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Independent Domination of Middle Graph and Line Graph–Laplacian Approach

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ABSTRACT

The Middle graph of a graph G , denoted by M(G), is a graph whose vertex set is V(G)UE(G) , and two vertices are adjacent if they are adjacent edges of G or one is a vertex and other is an edge incident with it. The Line graph of G , written L(G), is the simple graph whose vertices are the edges of G, with ef Є E(L(G)) when e and f have a common end vertex in G An independent dominating set in a graph is a set that is both dominating and independent.The independent domination number of G is the minimum size of an independent dominating set.In this paper, we prove the result for connected graph G, iM(G) ≥ Rank of the Laplacian matrix of G – Q((number of internal vertices in Longest path of G)/2) and For connected graph of G,iL(G)) ≥ number of vertices in G – Rank of the Laplacian matrix + Q((number of internal vertices in longest path in G)/3) where Q is the quotient.

INTRODUCTION

The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it because of its many and varied applications in such fields as linear algebra and optimization, design and analysis of communication networks, and social sciences and military surveillance. Many variants of dominating models are available in the existing literature. For a comprehensive bibliography of papers on the concept of domination , readers are referred to Hedetniemi and Laskar . The present paper is focused on independent domination number of middle graphs in terms of vertices and Rank of Laplacian matrix.

1.1. Preliminaries:

In this section, the basic definitions and known results related to our results are given.

1.1.1. Definition:

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G, called edges. The vertex set and the edge set of G are denoted by V (G) and E(G) respectively.

If e= { u,v} is an edge, we write e=uv; we say that e joins the vertices u and v ; u and v are adjacent vertices ; u and v are incident with e.If two vertices are not joined , then we say that they are not joined, then we say that they are non-adjacent . If two distinct edges are incident with a common vertex, then they are said to be adjacent to each other.

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1.1.2. Definition:

Let G = (V, E) be a graph. A set S ,subset of V, is called a dominating set if every vertex in V-S is adjacent to some vertex in S. The domination number Y (G) is the minimum cardinality taken over all minimal dominating sets in G.

1.1.3. Definition:

A set is independent dominating if it is independent and dominating. let i(G) denotes the minimum cardinality of an independent dominating set of G.

1.1.4. Definition:

The adjacency matrix A(G) of G is an nxn matrix with its row and columns indexed by V(G) and with the (i,j)-entry equal to 1 if vertices i,j are adjacent (i.e., joined by an edge) and 0 otherwise.Thus A(G) is a symmetric matrix with its i-th row (or column) sum equal to d_i(G),which by definition is the degree of the vertex i , i=1,2,..n.

Let D(G) denote the nxn diagonal matrix,whose i-th diagonal entry is d_i(G),i=1,2,…n.

The Laplacian matrix of G,denoted by L(G),is simply the matrix D(G)-A(G).

1.1.5. Definition:

A graph G is said to be connected if any two distinct vertices of G are joined by a path.

1.1.6. Definition:

A set of vertices in a graph G is called independent set if no two vertices in the set are adjacent.

1.1.7. Definition:

Let u and v be vertices of a graph G. A u-v walk of G is a finite,alternating sequence of vertices and edges beginning with vertex u and ending with vertex v such that $e_i = u_{i-1}u_i$,i=1,2,…n.The number n is called the length of the walk.The walk is said to be open if u and v are distinct vertices;it is closed otherwise. A walk u_0 ,e_1,u_1,e_2,u_2,..u_n is determined by the sequence u_0 ,u_1,u_2,..u_n of its vertices and hence we specify the walk by (u_0 ,u_1,u_2,..u_n).A walk in which all the vertices are distinct is called a path. A closed walk u_0 ,u_1,u_2,..u_n) in which u_0 ,u_1,u_2,..u_{n-1} are distinct is called a cycle.A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n.

1.1.8. Definition:

Let *v* □ *G* be a vertex a graph *G*. The neighbourhood of *v* is the set *NG*(*v*) = {*u* □ *G* | *vu* □ *G*}. The degree of *v* is the number of its neighbours: *dG*(*v*) = |*NG*(*v*)| .

If *dG*(*v*) = 0, then *v* is said to be isolated in *G*, and if *dG*(*v*) = 1, then *v* is a leaf of the graph. The minimum degree and the maximum degree of *G* are defined as *d*(*G*) = min{*dG*(*v*) | *v* □ *G*} and D(*G*) = max{*dG*(*v*) | *v* □ *G*} .

