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On The Approximate Solution of Linear Fuzzy Volterra-Integro Differential Equations of the Second Kind

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ABSTRACT

In this paper, the variation iteration method (VIM) is adopt to finding the approximate solution of the linear fuzzy Volterra-integro differential equation of second kind (LFVIDEs) .The (VIM) is to construct correction functional using general Lagrange multipliers(λ) identified optimally via the variational theory. We proving theorm study the convergence approximate solutions to the exact solutions. Finally, two examples are given and their results are shown in figures to illustrate the efficiency and accuracy of this method.

INTRODUCTION

The topics of fuzzy differential equation (FDE) and fuzzy integral equation (FIE) in both theoretical and numerical points of view have been developed in recent years. Prior to discussing fuzzy integro-differential and their numerical treatments (ZEINALI, M., S. *et al*., 2013; Osama, H., *et al*., 2013.; Zadeh, L.A. and S.S.L. Chang, 1972; Mizumoto, M. and K. Tanaka, 1979; Kaleva, O., 1987; Goetschel, R., W. Vaxman, 1986; Puri, M.L. and D. Ralescu, 1983; Mehrkanoon, S., *et al*., 2009; He, J.H., 1997), it is necessary to present and brief introduction to preliminary topics such as fuzzy numbers and fuzzy calculus (Osama, H., *et al*., 2013). The concept of fuzzy sets which was originally introduced by Zadah (1972) led to definition of the fuzzy number and its implementation in of the fuzzy control and approximate reasoning problems. The basic arithmetic structure for fuzzy numbers was later developed by Mizumoto and Tanaka (1979). Nohmias and Ralesu (1987) all of which observed the fuzzy number as a collection of r $-$ levels, $0 < r \leq 1$. In this paper, we used variational iteration method (VIM) for solving of linear fuzzy Volterra integro-differential equation of the second kind (LFVIDEs):

$$\tilde{y}'(x) = \tilde{f}(x) + \lambda \int_0^x k(x,t)\tilde{y}(t)dt \qquad (1)$$

2. Basic concept of Fuzzy Set Theory:

In this section the basic notations used in fuzzy calculus are introduced. We start by defining the fuzzy number.

Definition (2.1) (Goetschel, R., W. Vaxman, 1986): A fuzzy number is a map $u: \mathcal{R} \to I = [0,1]$ which satisfies?

i. u is upper semi-continuous,
ii. u(x)=0 outside some interval [c,d]
iii. There exist real numbers a,b such that $c \leq a \leq b \leq d$, where

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1. u(x) is monotonic increasing on [c,a]
2. u(x)is monotonic decreasing on [b,d]
3. u(x)=1,a$\leq x \leq b$.

The set of all the fuzzy numbers (as given in definition (2.1)) is denoted by E^1 an alternative definition which yield the same E^1 is given by[5].

Definition (2.2):A fuzzy number u is a pair ($\underline{u},\overline{u}$) of function $\underline{u}(r),\overline{u}(r)$; $0 \leq r \leq 1$ Which satisfying the following requirements:

1. $\underline{u}(r)$ is a bounded monotic increasing Left continuous function
2. $\overline{u}(r)$ is a bounded monotic decreasing Left continuous function
3. $\underline{u}(r) \leq \overline{u}$(r),$0 \leq r \leq 1$.

For arbitrary fuzzy numbers u=($\underline{u}(r),\overline{u}(r)$), $v = (\underline{v}(r),\overline{v}\,(r))$ and real constant we define addition u+v and scalar multiplication ku as

$$\underline{(u+v)}(r) = \underline{u}(r) + \underline{v}(r)$$

$$\overline{(u+v)}(r) = \overline{u}\,(r) + \overline{v}(r) \quad (2)$$

$$\left(\underline{ku}\right)(r) = k\underline{u}(r)\,,\ \left(\overline{ku}\right)(r) = k\overline{u}\,(r) \qquad \text{if } k \geq 0$$

$$\left(\underline{ku}\right)(r) = k\overline{u}\,(r)\,,\ \left(\overline{ku}\right)(r) = k\underline{u}(r) \qquad \text{if } k > 0 \quad (3)$$

The collection of all such fuzzy numbers with addition and multiplication as define by equation (2) and (3) is denoted by E^1 and is convex cone. Next, we will define the fuzzy function notation and a metric D in E^1.

