteristic strength of concrete, f_{cu}; breadth of steel section, B; steel column depth, D; bending strength, P_b; plastic modulus, S_X; and major

applied moment, M_x. Safety indices were obtained from the programs, and plots of the safety indices versus the varied design variables were as shown in Figs. 4 to 7, considering axial load; while Figs 8 to 13 when axial load and moment were considered. From the plots it can be observed that:

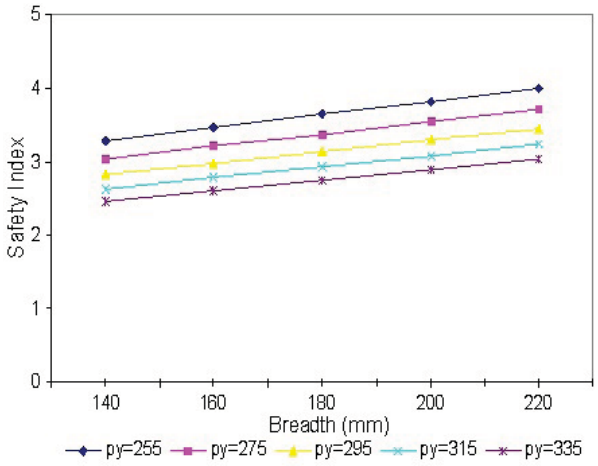

Fig. 4: Safety Index against Breadth (Axially Loaded Condition)

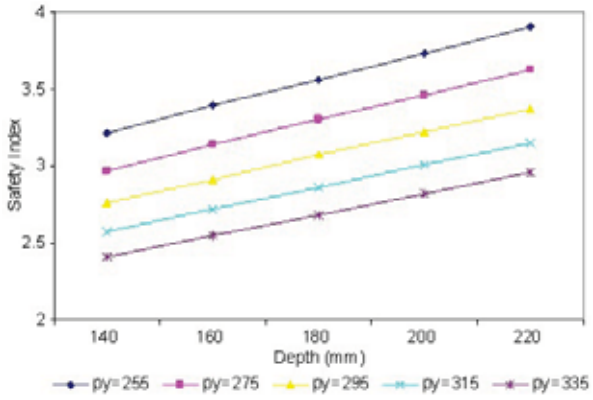

Fig. 5: Safety Index against Depth (Axially Loaded Condition)

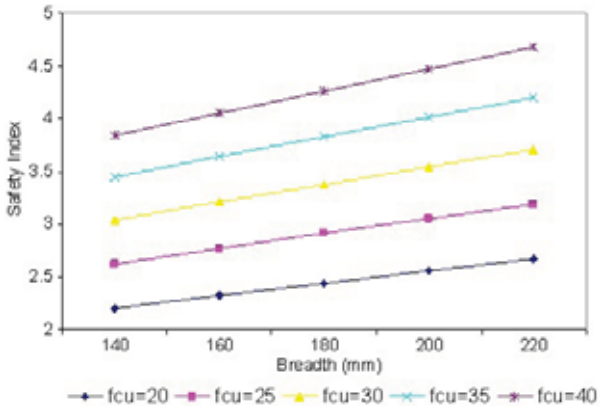

Fig. 6: Safety Index against Breadth (Axially Loaded Condition)

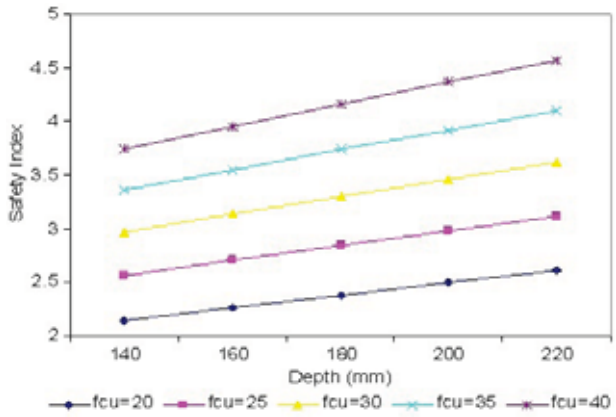

Fig. 7: Safety Index against Depth (Axially Loaded Condition)

