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Defuzzication through a Bi-Symmetrical Weighted Function

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Abstract: In this endeavor, the researchers discuss the problem of defuzzification by minimizing the weighted distance between two fuzzy quantities. Also, this study obtains the nearest point with respect to a fuzzy number and shows that this point is unique relative to the weighted distance. By utilizing this point, a method is presented for effectively ranking various fuzzy numbers and their images to overcome the deficiencies of the previous techniques. Finally, several numerical examples following the procedure indicate the ranking results to be valid.

Key words: Fuzzy number; Defuzzication; Ranking; Weighted distance; Nearest Weighted point.

INTRODUCTION

Representing fuzzy numbers by proper intervals is an interesting and important problem. An interval representation of a fuzzy number may have many useful applications. By using such a representation, it is possible to apply in fuzzy number approaches some results derived in the eld of interval number analysis. For example, it may be applied to a comparison of fuzzy numbers by using the order relations dened on the set of interval numbers. Various authors in (Saneifard, 2009; Grzegorzewski, 2002) have studied the crisp approximation of fuzzy sets. They proposed a rough theoretic denition of that crisp approximation, called the nearest interval approximation of a fuzzy set. Moreover, quite dierent approach to crisp approximation of fuzzy sets was applied in (Chakrabarty *et al.,* 1998). They proposed a rough theoretic denition of that crisp approximation, called the nearest ordinary set of a fuzzy set, and they suggested a construction of such a set. They discussed rather discrete fuzzy sets. Their approximation of the given fuzzy set not unique. Thus this article will not discuss this method.

Having reviewed the previous interval approximations, this article proposes here a method to nd the weighted interval approximation of a fuzzy number, that it is fullls two conditions. In the rst, this interval is a continuous interval approximation operator. In the second, the parametric distance between this interval and the approximated number is minimal. In continuance, by using this interval we obtain the nearest weighted point approximation respect to weighted distance and show that this point is unique. The main purpose of this article is that this nearest weighted point can be used as a crisp approximation of a fuzzy number, therefore by means of this approximation this article aims to present a new method for ranking of fuzzy numbers.

The paper is organized as follows: In Section 2, this article recalls some fundamental results on fuzzy numbers. In Section 3, a crisp set approximation of a fuzzy number is obtained. In this Section some remarks are proposed and illustrated. In Section 4, a crisp approximation of a fuzzy number is obtained. The proposed method for ranking fuzzy numbers is in Section 5. Discussion and comparison of this work and other methods are carried out in Section 6.

Preliminaries:

The basic denition of a fuzzy number given in (Dubois and Prade, 1987; Heilpern, 1992; Kauffman, 1991) as follows:

Let be the set of all real numbers. The researchers assume a fuzzy number u that can be expressed for all $x$in the form

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$$u(x)=\begin{cases} g(x) & when\, x\in[a,b), \\ 1 & when\, x\in[b,c], \\ h(x) & when\, x\in(c,d], \\ 0 & otherwise. \end{cases} \tag{2.1}$$

Where *abcd* are real numbers such as $a < b\ c < d$ and *g* is a real valued function that is increasing and right continuous and *h* is a real valued function that is decreasing and left continuous.

Denition 2.1.:

A fuzzy number u in parametric form is a pair $(\underline{u},\overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$ that $0 \le r \le 1$, which satisfy the following requirements:

1. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,
2. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,
3. $\underline{u}(r)\le\overline{u}(r), 0\le r\le 1.$

Denition 2.2.:

The trapezoidal fuzzy number u = *(x*0, *y*0, *σ, β*), with two defuzzier x0*, y*0, and left fuzziness *β*> 0 and right fuzziness > 0 is a fuzzy set where the membership function is as1

$$u(x)=\begin{cases} \dfrac{1}{\sigma}(x-x_0+\sigma) & x_0-\sigma\le x\le x_0, \\ 1 & x_0\le x\le y_0, \\ \dfrac{1}{\beta}(y_0-x+\beta) & y_0\le x\le y_0+\beta, \\ 0 & otherwise. \end{cases}$$

*If x*0 = *y*0*, then u* = (x_0, *σ, β*) is called trapezoidal fuzzy number. The parametric form of symmetric triangular fuzzy number is $\underline{u}(r)=x_0-\sigma+\sigma r, \overline{u}(r)=x_0+\beta-\beta r.$