Definition (2.3) (Goetschel, R., W. Vaxman, 1986): For arbitrary fuzzy numbers u= ($\underline{u}(r),\overline{u}(r)$), $v = (\underline{v}(r),\overline{v}\,r)$, the quantity $D(u.v) = max\{\sup_{0\leq\alpha\leq 1}|\underline{u}(r) - \underline{v}(r)|, \sup_{0\leq\alpha\leq 1}|\overline{u}(r) - \overline{v}\,r|\}$

Is the distance between u and v. This metric is equivalent to the one used by (Goetschel, R., W. Vaxman, 1986; Puri, M.L. and D. Ralescu, 1983). It is shown (Puri, M.L. and D. Ralescu, 1983) that (E^1, D) is a complete metric space.

Definition (2.4): A fuzzy function f: $R^1 \rightarrow E^1$ is said to be continuous if for arbitrary fixed $x_0 \in R^1$ and $\in\ > 0$ there exists $\delta > 0$ such that

$$|x - x_0| < \delta, then\ D(f(x), f(x_0)) < \in \quad (4)$$

Definition (2.5) (Mehrkanoon, S., *et al*., 2009): The Seikkala derivative $f'(x)$ of a fuzzy function f is defined by $[f'(x)]_r$=[$\underline{f}'$ (x;r),$\overline{f}'(x;r)$] ,$r \in [0,1]$, where prime symbol denoted the derivative with respect to x.

Definition(2.6): let f : I→ E^1 be a fuzzy function and $x_0 \in I \subseteq R$, then f is differentiable at x_0 , if (I) there exist an element $f'(x_0) \in E^1$, such that for all $h > 0$ sufficiently small, there are $f(x_0 + h) - f(x_0)$; $f(x_0) - f(x_0 + h)$ and

$$\left(\lim_{h\rightarrow 0^+}\right)\frac{f(x_0+h)-f(x_0)}{h} = \left(\lim_{h\rightarrow 0^+}\right)\frac{f(x_0)- f(x_0+h)}{h} = f'(x_0) \quad (5)$$

Or

(II) there exist an element$f'(x_0) \in E^1$, such that for all h < 0 sufficiently small, there are $f(x_0 + h) - f(x_0$; $f(x_0) - f(x_0 + h)$ and

$$\left(\lim_{h\rightarrow 0^-}\right)\frac{f(x_0+h)-f(x_0)}{h} = \left(\lim_{h\rightarrow 0^-}\right)\frac{f(x_0)- f(x_0+h)}{h} = f'(x_0) \quad (6)$$

where the relation (I) is the classical definition of the fuzzy *H-derivative*.

3- Fuzzy Integro- Differential Equation:

In this section, we consider the first order linear fuzzy Volterra integro-differential equation (LFVIDEs).

$$\tilde{y}'(x) = \tilde{f}(x) + \lambda\int_0^x k(x,t)\tilde{y}\,(t)dt \quad (7)$$

Where the initial $\tilde{y}_0 = \tilde{y}(0) = \left(\underline{y}(0,r),\overline{y}(0,r)\right) = (0,0)$,

Where $\tilde{y}'(x) = \frac{d}{dx}\tilde{y}(x)$, $\tilde{f}: [0,b] \rightarrow y(R)$ is continuous fuzzy number, k is arbitrary continuous function over the region $\Omega = \{(x,t)|0 \leq x \leq b\}$ and $\Delta = \{(x,t,\tilde{y}(x))|0 \leq x \leq b, \tilde{y}(x) \in y(R)\}$ and $\tilde{y}$ is not to be determined. Eq (7)

can be written in terms of r − Level set as:

$$[\underline{y}'(x;r),\overline{y}'(x;r)] = [\underline{f}(x;r),\overline{f}(x;r)] + \lambda\int_0^x k(x,t)[\underline{y}\,(t;r),\overline{y}(x;r)]dt \quad (8)$$

For $0 \leq r \leq 1$

Thus, Eq. (7) can be transformed into the following system

$$\underline{y}'(x;r) = \underline{f}(x;r) + \lambda \int_0^x k(x,t)\, \underline{y}\,(t;r)\, dt$$
$$\overline{y}'(x;r) = \overline{f}(x;r) + \lambda \int_0^x k(x,t) \overline{y}\,(t;r) dt \quad (9)$$

