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A New Ordering Policy for the Fixed Lifetime Inventory System

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ABSTRACT: The fixed-lifetime inventory system is one of the most critical aspects of operations management. This is because of the effect of outdating on products when not used to meet demand during their useful lifetime in inventory. Over the years, fixed lifetime inventory models have based the decision to reorder on the quantity of products on hand rather than the age of items on hand at the point of reordering. In this work, we propose a new fixed lifetime inventory model where the decision to reorder is based on the number of useful life remaining on the products rather than just the quantity of products on hand. In addition, the model assumes lost sales.

Keywords: Fixed lifetime, outdates, useful lifetime, inventory, lost sales, reorder

INTRODUCTION

The fixed lifetime inventory system consists of products with an expiration date. Fixed lifetime products have deterministic shelf life i.e. if a product remains unused by the end of its useful lifetime in inventory, it is considered outdated. Outdating refers to sudden death in value or usability of a product over time and outdated products must be discarded. However, all units which have not expired have constant utility, Omosigho (2002). Over the years, fixed lifetime inventory models have decided to reorder on the quantity of products on hand rather than the age of items at the point of reordering.

Authors in the literature on ordering policies based on the quantity of products on hand include; Chiu (1994), Bookbinder and Cakanyildirim (1999), Mohammad et al. (2007), Hariga (2010) and Siriruk (2012), all of whom considered the ordering policy (Q,r) which order Q whenever inventory on hand drops to the reorder point r. Nahmias (1978), Hollier et al. (1995), Liu and Lian (1999) and Silver et al. (2012) considered the ordering policy (s,S), which orders up to S when inventory on hand drops to the reorder point s. Schmidt and Nahmias (1985), Perry and Posner (1998), Kranenburg and Houtum (2007) and Olsson and Tydesjo (2010) considered the ordering policy $(S-1,S)$ in which an order is placed for precisely one item each time inventory is depleted by either demand or outdating. Nahmais and Pierskalla (1973) considered the ordering policy (x,y) where y the ordered quantity and x the sum of items (items with different age categories) on hand must be less than a certain quantity. Shen et al. (2012) considered an ordering policy that maintains a minimum volume of inventory; whenever inventory drops to this level, a new order is placed.
As a deviation from these ordering policies, we propose a new fixed lifetime inventory model where the decision to reorder is based on the number of useful lifetime remaining on the items on hand rather than just the quantity of items on hand. The ordering

policy of our model is based on the number of useful lifetime remaining on the items on hand when placing new orders and not the quantity. Our policy is $(y, m-1)$ interpreted as order y when the useful lifetime remaining on the items on hand is one period. One advantage of our ordering policy over existing policies is that the policy is not fixed. If the demand is high, the inventory manager can decide to reorder new items with two periods remaining on the items on hand instead of one period. This is very common during festive periods when sales are high. If the demand drops, the inventory manager can reverse back to placing new orders with one useful period remaining on the items. If new orders are placed with two periods remaining, our ordering policy will be $(y, m-2)$ ordered y when the useful lifetime remaining on the items on hand is two periods.

The other sections of this work are as follows; section2 presents the assumptions and notation of the model, section3 gives a description of the model, section4 derivation of costs components, section5 the difference between our model and existing models, section6 shows the convex property of our total cost function, section7 illustrating our model on two fixed lifetime products, bread and egg. Bread has a fixed lifetime of 4days and egg has a fixed lifetime of 5weeks; section 8 data analysis, section9 compares results from our model with the result from existing models, section10 research observation and recommendations and section11 conclusion.

