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Extended O(6) Dynamical Symmetry of Interacting Boson Model Including Cubic d-Boson Interaction Term

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ABSTRACT

In the interacting boson model (IBM), the low lying collective states in even-even nuclei are described by the interacting s-(L=0) and d-(L=2) bosons. In the original (IBM) the Hamiltonian was defined in terms of one and two body interactions. There are three group chains U(5), SU(3) and O(6) of which represent vibration, rotation and γ-unstable respectively of U(6) which ends in O(3). The cubic terms (three boson interaction terms) are considered in the IBM Hamiltonian in order to obtain a stable, triaxially shaped nucleus. The inclusion of single third order interaction between the d-bosons for interacting boson model (IBM) in the potential of the O (6) limit and the influence on the nuclear shapes are studied. The classical limit of the potential energy surface (PES) which represents the expectation value of the total Hamiltonian in the coherent state is described. The classical limit of the cubic term and the triaxiality is analyzed. A triaxial minimum of the nuclear PES has been obtained. The results are applied to the even-even $^{124-132}$Ba isotopic chain as example of illustrating the effect of the cubic d-boson interaction. A fitting procedure is performed to get the model parameters in order to obtain a minimum root mean square deviation between the calculated and the experimental some low lying energy levels.

INTRODUCTION

There are two different approaches to describe nuclear collective excitations: one approach is the geometric collective model (GCM) (Gneuss *et al.*,1969 andTroltenier,1992), essentially based on the quadrupole degree of freedom which is an extension of the Bohr-Mottelson model depending on the intrinsic deformation parameters β and γ(Rowe,D.J.,G.Thiamova,2005). The other approach is the interacting boson model (IBM) (Iachello*et al.*,1987), which provides us with an alternative description of nuclear collective excitation; it is of an algebraic nature and it exploits symmetry by using group theoretical methods. Its Hamiltonian is written in second quantized form in terms of creation and annihilation boson operators(Pavel,C.and J.Jolie.,2009). The s-d IBM exhibits three dynamical symmetries in the three vertices of Casten triangle, U(5), SU(3) and O(6) appropriate for vibrational nuclei with spherical form, an axially symmetric deformed prolate rotor and γ- unstable axially asymmetric rotor respectively. The relation between the GCM and the IBM was established on the basis of an intrinsic coherent state formalism (Dieperink *et al.*,1980,andGinocchio *et al.*1982).

Recently the nuclear shape phase transitions between the three dynamical limits of the IBM have gained much theoretical interest in terms of GCM (Khalaf *et al.*, 2013 and Khalaf *et al.*,2014) and IBM (Khalaf and Awwad, 2013 and Sirag, 2015). The authors used the Hamiltonian of GCM and the Hamiltonian of IBM in its three forms: creation –annihilation, multipole operators and Casimir operators to study the shape transitions from the limiting cases vibrator- rotator to γ- unstable. The resulted calculations were tested by observables that

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are often used to follow the evolution of shape transitions, such as energy ratios, E_γ over spin (EGOS), reduced B(E2) transition probabilities and the two- neutron separation energies(Proskurins,J.,2010).

The ratio of the energy E_{4+}/E_{2+} for the ground state band is used to investigate whether the nuclei are spherical or not. Three dynamical symmetries, U(5), SU(3) and O(6) of (IBM) are associated with these ratios; E_{4+}/E_{2+} =2.0, 3.33, 2.5 indicate spherical, axially deformed, γ- unstable nuclei, respectively. It could be getting information about the geometry of the nuclei by plotting potential energy surface in terms of deformation parameters β, γ.

Shape phase transitions came again to the forefront of nuclear structure physics in terms of Bohr Hamiltonian (Bohr and Mottelson,1975) when Ichello(Iachello,2000 and Iachello,2003) and Jolie (Jolie and Linnemann,2003) introduced new symmetries E(5), X(5), Y(5) and Z(5) that describe nuclei at the critical points.