With initial condition $\underline{y}(0;r) = \overline{y}(0;r) = 0$

4-Variational iteration Method:

The variational iteration method (VIM) is proposed by He (He, J.H., 1997) as a modification of a general Lagrange multiplier method .This method has been shown to solve effectively ,easily and accurately a large class of nonlinear problems with approximations converging rapidly to a accurate solutions. To illustrate its basic idea of the technique, we consider following general nonlinear system:

$$\mathrm{L}[\mathrm{u}(\mathrm{x})] + \mathrm{N}[\mathrm{u}(\mathrm{x})] = \mathrm{g}(\mathrm{x}) \quad (10)$$

Where L is linear operator, N is a nonlinear operator, and g(x) is given continuous function. The basic character of method is to a correction functional for system Eq.(10) which

$$\mathrm{u}_{n+1}(\mathrm{x}) = \mathrm{u}_n(\mathrm{x}) + \int_0^x \lambda(\tau)\{\mathrm{Lu}_n(\tau) + \mathrm{N}\tilde{\mathrm{u}}_n(\tau) - \mathrm{g}(\tau)\}d\tau \quad (11)$$

Where $\lambda(\tau)$ is a general Lagrangian multiplier (Kaleva, O., 1987) which can be identified optimally via variational theory, the subscript n denotes the nth-order approximation and $\tilde{\mathrm{u}}_n$ is consider a restricted variation, i.e. $\delta\tilde{\mathrm{u}}_n = 0$ where $L = \frac{d}{dt}$, we can construct the following correction functional

$$\mathrm{u}_{n+1}(\mathrm{x}) = \mathrm{u}_n(\mathrm{x}) + \int_0^x \lambda(\tau)\{\mathrm{u'}_n(\tau) + \mathrm{N}\tilde{\mathrm{u}}_n(\tau) - \mathrm{g}(\tau)\}d\tau \quad (12)$$

5- Variational Iteration Method Solving The Linear Fuzzy Volterra-integro Differential Equations (lFVIDEs):

Now, we consider the linear fuzzy Volterra-integro differential equation of second kinds: $\tilde{y}'(x) = \tilde{f}(x) + \lambda \int_0^x k(x,t)\tilde{y}\,(t)dt$

Then we have the following iteration sequences

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(\tau)\left\{\tilde{u}'_n(\tau) - \tilde{\mathrm{f}}(\tau) - \int_0^s \mathrm{k}(\mathrm{s},\mathrm{t})\tilde{\mathrm{u}}_n(\mathrm{s})d\mathrm{s}\right\}d\tau \quad (13)$$

To find optimal λ, we proceed as following:

$$\delta u_{n+1}(x) = \delta u_n(x) + \delta \int_0^x \lambda(\tau)\left\{\tilde{u}'_n(\tau) - \tilde{\mathrm{f}}(\tau) - \int_0^s \mathrm{k}(\mathrm{s},\mathrm{t})\tilde{\mathrm{u}}_n(\mathrm{s})d\mathrm{s}\right\}d\tau \quad (14)$$

And upon using the method of integration by parts, then Eq.(13) will be reduced to

$$\delta u_{n+1}(x) = \delta u_n(x) + \delta \int_0^x \lambda(\tau)\{\tilde{u}'_n(\tau)\}d\tau$$

Then the following stationary conditions are obtained:

$\acute{\lambda} = 0$ and $\lambda + 1 = 0$

The general Lagrange multipliers therefore, can be readily identified: $\lambda = -1$ and by substitute in Eq(13), the following iteration formula $n \geq 0$ is obtained

$$u_{n+1}(x) = u_n(x) - \int_0^x \left\{\tilde{u}'_n(\tau) - \tilde{\mathrm{f}}(\tau) - \int_0^s \mathrm{k}(\mathrm{s},\tau)\tilde{\mathrm{u}}_n(\mathrm{s})d\mathrm{s}\right\}d\tau \quad (15)$$

Therefore, we can write the following iteration formulas

$$\underline{y}_{n+1}(x,r) = \underline{y}_n(x,r) - \int_0^x \left\{\underline{y}'_n(\tau,r) - \underline{f}(\tau,r) - \int_0^s k(s,\tau)\underline{F_n}(s,r)ds\right\}d\tau$$

$\overline{y}_{n+1}(x,r) = \overline{y}_n(x,r) - \int_0^x \left\{\overline{y}_n{}'(\tau,r) - \overline{f}(\tau,r) - \int_0^s k(s,\tau)\overline{F}_n(s,r)ds\right\}d\tau$ with the initial approximations $\underline{y}_0(x,r) = 0$ and $\overline{y}_0(x,r) = 0$.