Assumptions and Notation of the model

Assumptions

(1) New product orders are based on the remaining useful lifetime of the items on hand. Therefore, we place an order for new items when the useful lifetime remaining on the items on hand is one period. One period in particular to the model presented in this work.
(2) Only one order is placed at a time. There are no outstanding orders.
(3) The new order arrives instantly whenever orders are placed.
(4) Order received is used to satisfy demand in period 1,2, . . . m .
(5) The fixed lifetime of the product is a positive integer m
(6) A complete cycle consists of two consecutive orders received. One at the beginning of the revolution and the other at the end of the cycle. The time in between is the cycle length.
(7) Shortage occurs whenever the on-hand inventory cannot satisfy all the demand in a period. Only a part of the demand is satisfied while the other part is lost. That is a lost sale is assumed and a shortage cost is charged against the inventory manager.
(8) Item(s) not used to meet demand by the end of period m , outdated and are discarded. An outdated cost is charged against the inventory manager.
(9) The issuing policy is FIFO. That is oldest units must be used to meet demand before the new ones are used.
(10) All units of the new order are of the same age and arrive in the inventory at age zero.
(11) Demand in each period is not known but assumed to be independent and identically distributed random variables $d_1, d_2, \ . \ . \ .$ with known distribution f. The demand $t = \sum_{i=1}^{m} d_i$ has a density f^* which is the n-fold convolution of f with itself.

Notation.

m = lifetime of the product. m is a positive integer

$d_i = demand \ \ in \ \ period \ \ i$

$t = \sum_{i=1}^{m} d_i, \ total \ \ demand \ \ with \ \ distribution \ \ f^*(t)$

x represent the quantity of products with one useful period remaining in them. That is $x = y - \sum_{i=1}^{m-1} d_i$

y = new products ordered/entering into inventory with age zero

$T_i = period \ \ i \ \ (periods \ \ can \ \ be \ \ in \ \ hours, days, weeks \ or \ \ years \ \ depending \ \ on \ the \ \ product)$

θ = outdate cost per unit
v = shortage cost per unit
h = holding cost per unit
k = fixed ordering cost per unit.

Model description.

The model involves a single fixed-lifetime product in a single location. New orders arrive with one useful lifetime remaining on the items on hand. The total cost function for the model consists of the following cost components:

(1) Ordering cost: cost of placing new orders.
(2) Holding cost: There is a holding cost per unit for every amount of products held in inventory.
(3) Shortage cost: for every demand that cannot be satisfied from the stock on hand, a shortage cost is charged against the inventory manager.
(4) Outdate cost: for every product not used to satisfy/meet demand at the end of its useful lifetime in inventory, an outdate cost is charged against the inventory manager.

Table 1 shows the model for a product with m a useful lifetime.

Table 1: Orders and Age distribution for a product with m lifetime.

Order	T_1	T_2	T_3	. . .	T_{m-1}	T_m	T_{m+1}	T_{m+2}
1	$(y-d_1)^+$ $age\ 1$	$(y-\sum_{i=1}^{2} d_i)^+$ $age2$	$(y-\sum_{i=1}^{3} d_i)^+$ $age3$	. . .	$(y-\sum_{i=1}^{m-1} d_i)^+$ $age\ m-1$	$x-d_m$ $age\ m$		
2					$(y-d_1)^+$ $age\ 1$	$(y-\sum_{i=1}^{2} d_i)^+$ $age\ 2$	....	$(y-\sum_{i=1}^{m-1} d_i)^+$ $age\ m-1$
3								y $age\ 0$

Table1 T_i shows the period i. The first order y arrives at the start of the period1 at age zero. At the end of period 1, it reduces by the demand in period 1 and becomes $(y-d_1)^+$ age1. Next, the amount of items brought into period 2 is $(y-d_1)^+$ and reduced (by demand) in period2 and becomes $(y-d_1-d_2)^+$ age2. This continues until the reordering period $m-1$ when the first order reduces to $y-\sum_{i=1}^{m-1} d_i$ and the second-order arrives. Finally, at the end of the m^{th} period items from the first order not used to meet demand outdated leaving only items from the second order. The process continues with the second, and third, orders. Next, we consider the outlook of two fixed lifetime products with the useful life of four and five periods, respectively.

Table 2 : Orders and Age distribution for a product with 4 lifetimes.

Order	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
1	$(y-d_1)^+$ $age1$	$(y-d_1-d_2)^+$ $age2$	$(y-\sum_{i=1}^{3} d_i)^+$ $age3$	$x-d_4$ $age4$				
2			T_1 $(y-d_1)^+$ $age1$	T_2 $(y-\sum_{i=1}^{2} d_i)^+$ $age2$	T_3 $(y-\sum_{i=1}^{3} d_i)^+$ $age3$	T_4 $x-d_4$ $age4$		
3					T_1 $(y-d_1)^+$ $age1$	T_2 $(y-\sum_{i=1}^{2} d_i)^+$ $age2$	.	.