The triaxial intrinsic state with $\gamma = 30^\circ$ produces, after angular momentum projection, the exact O(6) eigen functions for small numbers of bosons N. This triaxiality is reduced to γ- unstability. If higher order terms for the interactions between the d-bosons are included in the sd-IBM Hamiltonian, a triaxial minimum of the nuclear potential energy surface (PES) can be found. Effects of three-body interactions on the Hamiltonian of the O(6) limit were considered for a long time as a possible mechanism for generating states with a triaxial deformation (Heyde *et al*.,1984 and Thiamova,2009) corresponding to the classical energy functional with a minimum at the deformation parameters $\beta \neq 0$ and $\gamma \neq 0$.

The cubic $[\hat{Q}\hat{Q}\hat{Q}]^{(0)}$ terms, where $\hat{Q}$ is the O(6) symmetric quadrupole operator were also proposed (Van Isacker,1999and Thiamova and Cejnar,2006). This approach describes the rigid rotor states of Bohr- Mottelson model without using the SU(3) limit of IBM-1 and also allows the study of prolate- oblate shape phase transitions. The cubic term of the QQQ Hamiltonian breaks the O(5) symmetry and conserves the O(6) quantum number σ. This rotational spectrum can be generated only by quadratic and cubic O(6) invariants.

The aim of this work is to introduce the classical limit of the interaction cubic term to the O(6) limit of the IBM and to study its influence on shape transition(Böyükata, M. *et al*., 2014). The structure of the present paper is as follows: In the next section, the Hamiltonian of the original O(6) limit of sd-IBM is considered and the technique of intrinsic coherent formalism is outlined to derive the potential energy surfaces (PES`s) in the variables β and γ. The large boson number limit is considered and the reduced classical PES`s are expressed in the third section. In order to investigate triaxial shapes, the inclusion of the effective three-body potential between the interaction of d-boson and its influence on the nuclear shape is studied. The deviation from the O(6) limit is discussed in the fourth section. In fifth section a systematic study of Ba isotopic chain related to modified O(6) limit is finally given. Some concluding remarks are also given in the last section.

O(6) Limit of IBM:

We start by considering the most general O(6) dynamical symmetry Hamiltonian in the multipole from (Iachello and Arima,1987)

$$\hat{H} = a_0\hat{P}^\dagger.\hat{P} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 \tag{1}$$

where the pairing $\hat{P}$ angular momentum $\hat{L}$ and octupole $\hat{T}_3$ operators are given as (Iachello and Arima,1987)

$$\hat{P}^\dagger = \frac{1}{2}\,(d^\dagger.d^\dagger - s^\dagger.s^\dagger) \tag{2}$$

$$\hat{L} = \sqrt{10}\left[d^\dagger \otimes \check{d}\right]^{(1)} \tag{3}$$

$$\hat{T}_3 = \left[d^\dagger \otimes \check{d}\right]^{(3)} \tag{4}$$

The coefficients a_0, a_1 and a_3 are the model parameters.

We consider the intrinsic coherent state expressed in terms of the geometric deformation parameters β and γ and the total number of boson N (Ginocchio,1980)

$$|N\beta\gamma\rangle = \frac{1}{\sqrt{N!}}(B^\dagger)^N\,|0\rangle \tag{5}$$

With

$$B^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + \beta\, cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\beta sin\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right] \tag{6}$$

The expectation value of $\hat{H}$ is obtained from the evaluation of the expectation values of the single terms, yielding the energy functional

$$E(\beta,N) = \frac{1}{4}a_0N(N-1)\left(\frac{1-\beta^2}{1+\beta^2}\right)^2 + 6a_1N\frac{\beta^2}{1+\beta^2} + \frac{7}{5}a_3N\frac{\beta^2}{1+\beta^2}$$
$$= \lambda\frac{N\beta^2}{1+\beta^2} + \frac{1}{4}a_0N(N-1)\left(\frac{1-\beta^2}{1+\beta^2}\right)^2 \tag{7}$$

