Australian Journal of Basic and Applied Sciences, 9(35) November 2015, Pages: 126-129

ISSN:1991-8178

Australian Journal of Basic and Applied Sciences

Journal home page: www.ajbasweb.com

Investigation of Medium Domination Number of some Special Types of Graphs

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ARTICLE INFO

Article history:
Received 10 October 2015
Accepted 30 November 2015
Available online 24 December 2015

Keywords:
AMS Subject classification (2010): 05C69, ladder graph, Medium domination number (MD), peacock head graph, uniform t-ply graph.

ABSTRACT

The concept of Medium domination number was introduced by Duygu Vargor and Pinar Dundar in [1] with real life application to protect the pairs of vertices in a graph. If dom(u, v) = 1 for u, v∈ G, then u and v dominate each other. The total dominating vertices in the graph G is $\sum dom(u,v)$ for u, v∈ V(G). The medium domination number of the graph G is the average value of the total number of vertices that dominates all pairs of vertices. Investigating the medium domination number of some special types of graph plays an vital role for communication network. In this paper, we investigate and obtain the medium domination number as a vulnerability measure of some special type of graphs, $P_t(u, v)$ – uniform t-ply graph, ladder graph and PC(n, m) – Peacock head graph.

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To Cite This Article: G. Mahadevan, V. Vijayalakshmi and C. Sivagnanam., Investigation of Medium Domination Number of some Special Types of Graphs. ***Aust. J. Basic & Appl. Sci.,*** *9(35): 126-129, 2015*

INTRODUCTION

The graphs considered here are connected, undirected, loop less. Let G = (V, E) be a graph. The path graph with n vertices (p_1, p_2,…..p_n) is denoted by P_n and the initial and terminal vertices are p_1 and p_n respectively. The cycle graph with n vertices (c_1, c_2,……c_n) is denoted by C_n. If every vertex in V-D is adjacent to atleast one vertex in D, where D⊆V, then D is called a dominating set. If dom(u, v) = 1 for u, v∈ V(G), then u and v are dominate each other. $\sum dom(u,v)$ for u, v∈ V(G) is the total dominating vertices in the graph G and this value is called TDV(G). $\frac{TDV(G)}{\binom{p}{2}}$ is the medium domination number of the graph G, where p is the total number of vertices in the graph G. In this paper, we investigate the medium domination number for uniform t-ply graph, ladder graph and peacock head graph.

Definition 1.1:

Let G = (V, E) be a graph, V, E be the vertex set and edge set respectively. If dom(u, v) = 1, then u and v dominate each other.

Definition 1.2:

Let G be the graph. The total number of vertices that dominate every pair of vertices TDV(G) = $\sum dom(u,v)$ for u, v∈ V(G).

Definition 1.3:

In any simple graph G of p number of vertices, the medium domination number of G is defined as MD(G) = $\frac{TDV(G)}{\binom{p}{2}}$

Example 1.4:

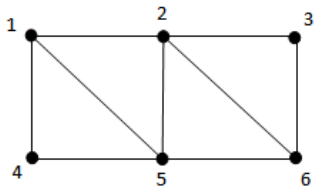

Fig. 1.1:

Form the above figure 1.1, we have, dom(1, 2) = 2; dom(1, 3) = 1; dom(1, 4) = 2; dom(1, 5) =3; dom(1, 6) = 2; dom(2, 3) = 2; dom(2, 4) = 2; dom(2, 5) = 3; dom(2, 6) = 3; dom(3, 5) = 2; dom(3, 6) = 2; dom(4, 5) = 2; dom(4, 6) = 1; dom(5, 6) = 2. TDV(G) = 29; MD(G) = $\frac{29}{15}$.

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Definition 1.5:

The peacock head graph is obtained by joining n pendent edges to any one vertex of the cycle C_m and it is denoted by PC(n, m).

Definition 1.6:

A uniform t-ply graph is a graph with t paths, each path of length at least two and no two paths have a common vertex other than u and v. Initial vertex of all paths joining to u and end vertices of all paths joining to v and it is denoted by $P_t(u, v)$.

Definition 1.7:

Let G = (V, E) be a ladder graph L_k such that V={A_1 , A_2,…….A_k, B_1, B_2,……B_k}, $E = E_1 \cup E_2 \cup E_3$, where E_1= {(A_i, A_{i+1}), 1≤ i≤ k-1}, E_2 = {(B_i, B_{i+1}), 1≤ i≤ k-1}, E_3 ={(A_i, B_i),1≤i≤k}.

II. Main Result:

In this section, we discuss the Medium domination number of some special type of graphs.