Theorem:

Let $\tilde{\mathrm{u}} \in (\mathrm{C}^2[\mathrm{a},\mathrm{b}], \|.\|_\infty)$ be the exact solution of the linear fuzzy Volterra-integro differential equation of (7) and $\tilde{\mathrm{u}}_n(\mathrm{x}) \in \mathrm{C}^2[\mathrm{a},\mathrm{b}]$ be the obtained solution of the sequence defined by eq.(15). If $\widetilde{\mathrm{E}}_n(\mathrm{x}) = \tilde{\mathrm{u}}_n(\mathrm{x}) - \tilde{\mathrm{u}}(\mathrm{x})$ and $|\mathrm{k}| < c$, $0 < c < 1$ then the sequence of approximate solutions $\{\tilde{\mathrm{u}}_n(\mathrm{x})\}$, n 0,1,... converges to the exact solution $\tilde{\mathrm{u}}(\mathrm{x})$.

Proof:- consider linear fuzzy Volterra integro-differential equation of the second kind:- $\tilde{y}'(x) = \tilde{f}(x) + \lambda \int_0^x k(x,t)\tilde{y}\,(t)dt$

Where the approximate solution using the VIM is given by $u_{n+1}(x) = u_n(x) - \int_0^x \left[\widetilde{\mathrm{u}'}_n(\tau) - \tilde{\mathrm{f}}(\tau) - \int_0^s \mathrm{k}(\mathrm{s},\tau)\tilde{\mathrm{u}}_n(\mathrm{s})d\mathrm{s}\right]d\tau$ (16)

and since $\tilde{u}$ is exact solution of the linear fuzzy Volterra-integro differential equation of the second kind

$$\tilde{u}(x) = \tilde{u}(x) - \int_0^x \left[\widetilde{\mathrm{u}'}_n(\tau) - \tilde{\mathrm{f}}(\tau) - \int_0^s \mathrm{k}(\mathrm{s},\tau)\tilde{\mathrm{u}}_n(\mathrm{s})d\mathrm{s}\right]d\tau \quad (17)$$

Now, subtracting Eq(16) from Eq.(17) $\widetilde{\mathrm{E}}_{n+1}(x) = \widetilde{\mathrm{E}}_n(x) - \int_0^x \left[\widetilde{\mathrm{E}}_n(\tau) - \tilde{f}(\tau) + \tilde{f}(\tau) + \int_0^s k(s,\tau)\tilde{u}_n(s)ds\right] d\tau$ $\widetilde{\mathrm{E}}_{n+1}(x) = \widetilde{\mathrm{E}}_n(x) - \int_0^x \widetilde{\mathrm{E}}_n(\tau)d\tau + \int_0^x \int_0^s k(s,\tau)\,\widetilde{\mathrm{E}}_n(\tau)\,dsd\tau$ $\widetilde{\mathrm{E}}_{n+1}(x) = \widetilde{\mathrm{E}}_n(x) - \widetilde{\mathrm{E}}_n(x) - \widetilde{\mathrm{E}}_n(0) - \int_0^x \int_0^s k(s,\tau)\,\widetilde{\mathrm{E}}_n(\tau)\,dsd\tau$

And since $\tilde{E}_n(0) = \tilde{u}_n(0) - \tilde{u}(0)$ which have the initial condition of the FVIDEs, the $\tilde{E}_n(0) = 0$. Hence

$\widetilde{\mathrm{E}}_{n+1}(x) = \int_0^x \int_0^s k(s,\tau)\,\widetilde{\mathrm{E}}_n(\tau)\,dsd\tau$ (18)

Now, taking the maximum-norm on both sides of Eq.(17)