With

$$\lambda = 6a_1 + \frac{7}{5}a_3 \tag{8}$$

Equation (7) can be written in another form as:

$$E(\beta, N) = \frac{A_2\beta^2 + A_4\beta^4}{(1+\beta^2)^2} + A_0$$

$$= \frac{A_2\beta^2 + (A_2+A_4)\beta^4 + A_4\beta^6}{(1+\beta^2)^3} + A_0 \tag{9}$$

where:

$$A_2 = [\lambda - a_0(N-1)]N \tag{10}$$

$$A_4 = \lambda N \tag{11}$$

$$A_0 = \frac{1}{4}a_0(N-1)N \tag{12}$$

The potential energy surface (PES) equation (9) is γ independent and has two independent parameters a_0, λ. The equilibrium value of β is obtained by calculating the β derivative of PES which leads to solving the equation $A_2 + (2A_4 - A_2)\beta_e^2 = 0$

We find that $\beta_e = 0$ for $\lambda > a_0(N-1)$ and $\beta_e = \sqrt{\frac{a_0(N-1)-\lambda}{a_0(N-1)+\lambda}}$ for $\lambda < a_0(N-1)$.

The critical point occurs when $A_2 = 0$, which yield to $x = \frac{a_0(N-1)}{\lambda} = 1$.

A sketch of PES as a function of β is illustrated in Figure (1a,1b,1c) for three values of the control parameter x, one value at the critical point x = 1, one below that value x = 0.5 and one above it x= 1.5.

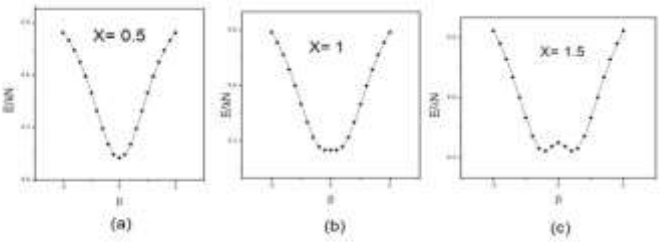

Fig. 1: The PES's as a function of deformation parameter β (a) for spherical shape (x=0.5), (b) for a flat β^4 surface (x=1) and (c) for deformed shape (x=1.5).

For x < 1, the equilibrium spherical shape is obtained, whereas for x > 1 the equilibrium shape is deformed. The value x = 1 gives a flat β^4 surface close to $\beta = 0$.

Pure O(6) dynamical symmetry occur when $\lambda = 0$ ($a_1=a_3=0$), and in order to remove the dependence on the number of bosons N, the Hamiltonian becomes

$$H = \frac{a_0}{N^2} P^\dagger P \tag{13}$$

In the classical limit, corresponding to large boson number N, the PES reads

$$E(\beta) = -a_0\frac{\beta^2}{(1+\beta^2)^2} + \frac{1}{4}a_0 \tag{14}$$

and the minimum occurs at $\beta_e=1$. This symmetry corresponds to a deformed nucleus with γ-unstable. The scaled PES E (β) as a function of β is illustrated in Figure (2).

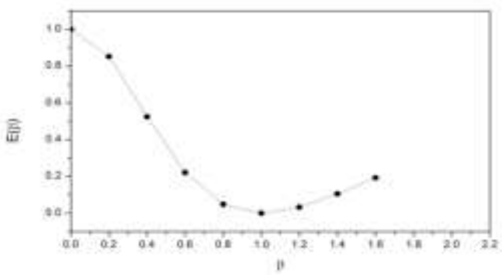

Fig. 2: The PES as a function of deformation parameter β for O(6) dynamical symmetrycorresponding to deformed shape with ɣ-unstable for large number of boson number N. The minimum occurs at β_e=1.