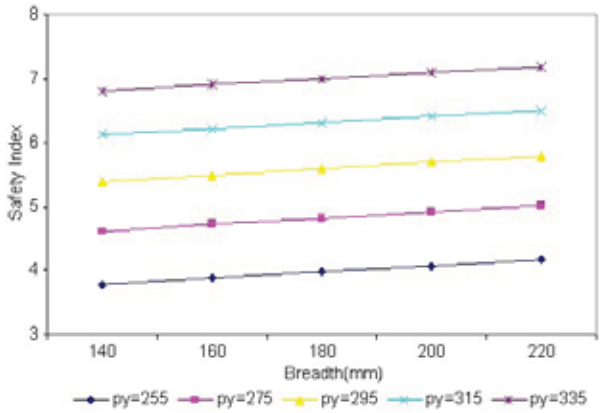

Fig. 8: Safety Index against Breadth (Axially Loaded and Bending)

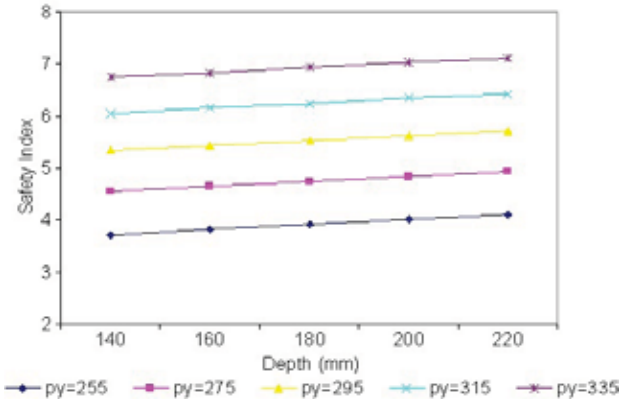

Fig. 9: Safety Index against Breadth (Axially Loaded and Bending)

- Generally, as breadth and depth of section was increased, the safety of the designed section increased.
- Also, as the characteristic concrete strength was increased, the safety of the designed element increased. This was due to fact that the carrying capacity of designed section is a function of the concrete strength[1].

- As the material strength P_y was increased, the safety of the designed section considering axially loaded decreased (Figs. 4 and 5).
- However, considering combined axial load and bending condition, as the material strength was increased, the safety of the section increased (Figs. 8 and 9). This was due to due to fact that capacity of the section in bending depends largely on the bending strength (eqn. 24), than preliminary check using material strength (eqn. 25).
- It was shown from Figs. 12 and 13 that as the bending strength of steel material increased, the safety indices increased.

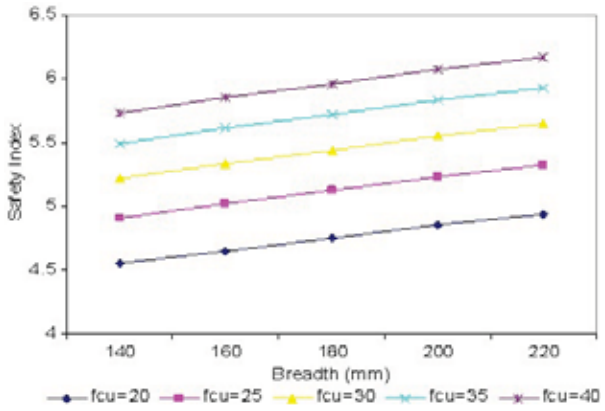

Fig. 10: Safety Index against Breadth (Axially Loaded and Bending)

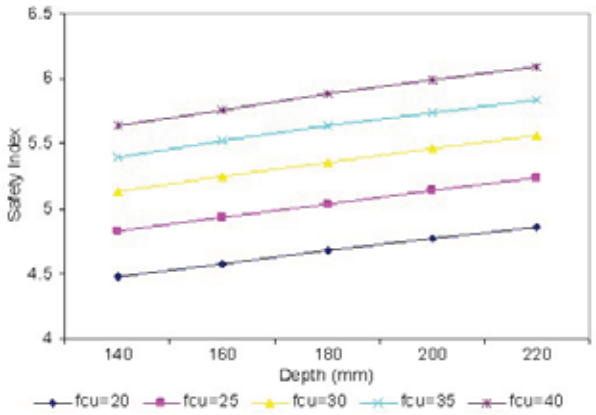

Fig. 11: Safety Index against Depth (Axially Loaded and Bending)

- Also, as the plastic modulus was increased, the safety of the section increased.
- It was shown from Figs. 4 to 13 that the BS5950[1] design criteria of steel cased columns are fairly consistent.
- Considering axially loaded section, at a constant concrete strength the safety indices were constant when either depth or breadth was kept constant (Figs. 6 and 7). However, with the application of bending moment (Figs. 10 and 11), the safety indices considering breadth of column section were higher than those considering depth of the steel section. This was attributed to the fact that the applied bending moment was applied in the direction of the depth (major axis).
- However, at constant yield strength of steel and a constant depth or breadth of UC, the safety indices were the same considering both axially loaded (Figs. 4 and 5) and combined axially loaded with bending (Figs. 8 and 9) respectively.
- Based on (i) and (j) above, it can therefore be deduced that applied bending moment has more significant effect on concrete strength of the cased column than on steel strength.