1.1.9. Definition:

The Line graph of G , written L(G), is the simple graph whose vertices are the edges of G, with ef Є E(L(G)) when e and f have a common end vertex in G

1.1.10. Definition:

Let G be a connected graph. The distance between two vertices u and v in G, denoted by d(u, v) is the length of a shortest path between them in G. The eccentricity of a vertex v is ecc(v) := maxx□V (G) d(v, x). The diameter of G is diam(G) := maxx□V (G) ecc(x).

The radius of G is rad(G) := minx□V (G) ecc(x). Distance between a vertex v and a set S □ V (G) is d(v, S) := minx□S d(v, x). The k-step open neighbourhood of a set S □ V (G) is Nk(S) := {x □ V (G)|d(x, S) = k}, k □ {0, 1, 2, · · · }. The degree of a vertex v is degree(v) := |N1({v})|. The minimum degree of G is δ(G) := minx□V (G) degree(x). A vertex is called pendant if its degree is 1 and isolated if its degree is 0.

1.1.11. Theorem:

For any graph, $i_M(G) \geq \left\lceil \frac{p}{2} \right\rceil$

1.1.12. Theorem:

If G is connected,then the rank of Laplacian matrix is n-1.

Australian Journal of Basic and Applied Sciences, 10(1) January 2016, Pages: 614-618

II. Main Results:

2.1. Theorem:

For Star graph, Cycle graph, Path graph, Tree, Unicyclic graph, $i_L(G)) \geq$ number of vertices in G – Rank of the Laplacian matrix + Q($\frac{number\ of\ internal\ vertices\ in\ longest\ path\ in\ G}{3}$).

2.1.1. Proof:

Let G= (p,q) graph. Let { $v_1, v_2, v_3, \ldots\ldots, v_n$ } be the vertices of G. Let { $e_1, e_2, \ldots, e_n$} be the edges of G.Let{ $f_1, f_2, f_3, \ldots\ldots, f_n$} be the corresponding vertices of L(G). If p≤ 4, the result is obvious. Assume that p>4

2.1.1.1. Case:

If Δ(G)=p-1.
If G contains an edge e∈ E(G) such that deg e = q-1.
Then the vertex having maximum degree dominates all the vertices of L(G)
Then i(L(G) =1.
Hence the result holds.

2.1.1.2. Case:

If G contains an edge e∈ E(G) such that Δ(G) < p-1 and if Δ(G) = 2

If G is cycle and p ≡1,2(mod 3)

Let $\{f_2, f_5, f_8 \ldots. f_i$ / $d(f_i, f_j)=3$,$1 \leq i<j \leq n, f_i$,f_j are not adjacent} which is independent dominating set of L(G) with cardinality Q($\frac{n}{3}$).

If p ≡0 (mod 3)

Choose I $=\{f_2, f_5, f_8 \ldots. f_i$ / $d(f_i, f_j)=3$,$1 \leq i<j \leq n, f_i$,f_j are not adjacent} ∪ v_n which is independent dominating set of L(G) with cardinality Q($\frac{n}{3}$)+1 vertices.

Consider, Q($\frac{n}{3}$)+1 ≥ Q($\frac{number\ of\ internal\ vertices\ in\ longest\ path\ in\ G}{3}$)+1
≥ Q($\frac{number\ of\ internal\ vertices\ in\ longest\ path\ in\ G}{3}$)+n-n+1.
≥ Q($\frac{number\ of\ internal\ vertices\ in\ longest\ path\ in\ G}{3}$)+n-(n-1).
≥n+ Q($\frac{number\ of\ internal\ vertices\ in\ longest\ path\ in\ G}{3}$)-Rank of the Laplacian matrix.

2.1.1.3. Case:

If G is a path,and if n≡2 (mod 3)
Choose I = $\{f_2, f_5, f_8 \ldots\ldots, f_n / d(f_i, f_{i+3})=3, 2 \leq i \leq n\}$ ∪ $\{f_n\}$
Clearly, I is an independent dominating set with cardinality Q($\frac{n}{3}$)+1 vertices.
If n≡0,1 (mod 3), Choose I= { $\{f_2, f_5, f_8 \ldots\ldots, f_n / d(f_i, f_{i+3})=3, 2 \leq i \leq n\}$
Clearly, I is an independent dominating set with cardinality Q($\frac{n}{3}$) vertices.