If we modify the Hamiltonian equation (1) by considering the effect of Casmir operator $C_2[O(6)] = N(N + 4) - 4P^{\dagger}.P$ and considering the coefficients A_2, A_4 and A_0 as functions of the number of bosons N, then it becomes immediately clear that the PES depends on the number of bosons and on the pairing operator (i.e. N, a_0). As an illustrative example to explain the role of boson number N, I show in Figure (3) the results of chosen set of parameters A_2, A_4 and A_0 listed in Table (1) to produce a shape transitions Fig.(3.a) for the total number of bosons N =2, (3.b) for N=5, (3.c) for N=8, (3.d) for N=11, (3.e) for N=13.

Table 1: A set of parameters A_2, A_4 and A_0 depending on the total number of bosons N to produce shape phase transition.

N	A2	A4	Ao
2	1.672	1.8744	0.0506
5	1.867	3.891	0.506
8	-0.7136	4.9536	1.4168
11	-6.0698	5.0622	2.783
13	-11.1826	4.6046	3.9468

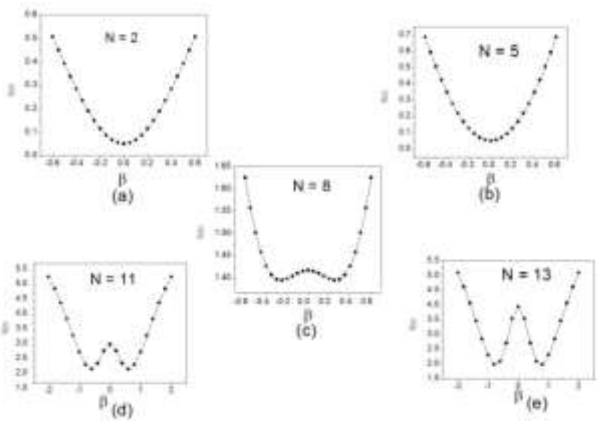

Fig. 3: The PES's as a function of deformation parameter β for shape phase transition obtained from parameters listed in Table (1) (a) for the total number of bosons N =2, (b) for N=5, (c) for N=8, (d) for N=11, (e) for N=13.

Excluding the dependence of the total number of bosons:

To remove the dependence on the boson number N in the original Hamiltonian, it is convenient to weigh the individual terms of the Hamiltonian by factor proportional to their large N expectation values. In this way the generalized Hamiltonian is redefined as:

$$H = \frac{1}{N^2} a_0 P^{\dagger} P + \frac{1}{N} a_1 L.L + \frac{1}{N} a_3 T_3.T_3 \quad (15)$$

which in the classical limit, leads to PES given by

$$E(\beta) = \frac{C_2\beta^2 + C_4\beta^4 + C_6\beta^6}{(1+\beta^2)^3} + C_0 \quad (16)$$

where: $C_2 = \lambda - a_0$, (17)

$C_4 = 2\lambda - a_0$, (18)

$C_6 = \lambda$ (19)

$C_0 = \frac{1}{4} a_0$ (20)

In Figure (4), I show the PES`s corresponding to modified O(6) limit, with varying the parameter a_0 to produce a shape transition. The corresponding values of a_0 are illustrated in the figure and the parameter λ = 1.

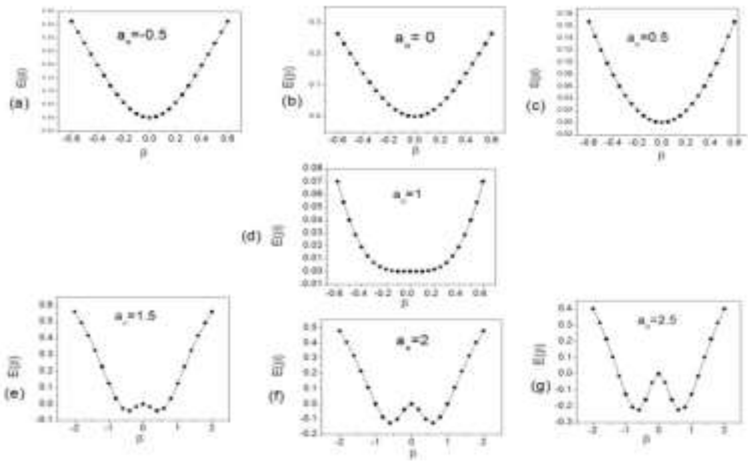

Fig. 4: The PES`s as a function of deformation parameter β for shape phase transition corresponding to modified O(6) limit with varying the control parameter (a) a_0= -0.5, (b) a_0= 0.0, (c) a_0= 0.5, (d) a_0= 1, (e) a_0= 1.5, (f) a_0= 2 and (g) a_0= 2.5.