Denition 2.3.:

[14]. A function *f* : [01]→[01] symmetric around $\frac{1}{2}, i.e. f(\frac{1}{2}-r)=f(\frac{1}{2}+r)$ for all $\in[0,\frac{1}{2}]$ which reaches its minimum in $\frac{1}{2}$ is called the bi-symmetrical weighted function. Moreover, the bi-symmetrical weighted function is called regular if

(1) $f(\frac{1}{2})=0,$

(2) $f(0)=f(1)=1,$

(3) $\int_0^1 f(r)dr=\frac{1}{2}$

Denition 2.4.:

For two arbitrary fuzzy numbers $u=(\underline{u},\overline{u})$ and $v=(\underline{u},\overline{u})$, the quantity

$$d_w(u,v)=\left(\int_0^1 f(r)[\underline{u}(r)-\underline{v}(r)]^2dr+\int_0^1 f(r)[\overline{u}(r)-\overline{v}(r)]^2dr\right)^{\frac{1}{2}} \tag{2.2}$$

were *f* : [01]→[01] is a bi-symmetrical (regular) weighted function is called the bisymmetrical (regular) weighted distance between *u* and *v* based on *f*.

One can, of course, propose many regular bi-symmetrical weighted functions and hence obtain dierent bi-symmetrical weighted distances. Further on we will consider mainly a following function

$$f(r)=\begin{cases}1-2r & when\ r\in[0,\frac{1}{2}),\\ 2r-1 & when\ r\in[\frac{1}{2},1],\end{cases} \tag{2.3}$$

Denition 2.5.:

(Grzegorzewski, 2002). An operator $I : F\rightarrow$ (*set of closed intervals in* $\Re$) is called an interval approximation operator if for any $u \in F$

$(a')\ I(u)\subseteq \sup pu,$

$(b')\ coreu \subseteq I(u),$

$(c')\ \forall(\varepsilon>0)\exists(\delta>0)\ \ S.t\ \ d(u,v)<\delta \Rightarrow d(I(u),I(v))<\varepsilon.$

Where d: F→ [0+∞) denotes a metric dened in the family of all fuzzy numbers.

Denition 2.6.:

(Grzegorzewski, 2002). An interval approximation operator satisfying in condition (c′) for any $u,v \in F$ is called the continuous interval approximation operator.

Nearest Weighted Interval of a Fuzzy Number:

Various authors in (Chakrabarty, *et al.,* 1998) and (Grzegorzewski, 2002) have studied the crisp approximation of fuzzy sets. They proposed a rough theoretic denition of that crisp approximation, called the nearest ordinary set and nearest interval approximation of a fuzzy set. In this section, the researchers will propose another approximation called the weighted interval-

value approximation. Let $u=(\underline{u},\overline{u})$ be an arbitrary fuzzy number. This article will try to nd a closed interval

$Id_w(u) = [I_L I_R]$, which is the weighted interval to u with respect to metric d_w. So, this article has to minimize

$$d_w(u,I_{dw}(u))=\left(\int_0^1 f(r)((\underline{u}(r)-I_L)^2+(\overline{u}(r)-I_R)^2)\right)^{\frac{1}{2}} \tag{3.4}$$

with respect to I_L and I_R. In order to minimize d_w it suces to minimize

$$\overline{D}_w(I_L,L_R)=d_w^2(I_L,I_R).$$

It is clear that, the parameters I_L and I_R which minimize Eq. (3.4) must satisfy in

$\nabla\overline{D}_w(I_L,I_R)=(\frac{\partial\overline{D}_w}{\partial I_L},\frac{\partial\overline{D}_w}{\partial I_R})=0$ Therefore, this article has the following equations:

$$\begin{cases}\frac{\partial\overline{D}_w(I_L,I_R)}{\partial I_L}=-2\int_0^1 f(r)(\underline{u}(r)-I_L)dr=0,\\ \frac{\partial\overline{D}_w(I_L,I_R)}{\partial I_R}=-2\int_0^1 f(r)(\overline{u}(r)-I_R)dr=0.\end{cases} \tag{3.5}$$

