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Global Conharmonic Concept Type Nerly Kahler Manifold

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ABSTRACT

The concept of permanence conharmonic type Nerely Kahler manifold conditions are obtained when the Nerely Kahler manifold is a manifold conharmonic constant type. Proved that the local conharmonic constancy of type Nerely Kahler manifold is equivalent to its global conharmonic constancy of type. And also proved that the Nerely Kahler manifold conharmonic constant type is a manifold of constant scalar curvature.

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INTRODUCTION

The notion of constancy of type Nerely Kahler manifolds was introduced A.Greem (Gray, 1970; Gray, 1976; Gray, 1969) and has proved very useful in the study of the geometry of nerly kahler manifolds. Next nerly kahler manifolds of constant type considered various mathematicians (see. Gray, 1970; Gray, 1969; Kirichenko, 1975; Kirichenko, 1976; Kirichenko, 1981). Comprehensive description of Nerely Kahler manifolds of constant type was obtained V.F.Kirichenko (Kirichenko, 1976). In (Kirichenko, 1976) it is shown that Nerely Kahler manifold of pointwise constant type description by the identity $B_{abh}B^{hcd} = B\delta_{ab}^{cd}$, where B - a function on Nerely Kahler manifold, $\delta_{ab}^{cd} = \delta_a^c\delta_b^d - \delta_a^d\delta_b^c$. Wherein local constancy of type Nerely Kahler manifold is equivalent to the local constancy of its type. Moreover, the covariant constancy of structure tensors of the first and second kind it follows immediately that, the constancy of type at only one point Nerely Kahler space implies global consistency of its type.

Recall the definitions.

Let M^{2n} - Nerely Kahler manifold with AH - structure {g, J} ∇ - Riemannian connection of the metric g.

Definition 1. (Gray, 1969). Nerely Kahler manifold M^{2n} has constant type at a point $p \in M^{2n}$ if $\forall X,Y,Z \in T(M^{2n})$: $\langle X,Y\rangle = \langle JX,Y\rangle = \langle X,Z\rangle = 0$;

$$\|Y\| = \|Z\| \Rightarrow \|\nabla_X (J)(Y)\| = \|\nabla_X (J)(Z)\|.$$

(Here $T(M^{2n})$ - tangent space M^{2n} in $p \in M^{2n}$.)

Definition 2 (Gray, 1969). Nerely Kahler -manifold having a constant type at each point, called NK-manifold of pointwise constant type.

Definition 3 [3]. Nerely Kahler manifold M^{2n} pointwise constant type is globally constant type if $\forall X,Y \in X(M^{2n})$: $\langle X,Y\rangle = \langle JX,Y\rangle = 0$;

$$\|X\| = \|Y\| = 1 \Rightarrow \|\nabla_X (J)(Y)\| = const.$$

1

These concepts have been summarized L. Vanhecke and Boutenom for arbitrary almost Hermitian manifolds (Vanhecke, 1977). As shown by L. Vanhecke, if the almost Hermitian manifold belongs to Nerely Kahler manifolds, these definitions are reduced to the above definitions A.Greya.

It is now natural to introduce the definition of constant type, similar definitions Vanhecke, replacing the tensor R Riemann-Christoffel tensor conharmonic curvature.

Let M^{2n} - Nerely Kahler manifold. To calculate the (X, Y, Z, W) and K (X, Y, JX, JY) on the space of the associated G-structure:

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$$K(X,Y,Z,W)=K_{ijkl}X^iY^jZ^kW^l=K_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^bZ^cW^{\hat{d}}+$$

$$+K_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^bZ^{\hat{c}}W^d+K_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Z^cW^d+K_{a\hat{b}c\hat{d}}X^aY^{\hat{b}}Z^cW^{\hat{d}}+$$

$$+K_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}Z^{\hat{c}}W^d+K_{ab\hat{c}\hat{d}}X^aY^bZ^{\hat{c}}W^{\hat{d}};$$

$$K(X,Y,JX,JY)=K_{ijkl}X^iY^j(JX)^k(JY)^l=K_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^bX^cY^{\hat{d}}+$$

$$+K_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^bX^{\hat{c}}Y^d-K_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}X^cY^d+K_{a\hat{b}c\hat{d}}X^aY^{\hat{b}}X^cY^{\hat{d}}+$$

$$+K_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}X^{\hat{c}}Y^d-K_{ab\hat{c}\hat{d}}X^aY^bX^{\hat{c}}Y^{\hat{d}}.$$

We show, $K(X,Y,JX,JY)-K(X,Y,X,Y)=-4K_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}X^cY^d$

In this way,. Given the specter tensor of curvature conharmonic last equation can be written as (Kirichenko, 2011):

$$K(X,Y,JX,JY)-K(X,Y,X,Y)=[-8B_{cd}^{ab}-\frac{2}{n-1}\{\delta_d^a\left(A_c^b+3B_c^b\right)-$$

$$-\delta_c^a\left(A_d^b+3B_d^b\right)+\delta_c^b\left(A_d^a+3B_d^a\right)-\delta_d^b\left(A_c^a+3B_c^a\right)\}]X^{\hat{a}}Y^{\hat{b}}X^cY^d.$$