O(6) Limit of IBM Including Cubic d-Bosons Interaction Term:

In order to introduce a degree of triaxiality (γ - dependent), the cubic term is added with three creation and annihilation operators of the d- bosons in the general form as (Heyde *et al*.,1984)

$$\hat{H}_{3d} = \sum_L \theta_L [d^\dagger x d^\dagger x d^\dagger]^{(L)} [\check{d} x \check{d} x \check{d}]^{(L)} \quad (21)$$

There are five linear independent combinations of type (21), which are determined uniquely by the value of L (L = 0, 2, 3, 4, 6). θ_L denotes interaction parameter for different L values.

For classical limit the expectation value of $\hat{H}_{3d}$ has the form (Van Isacker,1981)

$$V_{3d} = \langle N\beta\gamma | \hat{H}_{3d} | N\beta\gamma \rangle$$

$$= \sum_L \theta_L N(N-1)(N-2) \frac{\beta^6}{(1+\beta^2)^3} (A_L + B_L \cos^2 3\gamma) \quad (22)$$

The values of the coefficients A_L and B_L are listed in reference (VandenBerghe at al.,1985) as follows:

L	0	2	3	4	5
A_L	0	$\frac{1}{5}$	$\frac{-1}{7}$	$\frac{3}{49}$	$\frac{14}{55}$
B_L	$\frac{2}{35}$	0	$\frac{1}{7}$	$\frac{3}{25}$	$\frac{-8}{385}$

I choose all θ_L = 0 except θ_3, then yield (L = 3 cubic d-boson interaction or the third – order term $d^\dagger.\check{d}$).

$$V_{3d} = \theta_3 N(N-1)(N-2) \frac{1}{7} \frac{\beta^6}{(1+\beta^2)^3} (-1 + \cos^2 3\gamma) \quad (23)$$

Adding V_{3d} to the original PES equation (9), yield

$$E(\beta,\gamma) = \frac{D_2\beta^2 + D_4\beta^4 + D_6\beta^6}{(1+\beta^2)^3} + D_0 \quad (24)$$

where $D_2 = A_2$, (25)

$D_4 = A_2 + A_4$, (26)

$$D_6 = A_4 + \frac{\theta_3}{7} N(N-1)(N-2)(-1 + \cos^2 3\gamma) \quad (27)$$

$D_0 = A_0$. (28)

To remove the dependence on the boson number N I divide the term V_{3d} on N^3, so that the final classical limit of PES is given by the coefficients

$D_2 = \lambda - a_o$ (29)

$D_4 = -2\lambda - a_o$ (30)

$$D_6 = \lambda + \frac{\theta_3}{7}(-1 + \cos^2 3\gamma) \quad (31)$$

$D_o = ¼ a_o$ (32)

The optimized D's parameters which are linear combinations of the original parameters λ, ao, θ_3 and γ are adjusted by fitting procedure using a computer simulated search program in order to describe the gradual change in the structure as neutron number varied and to reproduce the experimental excitation energies of some selected reliable states. The entire procedure is repeated for a new set of the parameters until a reasonable compromise is found between theoretical and experimental ones. The mean square deviation is quantified with the common Chi squared.