The parameters I_L associated with the left bound and I_R associated with the right bound of the nearest weighted interval can be found by using Eq. (3.5) as follows:

$$\begin{cases}I_L=2\int_0^1 f(r)\underline{u}(r)dr,\\ I_R=2\int_0^1 f(r)\overline{u}(r)dr.\end{cases} \tag{3.6}$$

Remark 3.1.:

$$Since, \det\begin{bmatrix} \frac{\partial D_w^2(I_L,I_R)}{\partial I_L^2} & \frac{\partial D_w^2(I_L,I_R)}{\partial I_R \partial I_L} \\ \frac{\partial D_w^2(I_L,I_R)}{\partial I_L \partial I_R} & \frac{\partial D_w^2(I_L,I_R)}{\partial I_R^2} \end{bmatrix} = \det\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} => 0, and \frac{\partial D_w^2(I_L,I_R)}{\partial I_L^2} = 1 > 0,$$ therefore I_L and I_R given by (3.6)

minimize $d_w(u; I_{dw}(u))$. Therefore, the interval

$$I_{dw}(u) = \left[2\int_0^1 f(r)\underline{u}(r)dr, 2\int_0^1 f(r)\overline{u}(r)dr \right], \tag{3.7}$$

is the weighted nearest interval approximation of fuzzy number u with respect to d_w.

Now, suppose that this article wants to approximate a fuzzy number by a crisp interval. Thus the researchers have to use an operator $I_{dw} : \mathbb{F} \to (set\ of\ closed \text{ int } ervals\ in\ \square)$ which transforms fuzzy numbers into family of closed intervals on the real line.

Lemma 3.1.:

((Grzegorzewski, 2009))[14] $\left(2\int_0^1 f(r)g(r)dr\right)^2 \le 2\int_0^1 f(r)g^2(r)dr.$

Theorem 3.1.:

The operator $I_{dw} : \mathbb{F} \to (set\ of\ closed \text{ int } ervals\ in\ \square)$ *is an interval approximation operator, i.e* I_{dw} *is a continuous interval approximation operator.*

Proof.:

It is easy to verify that the conditions *(a')* and *(b')* are hold. For the proof of *(c'),* let *u* and *v* be two fuzzy numbers, then

$$d_w^2(I_{dw}(u), I_{dw}(v)) = \int_0^1 f(r)(\underline{u}(r) - \underline{v}(r))^2 dr + \int_0^1 f(r)(\underline{u}(r) - \underline{v}(r))^2 dr$$

$$= \frac{1}{2}(\underline{v}(r))^2 + \frac{1}{2}(\overline{u}(r) - \overline{v}(r))^2$$

$$= 2\left(\int_0^1 f(r)(\underline{u}(r) - \underline{v}(r))dr\right)^2 + 2\left(\int_0^1 f(r)(\overline{u}(r) - \overline{v}(r))dr\right)^2$$

Via to Lemma (3.1), there is

$$\le 2\int_0^1 f(r)(\overline{u}(r) - \overline{v}(r))^2 dr + 2\int_0^1 f(r)(\underline{u}(r) - \underline{v}(r))^2 dr$$

$$= 2\int_0^1 f(r)[(\overline{u}(r) - \overline{v}(r))^2 + (\underline{u}(r) - \underline{v}(r))^2]dr = 2d_w^2(u,v).$$

It means that $\forall \varepsilon > 0, \exists \delta = \frac{\sqrt{2\varepsilon}}{2} > 0, when\ d_w(u,v) < \delta,$ then we have $d_w(I_{dw})$ (u), I_{dw} (v)<ε. It shows that our weighted interval approximation is continuous interval approximation.

Nearest Weighted Point of a Fuzzy Number:

Let $u = (\underline{u}, \overline{u})$ be an arbitrary fuzzy number and $I_{dw}(u) = [I_L, I_R]$ be the nearest weighted interval of it. $I_L = I_R, then\ I_{dw}(u) = [I_L, I_R] = I_w(u)$ is the nearest weighted point approximation to fuzzy number *u,* and its unique. $I_w(u)$ value is as follows:

$$I_w(u) = \int_0^1 f(r)(\underline{u}(r) + \overline{u}(r))dr$$

The above equation introduces in the following Theorem.