Table 3: Orders and Age distribution for a product with 5 lifetime .

Order	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
1	$(y-d_1)^+$ $age1$	$(y-\sum_{i=1}^{2} d_i)^+$ $age2$	$(y-\sum_{i=1}^{3} d_i)^+$ $age3$	$(y-\sum_{i=1}^{4} d_i)^+$ $age4$	$x-d_5$ $age5$			
2				T_1 $(y-d_1)^+$ $age1$	T_2 $(y-\sum_{i=1}^{2} d_i)^+$ $age2$	T_3 $(y-\sum_{i=1}^{3} d_i)^+$ $age3$	.	.

The process described for m life time products also applies to the four and five periods of lifetime products.

Derivation of Costs Components.

Shortage Cost

Shortage, unsatisfied demand, run out, or stock out occurs when total demand exceeds on-hand inventory. The units on hand are wholly/completely used to satisfy only a part of the demand, while the unsatisfied demand is lost. This is because the model assumes lost sales of excess demand.

The expected unsatisfied demand per order for our model is

$$Expected \quad (unsatisfied\ demand) = \int_{x+y}^{\infty} (t-(x+y))\, f^*(t)dt \tag{1}$$

With v as the shortage cost per unit, our shortage cost is

$$shortage \quad \cos t = v \int_{x+y}^{\infty} (t-(x+y))\, f^*(t)dt \tag{2}$$

Outdate Cost.

Outdate or expiration of products occurs when the total demand is less than the on-hand inventory. Outdate is caused by products overstaying their useful lifetime in inventory because of low demand. The demand is satisfied by a part of the on-hand inventory while the other part not used to meet demand will outdate. Since new orders arrives with one useful lifetime left in the items from the previous order, only items from the previous order can outdate if not used to meet demand in their last useful period.

From Table1, to determine the outdates from order 1 our interest will be on the m^{th} period which is the last useful period for the order 1 and the second useful period for order 2. This imply that only items from order 1 not used to meet demand will outdate at this point, since the issuing policy is first in first out. The total amount of items on hand from order 1 at the end of the m^{th} period is $x-d_m$.

To obtain the outdate quantity, we consider the following cases.

Case1: if $d_m = x$. The amount of m periods old items from the order1 is equal to the demand in the m^{th} period . Since the issuing policy is FIFO, there will be no outdating as the demand is completely met by the m periods old items in inventory.

Case 2: if $d_m > x$. The amount of m periods old items from the order1 is less than, the demand in the m^{th} period . Part of the demand is satisfied by all of the m periods old items, while the other part is satisfied by items from the new order and there will be no outdating.

Case3: if $d_m < x$. The amount of the m periods old items from the order1 is more than the demand in the m^{th} period . Since the issuing policy is oldest units first, part of the m periods old items will be used to meet the demand, while the other part will outdate. Thus the expected outdate quantity per order will be $x-t$, that is

$$Expected \quad (outdate) \quad = \int_{0}^{x} (x-t)\, f(t)dt \tag{3}$$

With an outdate cost of θ per unit, our outdate cost is

$$outdate \ \ \cos t = \theta \int_{0}^{y} (y-t)\, f^*(t)dt \tag{4}$$

Holding Cost.

Using a similar argument as Nahmias and Pierskalla (1973), we define our on-hand inventory as the amount of items in inventory at the end of the period before outdating (at the m^{th} period), but after demands occur in the period.
For a product with m lifetime, we have

$$h_1(y+x-d_m)+h_2(y+x-d_m-d_1)+h_3(y+x-d_m-d_1-d_2)+\ldots+ h_m(y+x-d_m-d_1-d_2-\ldots-d_{m-1})$$

With a holding cost of $h>0$ per unit held in inventory and simplifying the expression above, our holding cost is

$$Holding\ \cos t = h\int_0^{x+y}(x+y-t)f^*(t)dt \qquad (5)$$

Ordering Cost

There is a fixed ordering cost k per unit ordered, so that our ordering cost is ky
Therefore our total cost function is

$$C(x,y)=\min_{y\geq 0}\{ky+h\int_0^{x+y}(x+y-t)f^*(t)dt+v\int_{x+y}^{\infty}(t-(x+y))f^*(t)dt+\theta\int_0^{y}(y-t)f^*(t)dt\} \qquad (6)$$

where

$f^*(t)=$ *distribution of total demand in periods* $1,2,\ldots m..$

The problem of obtaining optimal y and x

To find the optimal order quantity, we minimize (6) with respect to the order quantity.