An Application to the modified O(6) Limit:

The Barium isotopes along the mass region A ~ 130 represent excellent example for O(6) triaxial shapes. The validity of the present technique is examined for $^{124-132}$Ba isotopic chain. For each nucleus, the parameters D_0 , D_2 , D_4 and D_6 of the PES`S which depend on the original parameters of the Hamiltonian have been adjusted to fit the experimental excitation energies and B(E2) transition rates. A computer simulated search program has been used to get best set of parameters applying the common definition of chi:

$$\chi(x_i) = \sqrt{\frac{1}{N}\sum_i \left(x_i^{exp} - x_i^{cal}\right)^2}$$

to reproduce six positive parity experimental levels namely 2_1^+, 4_1^+,6_1^+, 8_1^+ , 2_2^+, 0_2^+ and four B(E2) reduced transition probabilities B(E2, $2_1^+ \to 0_1^+$), B(E2, $4_1^+ \to 2_1^+$), B(E2, $6_1^+ \to 4_1^+$) and B(E2,$8_1^+ \to 6_1^+$).The optimized parameters have been used to study the behavior of the energy ratios $R_{L/2} = E(L^+) / E(2_1^+)$ and the B(E2) ratios $B_{L+2\to L} = B(E2,L+2\to L) / B(E2,2_1^+\to 0_1^+)$.

The Hamiltonian for the O(6) symmetry equation (1) can be written in terms of Casimir operators as:

$$\hat{H} = \beta C_2[O(5)] + \gamma C_2[O(3)] + \eta C_2[O(6)]$$

In this case a basis $|N, \tau, L, \sigma\rangle$ consisting of the irreps of the U(6), O(5), O(3) and O(6) groups classifies the energies as follows:

$$E = E_o + 2\beta\tau(\tau+3) + 2\gamma L(L+1) + 2\eta\sigma(\sigma+4)$$

The lowest states are found by taking σ =N and the next class of the states have σ =N-2, N-4,…1 or 0. Within a given σ value, the values of τ are τ = 0,1,2,.., σ.

The electric quadrupole operator $\hat{Q}^\chi$ of the IBM for the O(6) structure has structure parameter χ=0 and then it represents a generator of O(6) limit and is given by

$$\hat{Q}^\chi = [s^\dagger \tilde{d} + d^\dagger s]^{(2)}$$

The electric quadrupole transition operator is simply chosen as:

$$\hat{T}(E2) = q_2 Q^\chi$$

where q_2 is the effective charge.

The reduced electric quadrupole transition probabilities are given by:

$$B(E2, I_i \to I_f) = \frac{1}{2I_i+1}|\langle I_f \| T(E_2) \| I_i \rangle|^2$$

For O(6) limit the quadrupole transitions are forbidden for Δσ ≠ 0 and Δτ ≠ 1. Using the operator B(E2), we obtain:

$$B(E2, [N], \sigma = N, \tau+1, L = 2\tau+2) = q_2^2 \frac{\tau+1}{2\tau+5}(N-\tau)(N+\tau+4)$$

$$(E2, [N], \sigma = N, \tau+1, L = 2\tau) = q_2^2 \frac{4\tau+2}{(2\tau+5)(2\tau-1)}(N-\tau)(N+\tau+4)$$

In Tables (2,3) and Figure (5.a,b), The calculated energy ratio $R_{I/2}$ for $I^\pi = 4_1^+, 6_1^+, 8_1^+, 2_2^+, 0_2^+$ are given for the Ba isotopes chain. Figure (5.c) shows the ratios of B(E2) in the ground state band $B_{4\to 2}$, $B_{6\to 4}$ and $B_{8\to 6}$ compared to the experimental ones ((NNDC). In most of the cases, the B(E2) strengths are in agreement with the experimental data within the standard deviations, except for ^{126}Ba (N=9) where we notice that B(E2) strength is the smallest one in the ground state band of that nucleus and it probably indicates experimental problem in the lifetime determination. In Figure (6) the PES`s for Barium isotopes Ba$^{124-132}$ are plotted as a function of the deformation parameter β, with γ = 45° and control parameter x = 1- θ_3 / (7λ).