Theorem 2.1 Let G be the peacock head graph PC(n, m) then MD(G) is $\frac{n^2+5n+4m}{2\binom{n+m}{2}}$

Proof:

Let (1, 2,…..i,….m) be the vertices of the cycle C_m. (x_1, x_2,….x_n) be the pendent vertices of the star $K_{1,n}$. Now attach the root vertex of the star $K_{1,n}$ to i^{th} vertex of the cycle C_m. TDV(G)=$\sum dom(u,v)$ for u, v∈ V(G). For any cycle C_m, TDV(C_m) = 2m for any m.

dom(i, x_j) = 1 for j = 1 to n; therefore $\sum_{j=1}^{n}$ dom(i, x_j) = n;

dom(i±1, x_j) = 1 for j = 1 to n; therefore $\sum_{j=1}^{n}$ dom(i±1, x_j) = 2n;

dom(x_1, x_i) = 1 for i = 2 to n; therefore $\sum_{i=2}^{n}$ dom(x_1, x_i) = n-1;

dom(x_2, x_i) = 1 for i = 3 to n; therefore $\sum_{i=2}^{n}$ dom(x_2, x_i) = n-2;…….

dom(x_{n-1}, x_n) = 1.

Therefore, TDV(G) = 2m + n + 2n + [(n-1)+(n-2)+……+1]

= $3n + 2m + \frac{n(n-1)}{2} = \frac{n^2+5n+4m}{2}$

MD(G) = $\frac{TDV(G)}{\binom{p}{2}} = \frac{n^2+5n+4m}{2\binom{n+m}{2}}$

Example 2.2, for the graph PC(4, 5),

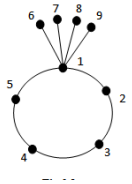

Fig. 2.1:

From the above figure 2.1, we have, dom(1, 2) = 1; dom(1, 3) = 1; dom(1, 4) = 1; dom(1, 5) = 1; dom(1, 6) = 1; dom(1, 7) = 1; dom(1, 8) = 1; dom(1, 9) = 1; dom(2, 3) = 1; dom(2, 4) = 1; dom(2, 5) = 1; dom(2, 6) = 1; dom(2, 7) = 1; dom(2, 8) = 1; dom(2, 9) = 1; dom(3, 4) = 1; dom(3, 5) = 1; dom(4, 5) = 1; dom(5, 6) = 1; dom(5, 7) = 1; dom(5, 8) = 1; dom(5, 9) = 1; dom(6, 7) = 1; dom(6, 8) = 1; dom(6, 9) = 1; dom(7, 8) = 1; dom(7, 9) = 1; dom(8, 9) = 1. TDV(G) = 28; MD(G) = $\frac{28}{36}$.

TDV(G) = $\frac{n^2+5n+4m}{2} = \frac{16+20+20}{2} = 28.$

MD(G)= $\frac{TDV(G)}{\binom{p}{2}} = \frac{28}{\binom{9}{2}} = \frac{28}{36}$.

Theorem 2.3 Let G be the uniform t-ply graph $P_t(u, v)$ then MD(G) is $\frac{t^2+2st}{\binom{st+2}{2}}$.

Proof:

Let (i1, i2,…..is) be the vertices of the i^{th} path P_s for i = 1 to t. Join the initial vertices (11, 21,…..t1) to the vertex u and the terminal vertices (1s, 2s,……..ts) to the vertex v. TDV(G) = $\sum dom(u,v)$ for u, v∈ V(G). For any path P_m, TDV(P_m) = 2m-3 for any m. we have t copy of P_s.

dom(u, i1) = 1 for i = 1 to t; therefore $\sum_{i=1}^{t}$ dom(u, i1) = t;

dom(u, i2) = 1 for i = 1 to t; therefore $\sum_{i=1}^{t}$ dom(u, i2) = t;

dom(v, is) = 1 for i = 1 to t; therefore $\sum_{i=1}^{t}$ dom(v, is) = t;

dom(v, i(s-1)) = 1 for i = 1 to t; therefore $\sum_{i=1}^{t}$ dom(v, i(s-1)) = t;

dom(11, i1) = 1 for i = 2 to t; therefore $\sum_{i=2}^{t}$ dom(11, i1) = t-1;

dom(21, i1) = 1 for i = 3 to t; therefore $\sum_{i=3}^{t}$ dom(21, i1) = t-2;……

dom((t-2)1, i1) = 1 for i = t-1,t; therefore $\sum_{i=t-1}^{t}$ dom((t-2)1, i1) = 2;
dom((t-1)1, i1) = 1 for i = t;
dom(1s, is) = 1 for i = 2 to t; therefore $\sum_{i=2}^{t}$ dom(1s, is) = t-1;
dom(2s, is) = 1 for i = 3 to t; therefore $\sum_{i=3}^{t}$ dom(2s, is) = t-2;……
dom((t-2)s, is) = 1 for i = t-1, t; therefore $\sum_{i=t-1}^{t}$ dom((t-2)s, is) = 2;
dom((t-1)s, is) = 1 for i = t.