Theorem 4.1.:

Let $u=(\underline{u},\overline{u})$ be a fuzzy number and *f*(*r*) be a weighted function. Then $I_w(u)$ is nearest weighted point to fuzzy number *u*.

Proof.:

For the proof of Theorem it suces that we replace $I_L = I_R = I_w(u)$ in (3.4) and then minimize function $\overline{D}_w(u,I_w(u)) = d_w^2(u,I_w(u))$ with respect $I_w(u)$. Thus this article has to minimize

$$\overline{D}_w(u,I_w(u)) = \int_0^1 f(r)((\underline{u}(r)-I_w(u))^2+(\overline{u}(r)-I_w(u))^2)dr, \tag{4.8}$$

with respect to $I_w(u)$. It is clear that, the parameter $I_w(u)$ which minimizes Eq. (4.8)

must satisfy in $\nabla\overline{D}_w(u,I_w(u)) = \frac{\partial\overline{D}_w}{\partial I_w(u)} = 0$ Therefore, this article has:

$$\frac{\partial\overline{D}_w(u,I_w(u))}{\partial I_L} = -2\int_0^1 f(r)[(\underline{u}(r)-I_w(u))+(\overline{u}(r)-I_w(u))]dr = 0, \tag{4.9}$$

the solution is

$$I_w(u) = \int_0^1 f(r)(\underline{u}(r)+\overline{u}(r))dr. \tag{4.10}$$

$Since\frac{\partial D_w^2(u,I_w(u))}{2} = 2 > 0,$ therefore $I_w(u)$ actually minimize $\overline{D}_w(u,I_w(u))$ and simultaneously minimize $d_w(u,I_w(u))$.

Theorem 4.2.:

The nearest weighted point approximation to a given fuzzy number u is unique.

Proof.:

To prove the uniqueness of the operator $I_w(u)$, we show that for any C ε $\mathbb{R}$ $\overline{D}_w(u,I_w(u)) \le \overline{D}_w(u,C)$ holds. We can write

$$\overline{D}_w(u,C) = \int_0^1 f(r)((\underline{u}(r)-C)^2+(\overline{u}(r)-C)^2\,dr$$

$$= \int_0^1 f(r)((\underline{u}(r)+I_w(u)-I_w(u)-C)^2+(\overline{u}(r)+I_w(u)-I_w(u)-C)^2)dr$$

$$= \int_0^1 f(r)((\underline{u}(r)-I_w(u))^2+(\overline{u}(r)-I_w(u))^2)dr+\int_0^1 f(r)((I_w(u)-C)^2+(I_w(u)-C)^2)dr$$

$$+2(I_w(u)-C)\int_0^1 f(r)[(\underline{u}(r)-I_w(u))+(\overline{u}(r)-I_w(u))]dr.$$

The last sentence of the above diction is zero, hence

$$\overline{D}_w(u,C) = \overline{D}_w(u,I_w(u))+((I_w(u)-C)^2$$

Consequently $\overline{D}_w(u,C)-\overline{D}_w(u,I_w(u)) = ((I_w(u)-C)^2 \ge 0,$ then we have $\overline{D}_w(u,C) \ge \overline{D}_w(u,I_w(u))$which completes the proof of theorem.

Remark 4.1.:

Let *u* and *v* be two fuzzy numbers and λ and μ be positive numbers. Then we have

$$I_w(\lambda u \pm \mu v) = \lambda I_w(u) \pm \mu I_w(v).$$

Proof.:

Let $u=(\underline{u},(r),\overline{u}(r))\ and\ v=(\underline{v}(r),\overline{v}(r))\ \ for\ all\ 0\le r\le and\ f(r)$is weighted function. Then,

$$\lambda u\pm\mu v=\lambda(\underline{u}(r),\overline{u}(r))\pm\mu(\underline{v}(r),\overline{v}(r))=[\lambda\underline{u}(r)\pm\mu\underline{v}(r),\pm\mu\overline{v}(r)].$$

Then

$$I_w(\lambda u\pm\mu v)=\int_0^1 f(r)[\lambda\underline{u}(r)\pm\mu\underline{v}(r)+\lambda\overline{u}(r)\pm\mu\overline{v}(r)]dr$$

$$=\lambda\int_0^1 f(r)[\underline{u}(r)+\overline{u}(r)]dr\pm\mu\int_0^1 f(r)[\underline{v}(r)+\overline{v}(r)dr=\lambda I_w(u)\pm\mu I_w(v).$$