$\left\|\widetilde{\mathrm{E}}_{n+1}(x)\right\|_\infty = \left\|\int_0^x \int_0^s k(s,\tau)\,\widetilde{\mathrm{E}}_n(\tau)\,dsd\tau\right\|_\infty \left\|\widetilde{\mathrm{E}}_{n+1}(x)\right\|_\infty \le \int_0^x \int_0^s \|k\|_\infty \left\|\widetilde{\mathrm{E}}_n(\tau)\right\|_\infty dsd\tau$

Since K is function bounded by c, $c \in (0,1)$, then

$\left\|\widetilde{\mathrm{E}}_{n+1}(x)\right\|_\infty \le c\int_0^x \int_0^s \left\|\widetilde{\mathrm{E}}_n(\tau)\right\|_\infty d\tau$

$\left\|\widetilde{\mathrm{E}}_{n+1}(x)\right\|_\infty = c\,x\int_0^s \left\|\widetilde{\mathrm{E}}_n(\tau)\right\|_\infty d\tau$

Therefore

$\left\|\widetilde{\mathrm{E}}_{n+1}(x)\right\|_\infty \le c\,x\int_0^s \left\|\widetilde{\mathrm{E}}_n(\tau)\right\|_\infty d\tau \qquad \forall n = 0,1,..$ (19)

Now, if n=0, then inequality (18) yield to

$\left\|\widetilde{\mathrm{E}}_1(x)\right\|_\infty \le c\,x\int_0^s \left\|\widetilde{\mathrm{E}}_0(\tau)\right\|_\infty d\tau$

$\left\|\widetilde{\mathrm{E}}_1(x)\right\|_\infty \le c\,x\int_0^s \underset{\tau}{\mathrm{Max}}\left|\widetilde{\mathrm{E}}_0(\tau)\right| d\tau$

$\left\|\widetilde{\mathrm{E}}_1(x)\right\|_\infty \le c\,x(s-0)\underset{(\tau)}{\mathrm{Max}}\left|\widetilde{\mathrm{E}}_0\right|$ (20)

Also, if n=1, then form inequality (19) and (20) we have

$\left\|\tilde{E}_2(x)\right\|_\infty \le c\,x\int_0^s \left\|\tilde{E}_1(\tau)\right\|_\infty d\tau$

Substituting (20), in this inequality we get

$\left\|\tilde{E}_2(x)\right\|_\infty \le c\,x\int_0^s cxs\,Max\left|\tilde{E}_0\right| d\tau$

$\left\|\tilde{E}_2(x)\right\|_\infty = c^2x^2\,\frac{s^2}{2}\,\mathrm{Max}\left|\tilde{E}_0\right|$ (21)

Similarly, for n=2 and from inequality (18) and (20), we have

$\left\|\tilde{E}_3(x)\right\|_\infty \le c^2x^2\int_0^s \left\|\tilde{E}_2(\tau)\right\|_\infty d\tau$

Substituting (13), in this inequality we get

$\left\|\tilde{E}_3(x)\right\|_\infty \le cx\int_0^s c^2x^2\,\frac{s^2}{2}\,\mathrm{Max}\left|\tilde{E}_0\right| d\tau$

$\left\|\tilde{E}_3(x)\right\|_\infty = c^3x^3\,\frac{s^3}{3!}\,\mathrm{Max}\left|\tilde{E}_0\right|$

And so, in general and using mathematical induction we get:

$\left\|\tilde{E}_n(x)\right\|_\infty = c^nx^n\,\frac{s^n}{n!}\,\mathrm{Max}\left|\tilde{E}_0\right|$ (22)

And since $c \in (0,1)$ and as $n \to \infty$, then we will have the right hand side of inequality Eq.(22) tends to zero, i.e, $\left\|\widetilde{\mathrm{E}}_n(x)\right\|_\infty \to 0$ as $n \to \infty$ This implies to $\tilde{u}_n(x) - \tilde{u}(x)$ as $n \to \infty$, i.e., the sequence of solution obtained from the VIM converge of the exact solution $\tilde{u}(x)$.

6- Numerical Examples:

In the section, we apply variational iteration method for solving linear fuzzy volterra- integro differential equation of the second kind.