TDV(G) = t(2s-3) + 4t + [1+2+…….+(t-1)] + [1+2+……+(t-1)]
= 2st - 3t + 4t + $2\left(\frac{t(t-1)}{2}\right)$ = $2st+t^2$

MD(G)= $\frac{TDV(G)}{\binom{p}{2}}$ = $\frac{t^2+2st}{\binom{st+2}{2}}$

Example 2.4, for the graph $P_3(u, v)$:

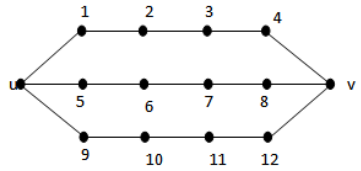

Fig. 2.2:

From the above figure 2.2, we have dom(u, 1) = 1; dom(u, 5) = 1; dom(u, 9) = 1; dom(u, 2) = 1; dom(u, 6) = 1; dom(u, 10) = 1; dom(v, 4) = 1; dom(v, 8) = 1; dom(v, 12) = 1; dom(v, 3) = 1; dom(v, 7) = 1; dom(v, 11) = 1; dom(1, 2) = 1; dom(1, 3) = 1; dom(2, 3) = 1; dom(2, 4) = 1; dom(3, 4) = 1; dom(5, 6) = 1; dom(5, 7) = 1; dom(6, 7) = 1; dom(6, 8) = 1; dom(7, 8) = 1; dom(9, 10) = 1; dom(9, 11) = 1; dom(10, 11) = 1; dom(10, 12) = 1; dom(11, 12) = 1; dom(1, 5) = 1; dom(1, 9) = 1; dom(5, 9) = 1; dom(4, 8) = 1; dom(4, 12) = 1; dom(8, 12) = 1.
TDV(G) = 33; MD(G) = $\frac{33}{91}$.
TDV(G) = $2st + t^2$ = $2(3)(4)+3^2$ = 33
MD(G) = $\frac{TDV(G)}{\binom{p}{2}}$ = $\frac{33}{\binom{14}{2}}$ = $\frac{33}{91}$.

Theorem 2.5 Let G be a ladder graph L_k then MD(G) is $\frac{9k-10}{\binom{2k}{2}}$

Proof:

Let (A1, A2,……Ak) and (B1, B2,….Bk) be the vertices of two distinct paths with k vertices. Now join the vertices Ai to Bi for i = 1 to k. TDV(G) = $\sum dom(u,v)$ for u, v∈ V(G). For any path P_m, TDV(P_m) = 2m-3 for any m. we have two paths P_k.
dom(Ai, Bi) = 1 for i = 1 to k; therefore $\sum_{i=1}^{k}$ dom(Ai, Bi) = k;
dom(Ai, Bi+1) = 2 for i = 1 to k-1; therefore $\sum_{i=1}^{k-1}$ dom(Ai, Bi+1) = 2(k-1);
dom(Ai, Bi-1) = 2 for i = 2 to k; therefore $\sum_{i=2}^{k}$ dom(Ai, Bi-1) = 2(k-1).
TDV(G) = 2(2k-3) + k + 4(k-1) = 9k - 10.
MD(G) = $\frac{TDV(G)}{\binom{p}{2}}$ = $\frac{9k-10}{\binom{2k}{2}}$

Example 2.6, for the graph $L_{5,:}$

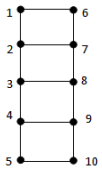

Fig. 2.3:

From the above figure 2.3, we have dom(1, 2) = 1; dom(1, 3) = 1; dom(2, 3) = 1; dom(2, 4) = 1; dom(3, 4) = 1; dom(3, 5) = 1; dom(4, 5) = 1; dom(6, 7) = 1; dom(6, 8) = 1; dom(7, 8) = 1; dom(7, 9) = 1; dom(8, 9) = 1; dom(8, 10) = 1; dom(9, 10) = 1; dom(1, 6) = 1; dom(2, 7) = 1; dom(3, 8) = 1; dom(4, 9) = 1; dom(5, 10) = 1; dom(1, 7) = 2; dom(2, 6) = 2; dom(2, 8) = 2; dom(3, 7) = 2; dom(3, 9) = 2; dom(4, 8) = 2; dom(4, 10)=2; dom(5, 9) = 2.
TDV(G) = 35; MD(G) = $\frac{35}{45}$.
TDV(G) = 9k-10 = 9(5)-10 = 35.
MD(G)= $\frac{TDV(G)}{\binom{p}{2}}$ = $\frac{35}{\binom{10}{2}}$ = $\frac{35}{45}$

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