$$\frac{\partial C}{\partial y}=k+h\{\frac{x^2f^*}{2}+xyf^*+\frac{y^2f^*}{2}\}-v\{1-\frac{x^2f^*}{2}-xyf^*-\frac{y^2f^*}{2}\}+\frac{\theta y^2f^*}{2} \qquad (7)$$

where $f^*(t)=\lambda e^{-\lambda t}$

$$(\frac{hf^*}{2}+\frac{vf^*}{2}+\frac{\theta f^*}{2})y^2+(hxf^*+vxf^*)y+k+\frac{hx^2f^*}{2}-v+\frac{vx^2f^*}{2}=0 \qquad (8)$$

$$(\frac{h}{2}+\frac{v}{2}+\frac{\theta}{2})f^*y^2+(hx+vx)f^*y+k+\frac{hx^2f^*}{2}-v+\frac{vx^2f^*}{2}=0 \qquad (9)$$

Using (9) with $y=0$, we have,

$$x=\sqrt{\frac{2v-2k}{(h+v)f^*}} \qquad (10)$$

Now, an inventory policy answers two questions; what quantity to order and when to order the quantity. The answer to the second question is already known with our model (when the useful lifetime remaining on the items from the previous order is one period). To answer the first question, we solve equation (9) for y. observed that equation (9) is a polynomial and one of the roots of the equation is picked as the order quantity. We follow the steps below to determine which of the roots is the order quantity.

Step 1: Eliminate all the complex roots, as the optimal order quantity cannot be represented by a Complex number, but by a real number.
Step 2: Eliminate all negative roots since the order quantity cannot be a negative number.
Step 3: When there is only one positive root, it is taken as the optimal order.

Step 4: If there is more than one positive root, we pick the minimum values from the positive roots as our Optimal quantity since higher values can increase the outdating quantity.
Step 5: If zero is the polynomial's only non-negative and non-complex root, we use the aspiration Level criterion to determine the optimal order quantity. The aspiration level is the degree/level of Performance (determined by past experience) that a firm desires to attain or feels it can achieve. For details on aspiration level criterion, see Vahid et al. (1992).

DISCUSSION

During the course of this research, the following were our observations. Firstly, many inventory managers are involved in repackaging outdated products to maximize profit. Repackaging is exchanging the original pack of a product with the right expiration date with a new pack carrying a false or wrong expiration date. This is very common with inventory managers in the food and pharmaceutical sectors. In response to this, we recommend that the expiration date be written on the product and not on the park.

Secondly, some products (especially pharmaceuticals) carry verification codes on them. The code is sent to the producing or manufacturing company and an instant message is sent back saying the product is authentic and safe for consumption. It, however, does not tell you whether the product has expired or when it will expire. Therefore, we recommend adding an expiration code to the verification code. The expiration code can be sent via text message to the producing company and an instant message sent back to the consumer with either one of the following messages (i) expired on the 2nd April 2015, for an expired product or (ii) Not expired, will expire on 2nd April 2017, for a product that has not expired. The expiration code should be on the product and not on the pack. Introducing the expiration code and the verification code (already being practice) will make the products much safer for Nigerians. The codes will tell you that the product is authentic and when it will expire.

Thirdly, the verification and expiration codes can be combined into one code called the verification and expiration code or simply (VEC). Reply message from the producing company should read 'product is authentic and will expire on 2nd April, 2017'. This will go a long way in reducing the problem of repackaging. Please note that having an expiration code on a product is not the same as having an expiration date on the product pack, which is currently being practiced.

CONCLUSION.

We have used the remaining useful life of a product as a decision variable for developing an ordering policy for the fixed lifetime inventory system. The model is easy to implement and can be used for products with a useful life greater than 2. The problem of rebranding can be eliminated by introducing the expiration code on the development and not on the park.

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CONFLICT OF INTEREST

The authors confirm no conflict of interest.

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