Example (1):

Consider the fuzzy volterra integro –differential equation of the second kind

$\check{y}'(x;r) = \check{f}(x;r) + \int_0^x k(x,t)\check{y}(t;r)dt$

Where

$k(x,t) = xt$

$\underline{f}(x;r) = r\left(1 - \frac{1}{3}x^4\right)$

$\overline{f}(x;r) = 2 - r - \frac{1}{3}x^4\,(2-r)$

With the initial approximation $\underline{y}_0(x;r) = 0$ *and* $\overline{y}_0(x;r) = 0$

By using iteration formulas, we obtain

$\underline{y}_1(x;r) = \frac{-rx(x^4-15)}{15}$

$\overline{y}_1 = \frac{x^4(r-2)}{3-(r+2)}$

$$\underline{y}_2(x;r) = \frac{-rx(x^8 - 945)}{945}$$

$$\overline{y}_2(x;r) = \frac{-x(r-2)(-5x^7 - 16x^4 + 30x^3 + 240)}{240}$$

$$\vdots$$

And so on .Therefor, it is obvious that this solution is converage to exact solution

$$\underline{y}(x;r) = rx$$

$$\overline{y}(x;r) = (2-r)x$$

Following figure (1) representing the upper and Lower solution of example (1) using different value of r.

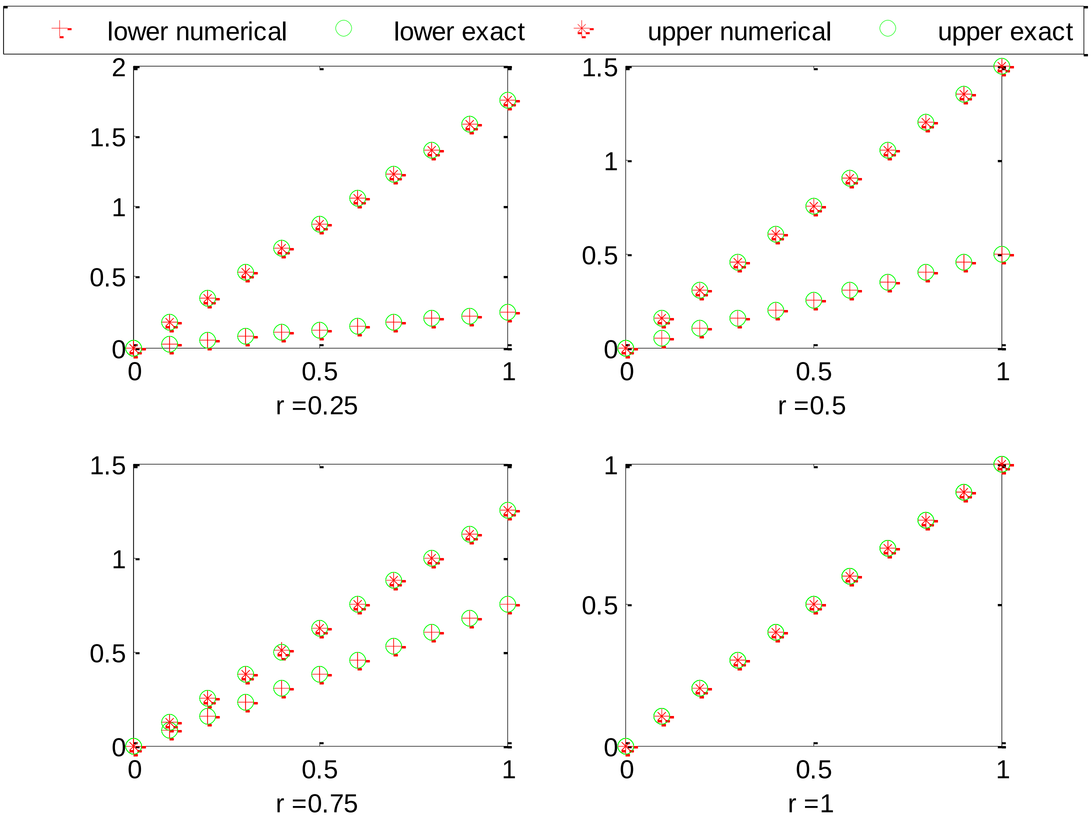

Fig. 1: Upper and lower solution of Example (1) for different value of r

Example (2):