- Safety indices were more sensitive to increase in applied moment at a constant bending strength, than to increase in sectional plastic modulus than at a constant bending strength (see Figs. 12 and 13). Thus the performance of a cased column to a large extent depends on its ability to withstand applied bending moment whereby the plastic modulus is a function.

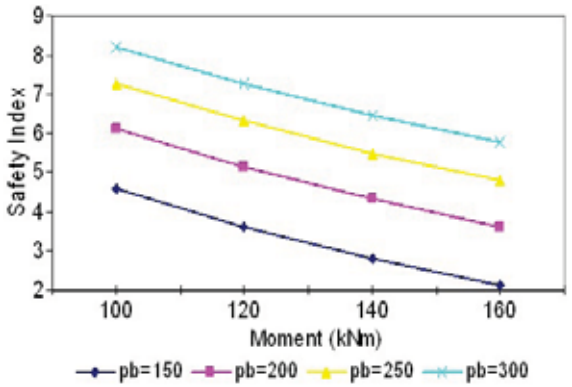

Fig. 12: Safety Index against Applied Moment (Axially Loaded and Bending)

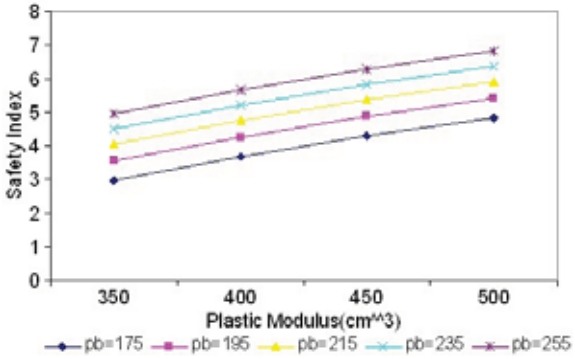

Fig. 13: Safety Index against Plastic Modulus (Axially Loaded and Bending)

Conclusion:

Reliability analysis of steel cased column considering axial load on one hand, and axial load and moment on the other, was investigated using FORM. The results of investigation showed that the BS5950 design procedure of steel cased columns is fairly consistent. It was shown among other findings that, as the material strength P_y was increased, the safety of the designed section considering axially loaded decreased. However, considering combined axial load and bending condition, as the material strength was increased, the safety of the section increased. It was also shown that applied bending moment has more significant effect on concrete strength of the cased column than on steel strength.

REFERENCES

BS5950, 1990. "Structural Use of Steelwork, Part 1." The majesty's Stationery Office, London.

Clarke, A.B. and S.H. Coverman, 1987. "Structural Steelwork, limit State Design." pp: 59-62, J.W. Arrowsmith Ltd, Bristol.

Steel Construction Institute, 1991. "Steelwork Design; Guide to BS5950: Worked Examples". Part 1, Vol. 2, Berkshire.

McGinley, T.J. and T.C. Ang, 1990. "Structural Steelwork: Design to Limit State Theory."

Melchers, R.E., 1999. "Structural Reliability Analysis and Prediction" John Wiley and Sons, New York

Madsen, H.O., T. Addo and N.C. Lind, 1986. "Methods of Structural Safety" Prentice Hall

Morris, L.J. and D.R. Plum, 1987. "Steelwork Design, Volume 1." Section Properties Member Capacities, Steel Construction Institute

Carino, N.J., K.A. Woodword, E.V. Leyendecker and S.G. Fattal, 1983. "A Review of Skyline Plaza Collapse" Journal of Concrete International, pp: 35-41.

Igba, P.O.E., 1996. "Societal Mentality and Engineering Practice in Nigeria" The Nigerian Engineer, 34(2): 24-32

Abubakar, I. and J. Sanusi, 2006. "Reliability Analysis of Crane Girders at Ultimate Limit State" J. Eng. & Appl. Sci., 25(1): 39-45, Peshawar, Pakistan.

Thoft-Christensen, P. and M.J. Baker, 1982. "Structural Reliability Theory and Its Applications" Springer-Verlag, Germany, pp: 82.

Gollwitzer, S., T. Abdo and K. Rackwiz, 1988. "First Order Reliability Method (FORM)" Users Manual, RCP-GMBH, Munich, West Germany.