Remark 4.2.:

If$u=(x_0,y_0,\sigma,\beta)$ be *a* trapezoidal fuzzy number, then the nearest weighted point to it is

$$I_w(u)=\frac{x_0+y_0+\beta-\sigma}{2}+\int_0^1 f(r)rdr.$$

Ordering of Fuzzy Numbers by the Nearest Weighted Point:

In this section, the researchers will propose the ranking of fuzzy numbers associated with the nearest weighted point approximation. Ever, the nearest weighted point can be used as a crisp approximation of a fuzzy number, therefore the resulting approximation is used to rank the fuzzy numbers. Thus, $I_w(.)$ is used to rank fuzzy numbers.

Denition:

Let u and $v\ \varepsilon\ F$ be two fuzzy numbers, and Iw(u) and $I_w(v)$ be the nearest weighted point of their. Dene the ranking of u and v by I_w on F, *i.e.*

$I_w(u) < I_w(v)$ *if only if* $u \prec v$,
$I_w(u) > I_w(v)$ *if only if* $u \prec v$,
$I_w(u) = I_w(v)$ *if only if* $u \prec v$.

Then, this article formulates the order $\succeq$ and $\preceq$ as $u \succeq v$ if and only if $u \succ v$ or $u \sim v$, $u \preceq v$ if and only if $u \prec v$ or $u \sim v$.

This article considers the following reasonable axioms that Wang and Kerre (2001) proposed for fuzzy quantities ranking.

Let I be an ordering method, S the set of fuzzy quantities for which the method I can be applied, and A a nite subset of S. The statement two elements u and v in A satisfy that u has a higher ranking than v when I is applied to the fuzzy quantities in Awill be written as "$u \succ v$ by I on A ". "$u \sim v$ by I on A ", and "$u \succeq v$ by I on $A$are similarly interpreted. The following proposition shows the reasonable properties of the ordering approach, I.

Let S be the set of fuzzy quantities for which the nearest point method can be applied, and A and $A$are two arbitrary nite subsets of S. The following axioms hold.

$\mathbf{A_1}$ For $u\ \varepsilon\ A$, $u \preceq u$by I on$A$.
$\mathbf{A_2}$ For $(u, v)\ \varepsilon\ A^2$, $u \preceq v$ and $v \ u$ by I on A, we should have $u \sim v$ by I on A.
$\mathbf{A_3}$ For $(u,v,w)\ \varepsilon\ A^3$, $u \preceq v$ and $v \preceq w$ by I on A, we should have $u \preceq w$ by I on A.
$\mathbf{A_4}$ For $(u, v)\ \varepsilon\ A^2$, *inf supp(v)>sup supp(u),* we should have $u \preceq v$ by I on A.
$\mathbf{A_4}$ For $(u, v)\ \varepsilon\ A^2$, *inf supp(v)>sup supp(u),* we should have $u \preceq v$ by I on A.
$\mathbf{A_5}$ Let $(u, v)\ \varepsilon\ (A \cap A)^2$. We obtain the ranking order $u \preceq v$ by I on A' if and only if $u \preceq v$ by I on A.
$\mathbf{A_6}$ Let u, v, $u+w$ and $v+w$ be elements of S. If $u \preceq v$ by Ion $\{u,v\}$, then $u+w \preceq v+w$ by Ion $\{u+w,v+w\}$.
$\mathbf{A_6}$ Let u, v, $u+w$ and $v+w$ be elements of S. If $u \preceq v$ by Ion $\{u,v\}$, then $u+w \preceq v+w$ by Ion $\{u+wv+w\}$.
$\mathbf{A_7}$ Let u, v, uw and vw be elements of S and $w \ge 0$. If $u \preceq v$ by I on $\{u,v\}$, then $uw \preceq vw$ by I on $\{uw, vw\}$.

Remark 5.1.:

Ranking order Iw has the axioms $\mathbf{A}_1$ $\boldsymbol{A}_2$,..., $\boldsymbol{A}_7$

Remark 5.2.:

If $A \preceq B$, then - $A \succeq -B$.