2.1.1.4. Case

If 2<Δ(G)<p-1 , If G is a tree,
Let $\{u_1, u2, u3, \ldots u_{n-1}\}$ be the cutvertices of L(G)
Let $S=\{u_i, u_j / u_i$ and u_j are not adjacent $1 \leq i<j \leq n-1\}$
Let I = $\{f_i, f_j / u_i \in S$ and $d(u_i, x_i)=2, u_i, x_i$ are not adjacent and x_i, x_j are not adjacent $1 \leq i<j \leq n-1\}$
Clearly, H=SUI is an independent dominating set with cardinality $|H|$ = diam G-1
Now, Number of internal vertices of G in longest path is diam G-1
$|H| \geq diam\ G$ ≥ 1+ Q($\frac{Number\ of\ internal\ vertices\ in\ longest\ path\ of\ G}{3}$)
≥1+ Q($\frac{Number\ of\ internal\ vertices\ in\ longest\ path\ of\ G}{3}$)
≥ Q($\frac{Number\ of\ internal\ vertices\ in\ longest\ path\ of\ G}{3}$)+n-(n-1).

2.1.1.5. Case:

Let G be Unicyclic graph. Let C= { e1,e2,e3…..en} be the cycle of G.

Let C1={ei,ej/d(ei,ej) are not adjacent,1≤i<j≤n} be the edges of cycle with cardinality atmost Q($\frac{n}{2}$) edges.

Consider the connected spanning tree T of G by removing the Q($\frac{n}{2}$) edges of C1

Clearly, T is tree of G with atmost p- Q($\frac{n}{2}$) vertices

L(T) has independent dominating set I with

$|I| \geq$ Q($\frac{Number\ of\ internal\ vertices\ in\ longest\ path\ of\ G}{3}$)+n-(n-1).

Now, Adding the edges ,

If edges of C1 adjacent to some vertex of L(T),then there is nothing to prove.

If not, i(L(G) ≥ Q($\frac{Number\ of\ internal\ vertices\ in\ longest\ path\ of\ G}{3}$)+n-(n-1)+ Q($\frac{n}{2}$)

i_L(G)) ≥ number of vertices in G – Rank of the Laplacian matrix + Q($\frac{number\ of\ internal\ vertices\ in\ longest\ path\ in\ G}{3}$)

2.2. Theorem:

For Star graph, Cycle,Path, G, i_M(G) ≥ Rank of the Laplacian matrix of G – Q($\frac{number\ of\ internal\ vertices\ in\ Longest\ path\ of\ G}{2}$) where Q is the quotient .

2.2.1. Proof:

Let G= (p,q) graph. Let { v_1, v_2 , v_3,……,v_n } be the vertices of G. Let { e_1,e_2, …,e_n} be the edges of G.Let { u_1,u_2,…..u_n,f_1,f_2,f_3,……,f_n} be the corresponding vertices of M(G). where f_1,f_2,f_3,……,f_n are subdividing edges of M(G). Let us prove the theorem by induction on p, the number of vertices of G. If n ≤ 4, the result is obvious. Assume that the result is true for less than n. Now we prove for n

2.2.1.1. Case:

If G contains an edge e∈ E(G) such that deg e = q-1.

Hence , if deg e = q-1,

Then exactly subdividing edge e_i, 1≤i≤n, dominate all the subdividing edges and one end and non end vertex.

Also, remaining n-2 end vertices are dominated bt themselves.

I = {e_i,(n-2) end vertices , 1 ≤i≤n,}

I is an independent dominating set with cardinality $|n-1|$.

The number of internal vertices of G is 1

Hence, i_M(G) ≥ Rank of the Laplacian matrix of G – Q($\frac{number\ of\ internal\ vertices\ in\ Longest\ path\ of\ G}{2}$).

2.2.1.2. Case:

If G contains an edge e∈ E(G) such that deg e < p-1

If deg e ≤ 2

If n is even,

Choose , I = { f_1,f_3,…f_{i-1} /d(f_i,f_{i-1})=2,1≤i≤n-1, f_i, f_{i-1} are not adjacent}

Clearly, I is an independent dominating set with cardinality atleast Q$\left(\frac{n}{2}\right)$ vertices.

If n is odd,

Choose I = { f_1,f_3,…f_{i-1} /d(f_i,f_{i-1})=2,1≤i≤n-1, f_i, f_{i-1} are not adjacent} U v_n which is an independent dominating set with cardinality atleast Q$\left(\frac{n}{2}\right)+1$ vertices.