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Table 2: Calculated and experimental energy ratios $R_{L/2}$=E (L^+) / E (2_1^+) for Ba isotopes

Nucleus	N		$R_{4_1^+/2_1^+}$	$R_{6_1^+/2_1^+}$	$R_{8_1^+/2_1^+}$	$R_{2_2^+/2_1^+}$	$R_{0_2^+/2_1^+}$
^{132}Ba	6	Cal.	2.415	4.128	6.081	2.119	3.359
		Exp.	2.425	4.156	6.023	2.219	3.234
^{130}Ba	7	Cal.	2.512	4.404	6.592	2.280	3.476
		Exp.	2.523	4.462	6.708	2.540	3.302
^{128}Ba	8	Cal.	2.649	4.810	7.384	2.738	3.848
		Exp.	2.686	4.954	7.707	3.116	3.320
^{126}Ba	9	Cal.	2.778	5.194	8.132	3.186	4.237
		Exp.	2.773	5.207	8.164	3.414	3.843
^{124}Ba	10	Cal.	2.921	5.612	8.953	3.705	4.815
		Exp.	2.830	5.339	8.356	3.791	3.904

Table 3: Calculated and experimental ratios of reduced transition probabilities $B_{L+2\to L} = \frac{B(E2,L+2\to L)}{B(E2,21+\to 01+)}$ for Ba isotopes.

Nucleus	N		$B_{4\to 2}$	$B_{6\to 4}$	$B_{8\to 6}$
^{132}Ba	6	Cal.	1.511	1.850	2.109
		Exp.			
^{130}Ba	7	Cal.	1.487	1.763	1.950
		Exp.			
^{128}Ba	8	Cal.	1.521	1.793	1.960
		Exp.	1.491	1.928	1.587
^{126}Ba	9	Cal.	1.535	1.798	1.980
		Exp.	1.434	1.615	0.980
^{124}Ba	10	Cal.	1.502	1.750	1.925
		Exp.	1.572	1.600	1.615

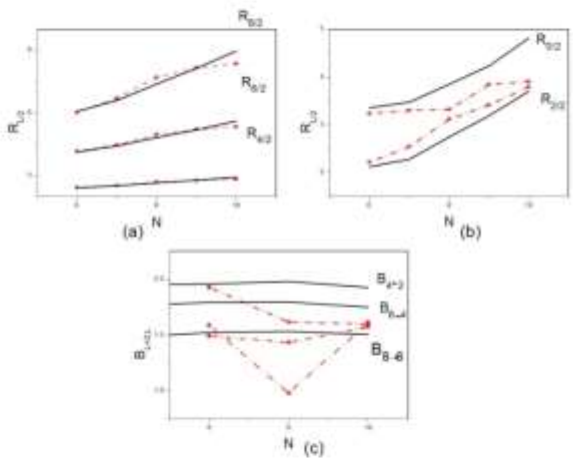

Fig. 5: (a),(b)Comparison between the calculated (solid lines) and experimental (dots with dashed lines) energy ratios $R_{L/2}$=E(L^+)/E(2_1^+) and (c) the reduced transition probability ratios between the ground state band $B_{L+2}\to$L=B(E2,L +2→L)/B(E2,2_1+→0_1^+) for Ba isotopes. N is the total number of bosons for the isotope. The experimental data are taken from Ref.(VandenBerghe *et al.*,1985)

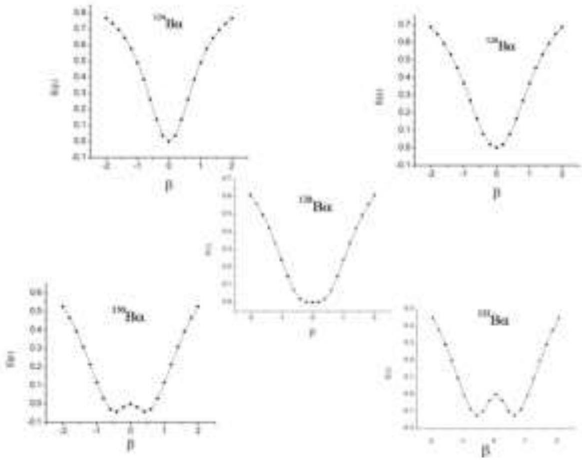

Fig. 6: The PES`s as a function of deformation parameter β for the even-even Ba isotopic chain $^{124-132}$Ba obtained from the IBM modified O(6) limit, including cubic d-boson interaction term at θ_3 =0.055 and γ = 45̊.