Consider the linear fuzzy volterra integro –differential equation of the second kind

$$\check{y}'(x;r) = \check{f}\,(x;r) + \int_0^x k(x,t)\check{y}\,(t;r)dt$$

where

$$k(x,t) = 1$$

$$\underline{f}\,(x;r) = (r+1)(x+1)$$

$$\overline{f}\,(x;r) = (1+x)(2-r)$$

With the initial approximation $\underline{y}_0(x;r) = 0$ *and* $\overline{y}_0(x;r) = 0$

By using iteration formulas, we obtain

$$\underline{y}_1\,(x;r) = \frac{x(r+1)(x+2)}{2}$$

$$\overline{y}_1(x;r) = (x+1)(r-2)$$

$$\underline{y}_2(x;r) = \frac{x(r+1)(x^3 + 4x^2 + 12x + 24)}{24}$$

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$$\overline{y}_2(x;r) = \frac{x(r-2)(x^2 + 3x + 3)}{3}$$

$$\vdots$$

And so on .Therefor, it is obvious that this solution is converge to exact solution

$$\underline{y}(x;r) = (r+1)(e^x - 1)$$

$$\overline{y}(x;r) = (2-r)(e^x - 1)$$

Following figure (2) representing the upper and Lower solution of example (2) using different value of r.

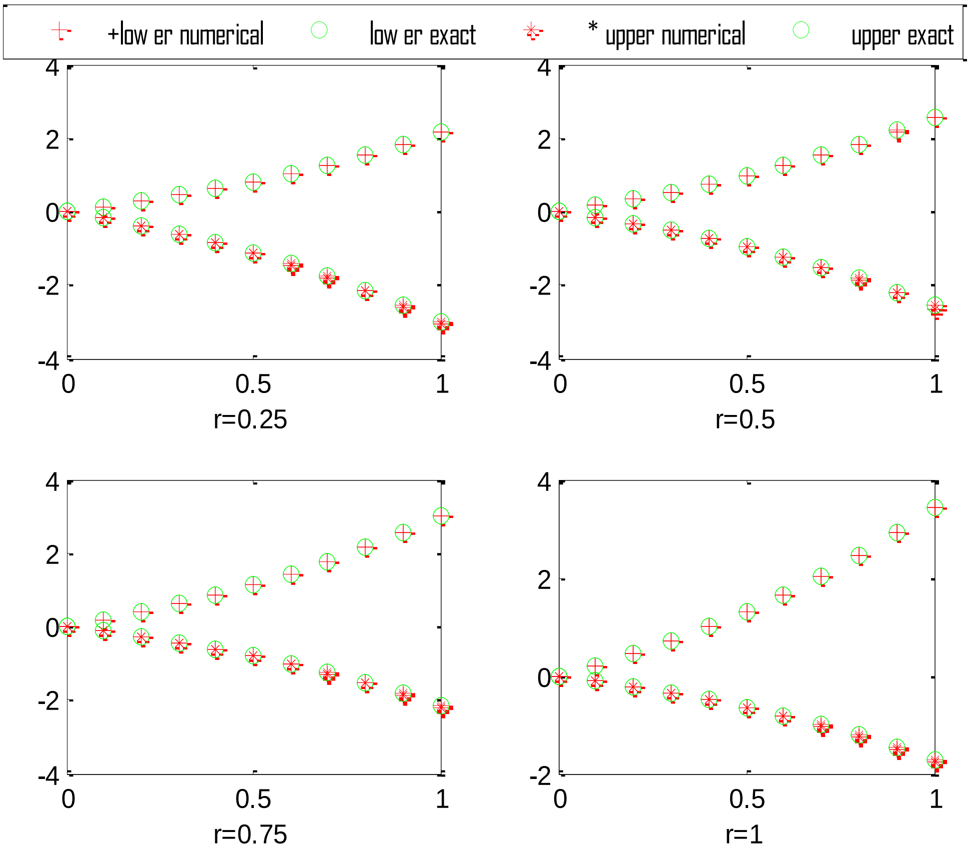

Fig. 2: Upper and lower solution of Example (2) for different value of r

Conclusion:

In this paper the variational iteration method (VIM) is used to solve the linear fuzzy volterra- integro differential equation of second kind. The results showed that the convergence and accuracy of variational iteration method for numerically solution for (LFVIDEs) where in good agreements with analytical solution. The computations associated with examples and graphing in this paper performed using matlab (V.7).

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