Now, Q$\left(\frac{n}{2}\right)$+1-n ≥ -1- Q$\left(\frac{nnumber\ of\ internal\ vertices\ in\ Longest\ path\ of\ G}{2}\right)$

≥Rank of the Laplacian matrix - Q$\left(\frac{nnumber\ of\ internal\ vertices\ in\ Longest\ path\ of\ G}{2}\right)$

Q$\left(\frac{n}{2}\right)$≥Rank of the Laplacian matrix - Q$\left(\frac{nnumber\ of\ internal\ vertices\ in\ Longest\ path\ of\ G}{2}\right)$

Hence the result is true.

2.3. Theorem:

For complete bipartite graph $k_{m,n}$, Y_c(M($k_{m,n}$))=m+n-1 for any m ,n.

2.3.1. Proof:

Let (X,Y) be a bipartition of $k_{m,n}$. Let │X│ = m and │Y│ = n

Let X = { x_1,x_2,x_3,…..x_m} and Y = {y_1,y_2,y_3,….y_n}

Let { u_{11},u_{12},……u_{ij} , 1 ≤ i ≤ m , 1 ≤ j ≤ n } be the added vertices corresponding to the edges e_{11},e_{12},…..e_{ij} 1 ≤ i ≤ m , 1≤ j ≤ n of $k_{m,n}$ to obtain M($k_{m,n}$).

Thus V(M($k_{m,n}$))) ={ x_1,x_2,x_3,…..x_m, y_1,y_2,y_3,….y_n , u_{11},u_{12},……u_{ij}1 ≤ i ≤ m , 1 ≤ j ≤ n }

Therefore │ V(M($k_{m,n}$)))│ = mn + m + n

Consider a set F ={ $e_{m-i,1}$, e_{mj} for i=1,2,3….m-1,j=1,2,3,…n} is a connected dominating set with│F│ = m + n 1.

Since each vertex in M($k_{m,n}$) is either in F or is adjacent to an vertex in F ,

Therefore, F is connected dominating set. Since , m number of vertices can dominate mn + m + 1 vertices and other vertices can be dominated by n - 1 vertices.

Therefore , any set containing edges less than that of F cannot be connected domainting set of $M(k_{m,n})$.
This implies that F is connected dominating set with minimum cardinality.
Therefore, $\Upsilon_c(M(k_{m,n})) = m + n - 1$.

2.4. Theorem:

For any connected (p,q) graph, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \Upsilon_c(G)$

2.4.1. Proof:

We have, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \Upsilon(G)$
But, $\Upsilon(G) \leq \Upsilon_c(G)$
Hence, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \Upsilon_c(G)$

2.5. Theorem:

$i(M(C_p)) = \lceil \frac{2p}{5} \rceil$.

2.5.1. Proof:

Let G be a (p,q) graph.
we know that, $\Upsilon(G) \leq i(G)$ and For any k-regular graph, $\Upsilon(G) \geq \frac{p}{k+1}$.
clearly , $\frac{p}{k+1} \leq \Upsilon(G) \leq i(G)$ $\frac{p}{k+1} \leq i(G)$.
$M(C_p)$) has p+p vertices.since number of vertices in M(G) is equal to twice the number of vertices in G and $M(C_p)$) is 4-regular graph.
$i(M(C_p)) = \lceil \frac{2p}{5} \rceil$.

2.6. Theorem:

If G is connected graph on n vertices other than star graph, then $i(M(G) \leq 3n - \Upsilon(G) - \left\lceil \frac{3n}{\Upsilon(G)} \right\rceil$.

III. Conclusion:

The Middle graph of a graph G , denoted by M(G), is a graph whose vertex set is V(G)UE(G) , and two vertices are adjacent if they are adjacent edges of G or one is a vertex and other is an edge incident with it. The Line graph of G , written L(G), is the simple graph whose vertices are the edges of G, with ef Є E(L(G)) when e and f have a common end vertex in G An independent dominating set in a graph is a set that is both dominating and independent.The independent domination number of G is the minimum size of an independent dominating set.In this paper, we prove the result for connected graph G, $i_M(G) \geq$ Rank of the Laplacian matrix of G – Q($\frac{\text{number of internal vertices in Longest path of G}}{2}$) and For connected graph of G,$i_L(G)) \geq$ number of vertices in G – Rank of the Laplacian matrix + Q($\frac{\text{number of internal vertices in longest path in G}}{3}$) where Q is the quotient.

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