Hence, this article can infer ranking order of the images of the fuzzy numbers.

Examples:

In this section, we want compare proposed method with others in (Abbasbandy, 2006; 2009; Asady and Zendehnam, 2007; Choobineh and Li, 1993; Saneifarty, 2009; 2007;)Cheng, 1999; Chu and lee-Kwang, 1994; Baldwin and Guild, 1979; Chu and Tsao, 2002; Chen, 1985).

Example 6.1.:

Consider the data used in (Abbasbandy and Asady, 2006), *i.e*. the three fuzzy numbers, A = (611), *B* = (6011), *C* = (601), as shown in Fig. (6.1).

According to Eq. (4.10), the ranking index values are obtained i.e. I_w(A) = 6, $I_w(B)$ = 615 and $I_w(C)$ = 616. Accordingly, the ranking order of fuzzy numbers is $C \succ B \succ A$. However, by Chu and Tsaos approach (Chu and Tsao, 2002), the ranking order is $B \succ C \succ A$. Meanwhile, using CV index proposed (Cheng, 1999), the ranking order is $A \succ B \succ C$. From Fig. (6.1), it easy to see that the ranking results obtained by the existing approaches (Cheng, 1999; Chu and Tsao, 2002) are unreasonable and are not consistent with human intuition. On the other hand, in (Abbasbandy and Asady, 2006), the ranking result is $C \succ B \succ A$, which is the same as the one obtained by our approach. However, our approach is simpler in the computation procedure. Based on the analysis results from (Abbasbandy and Asady, 2006), the ranking results using our approach and other approaches are listed in Table (6.1).

Table 6.1: Comparative results of Example (6.1)

fuzzy number	new approach	Sign Distance p=1	Sign Distance p=2	Chu- Tsao	Cheng Distance	CV index
A	6.00	6.12	8.52	3	6.021	0.028
B	6.15	12.45	8.82	3.126	6.349	0.0098
C	6.16	12.5	8.85	3.085	6.3519	0/0089
Results	$C \succ B \succ A$	$C \succ B \succ A$	$C \succ B \succ A$	$B \succ C \succ A$	$C \succ B \succ A$	$A \succ B \succ C$

Example 6.2.:

Consider the following sets: A = (213), *B* = (331) *and C* = (250505), (see Fig. 6.2).

By using this new approach, $I_w(A)$ = 233, $I_w(B)$ = 266 *and* $I_w(C)$ = 25. Hence, the ranking order is $A \prec C \prec B$ too. It looks that, the result obtained by Distance Minimizationmethod is unreasonable. To compare with some of the other methods in (Chu and Tsao, 2002), the readers can refer to Table (6.2). Furthermore, to aforesaid example $I_w(A)$ = 233, $I_w(B)$ = 266 and $I_w(C)$ = 25, consequently the ranking order of the images of three fuzzy number is $B \prec C \prec A$. Clearly, this proposed method has consistency in ranking fuzzy numbers and their images, which could not be guaranteed by C V-index method. Through Fig. (6.2), it easy to see that neither of them is consistent with human intuition.

Table 6.2: Comparative results of Example (6.2)

Fns	new approach	Sign distance p=2	Distance Minimization	Chu and Tsao (Revisited)	CV index	Magnitude method
A	2.33	3.9157	2.5	0.74	0.32	2.16
B	2.66	3.9157	2.5	0.74	0.36	2.83
C	2.50	3.5590	2.5	0.75	0.08	2.5
Results	$A \prec C \prec B$	$C \sim A \sim B$	$C \sim A \sim B$	$A \sim B \sim C$	$B \prec A \prec C$	$A \prec C \prec B$

Example 6.3.:

Consider the following sets, see Yao and Wu (2000).

Set1: A=(0.5,0.1,0.5), B=(0.7,0.3,0.3), C=(0.9,0.5,0.1).

Set2: A=(0.4, 0.7,0.1,0.2) (trapezoidal fuzzy number), B=(0. 7,0.4,0.2), C=(0. 7,0.2,0.2).

Set3: A =(0. 5,0.2,0.2), B =(0. 5,0.8,0.2,0.1) (trapezoidal fuzzy number), C =(0. 5,0.2,0.4).