Conclusion:

In the present paper exact numerical results of O(6) limit of IBM Hamiltonian including three order interactions between the bosons are obtainedconsidering the most general O(6) dynamical symmetry Hamiltonian in the multipole from and considering the intrinsic coherent state expressed in terms of the geometric deformation parameters β and γ and the total number of boson N,the expectation value of $\hat{H}$ is obtained from the evaluation of the expectation valuesof the single terms.The potential energy surface (PES) equation (9) is γ independent and has two independent parameters a_0, λ. The equilibrium value of β is obtained by calculating the β derivative of PES.For three values of the control parameter x, one value at the critical point x = 1, one below that value x = 0.5 and one above it x= 1.5. For x < 1, the equilibrium spherical shape is obtained, whereas for x > 1 the equilibrium shape is deformed. The value x = 1 gives a flat β^4 surface close to β = 0.

The PES as a function of deformation parameter β for O(6) dynamical symmetrycorresponding to deformed shape with γ-unstable for large number of boson number N Fig.(2). The minimum occurs at β_e=1.As an illustrative example to explain the role of boson number N, is shown in Figure (3) the results of chosen set of parameters A_2, A_4 and A_0 are listed in Table (1) to produce a shape transitions.

The generalized Hamiltonian is redefined to remove the dependence on the boson number N in the original Hamiltonian eq.(15).PES`s corresponding to modified O(6) limit, with varying the parameter a_0 to produce a shape transition is shown in Fig.(4).

In order to introduce a degree of triaxiality (γ - dependent), the cubic term is added with three creation and annihilation operators of the d- bosons in the general form, eq.'s (21-24). The optimized parameters are adjusted by fitting procedure using a computer simulated search program in order to describe the gradual change in the structure as neutron number varied and to reproduce the experimental excitation energies of some selected reliable states. The entire procedure is repeated for a new set of the parameters until a reasonable compromise is found between theoretical and experimental ones. The mean square deviation is quantified with the common Chi squared.

The model is applied to the barium isotopes with A=124-132. The Hamiltonian have been adjusted to fit the experimental excitation energies and B(E2) transition rates. For each nucleus the model parameters were optimized to fit the energy ratios between selected low lying states and B(E2) transition rates. The agreement between theoretical excitation energies and the experimental ones presents reasonable support of the model. The geometric character of Ba isotopic chain has been visualized by deriving the potential energy surfaces BES's and the role of non-axial contribution (effect of γ softness) to the PES's has been investigated. Throughout the Ba isotopic chain, the stiffness in the β- direction is always larger than that in γ, this means that these nuclei are γ-softness.

Prolate – oblate shape transition is induced by another way (Van Isacker,1999andThiamova,2006) by adding the three-body interactions between the O(6) quadrupole operators $[QQQ]^{(0)}$ in the sdIBM version. A triple point of spherical, prolate and oblate phases is identified within the O(6) limit. This results of this $[QQQ]^{(0)}$ model is completely analogous to those obtained from our procedure by using the cubic d-boson interaction term $H_{3d} = \theta_3\,[d^\dagger\, d^\dagger d^\dagger\,]^{(3)}.[\tilde{d}\ \tilde{d}\tilde{d}\ \,]^{(3)}$ as another way.

In the present work L=3 (eq. 21) is chosen, for the future work we can take L=4,5,… and modify the Hamiltonian to predict the shape transition.

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