Set4: A=(0.4, 0.7,0.4,0.1) (trapezoidal fuzzy number), B =(0. 5,0.3,0.4), C=(0. 6,0.5,0.2).

To compare with other methods authors refer the reader to Table (6.3).

Table 6.3: Comparative results of example (6.3).

Authors	Fuzzy number	Set1	Set2	Set3	Set4
	A	0.566	0.5666	0.500	0.500
Proposed method	*B*	0.700	0.666	0.633	0.516
	C	0.830	0.7	0.530	0.550
Results		*A ≺B ≺C*	*A ≺B ≺C*	*A ≺C ≺B*	*A ≺B ≺C*
Sing Distance method	*A*	1.2000	1.1500	1.0000	0.0950
with p=1	*B*	1.4000	1.3000	1.2500	1.0500
	C	1.6000	1.4000	1.1000	1.0500
Results		*A ≺B ≺C*	*A ≺B ≺C*	*ACB*	*A ≺B ∼C*
Sing Distance method	*A*	0.8869	0.8756	0.7257	0.7853
with p=2	*B*	1.0194	0.9522	0.9416	0.7958
	C	1.1605	1.0033	0.8165	0.8386
Results		*A ≺B ≺C*	*A ≺B ≺C*	*ACB*	*A ≺B ≺C*
Distance Minimization	*A*	0.6	0.575	0.5	0.475
	B	0.7	0.65	0.625	0.525
	C	0.9	0.7	0.55	0.525
Result		*A ≺B ≺C*	*A ≺B ≺C*	ACB	A ≺B ≺C
Abbasbandy and Hajjari	*A*	0.5334	0.5584	0.5000	0.5250
(Magnitude method)	*B*	0.7000	0.6334	0.6416	0.5084
	C	0.8666	0.7000	0.5166	0.5750
Result		*A ≺B ≺C*	*A ≺B ≺C*	*A ≺C ≺B*	*B ≺A ≺C*
Chen	*A*	0.3375	0.4315	0.3750	0.5200
	B	0.5000	0.5625	0.4250	0.5700
	C	0.6670	0.6250	0.5500	0.6250
Results		*A ≺B ≺C*	*A ≺B ≺C*	A ≺B ≺C	A ≺B ≺C
Chu and Tsao	*A*	0.2990	0.2847	0.2500	0.2440
	B	0.3500	0.3247	0.3152	0.2624
	C	0.3993	0.3500	0.2747	0.2619
Results		*A ≺B ≺C*	*A ≺B ≺C*	A ≺C ≺B	A ≺C ≺B
Yao and Wu	*A*	0.6000	0.5750	0.5000	0.4750
	B	0.7000	0.6500	0.6250	0.5250
	C	0.8000	0.7000	0.5500	0.5250
Results		*A ≺B ≺C*	*ABC*	*ACB*	*A ≺B ∼C*
Cheng distance	*A*	0.7900	0.7577	0.7071	0.7106
	B	0.8602	0.8149	0.8037	0.7256
	C	0.9268	0.8602	0.7458	0.7241
Results		*A ≺B ≺C*	*ABC*	*ACB*	*A ≺C ≺B*
Cheng CV uniform	*A*	0.0272	0.328	0.0133	0.0693
distribution	*B*	0.0214	0.0246	0.0304	0.0385
	C	0.0225	0.0095	0.2750	0.0433
Results		*A ≺C ≺B*	*A ≺B ≺C*	*B ≺C ≺A*	*A ≺C ≺B*
Cheng CV proportional	*A*	0.1830	0.0260	0.0080	0.0471
distribution	*B*	0.0128	0.0146	0.0234	0.0236
	C	0.0137	0.0057	0.0173	0.0255
Results		*A ≺C ≺B*	*A ≺B ≺C*	*B ≺C ≺A*	*A ≺C ≺B*

All the above examples show that this method is more consistent with institution than the previous ranking methods.

Conclusion:

In this paper, the researchers proposed a defuzzication using minimizer of the weighted distance between two fuzzy number and by using this defuzzication we proposed a method for ranking of fuzzy numbers . Roughly, there not much dierence in our method and theirs. The modied method can eectively rank various fuzzy numbers and their images.

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