Since $c\|X\|^2\|Y\|^2=4c\delta_{cd}^{ab}X^cY^dX_aY_b$ then

$$-8B_{cd}^{ab}-\frac{2}{n-1}\{\delta_d^a\left(A_c^b+3B_c^b\right)-\delta_c^a\left(A_d^b+3B_d^b\right)+\delta_c^b\left(A_d^a+3B_d^a\right)-$$

$$-\delta_d^b\left(A_c^a+3B_c^a\right)\}=4c\delta_{cd}^{ab},$$ ie

$$B^{abh}B_{hcd}+\frac{1}{4(n-1)}\{\delta_d^a\left(A_c^b+3B_c^b\right)-\delta_c^a\left(A_d^b+3B_d^b\right)+$$

$$+\delta_c^b\left(A_d^a+3B_d^a\right)-\delta_d^b\left(A_c^a+3B_c^a\right)\}=-\frac{1}{2}c\delta_{cd}^{ab}.$$ (1)

We recall some useful formulas, we need, is easily obtained from the first group of structural equations Nerely Kahler manifolds (Kirichenko, 1976; Kirichenko, 1981):

$$1)\ dA_b^a=dA_{bh}^{ah}=A_{bhc}^{ah}\omega^c+A_{bh}^{ahc}\omega_c+A_c^a\theta_b^c-A_b^c\theta_c^a;$$

$$2)\ dB_{bc}^{ad}=-B_{bc}^{hd}\theta_h^a-B_{bc}^{ah}\theta_h^d+B_{hc}^{ad}\theta_b^h+B_{bh}^{ad}\theta_c^h;$$ (2)

$$3)\ dB_b^a=B_c^a\theta_b^c-B_b^c\theta_c^a.$$

Theorem 1. Nerely Kahler manifold is a manifold conharmonic constant type if and only if

$$B^{abh}B_{hcd}+\frac{1}{4(n-1)}\{\delta_d^a\left(A_c^b+3B_c^b\right)-\delta_c^a\left(A_d^b+3B_d^b\right)+$$

$$+\delta_c^b\left(A_d^a+3B_d^a\right)-\delta_d^b\left(A_c^a+3B_c^a\right)\}=-\frac{1}{2}c\delta_{cd}^{ab}.$$

The proof:

We differentiate externally with regard to (2) Equation (1):

$$
\begin{aligned}
&- B^{fdh}B_{abh}\theta_f^c - B^{cfh}B_{abh}\theta_f^d + B^{cdh}B_{fbh}\theta_a^f + B^{cdh}B_{bfh}\theta_b^f + \\
&+\frac{1}{4(n-1)}\delta_a^c\left\{A_{bhg}^{dh}\omega^g + A_{bh}^{dhg}\omega_g + \left(A_f^d + 3B_f^d\right)\theta_b^f - \left(A_b^f + 3B_b^f\right)\theta_f^d\right\}+ \\
&+\frac{1}{4(n-1)}\delta_b^d\left\{A_{ahg}^{ch}\omega^g + A_{ah}^{chg}\omega_g + \left(A_f^c + 3B_f^c\right)\theta_a^f - \left(A_a^f + 3B_a^f\right)\theta_f^c\right\}- \\
&-\frac{1}{4(n-1)}\delta_a^d\left\{A_{bhg}^{ch}\omega^g + A_{bh}^{chg}\omega_g + \left(A_f^c + 3B_f^c\right)\theta_b^f - \left(A_b^f + 3B_b^f\right)\theta_f^c\right\}- \\
&-\frac{1}{4(n-1)}\delta_b^c\left\{A_{ahg}^{dh}\omega^g + A_{ah}^{dhg}\omega_g + \left(A_f^d + 3B_f^d\right)\theta_a^f - \left(A_a^f + 3B_a^f\right)\theta_f^d\right\}= \\
&\qquad = -\frac{1}{2}\delta_{ab}^{cd}dc.
\end{aligned}
\tag{3}
$$

Since, $c \in C^\infty(M)$ dc (where, $d(\pi^* c) = \pi^*(dc)$) is a horizontal form, and $dc = c_a\omega^a + c^a\omega_a$ therefore. Then (3) can be written as

$$
\begin{aligned}
&- B^{fdh}B_{abh}\theta_f^c - B^{cfh}B_{abh}\theta_f^d + B^{cdh}B_{fbh}\theta_a^f + B^{cdh}B_{bfh}\theta_b^f + \\
&+\frac{1}{4(n-1)}\delta_a^c\left\{A_{bhg}^{dh}\omega^g + A_{bh}^{dhg}\omega_g + \left(A_f^d + 3B_f^d\right)\theta_b^f - \left(A_b^f + 3B_b^f\right)\theta_f^d\right\}+ \\
&+\frac{1}{4(n-1)}\delta_b^d\left\{A_{ahg}^{ch}\omega^g + A_{ah}^{chg}\omega_g + \left(A_f^c + 3B_f^c\right)\theta_a^f - \left(A_a^f + 3B_a^f\right)\theta_f^c\right\}- \\
&-\frac{1}{4(n-1)}\delta_a^d\left\{A_{bhg}^{ch}\omega^g + A_{bh}^{chg}\omega_g + \left(A_f^c + 3B_f^c\right)\theta_b^f - \left(A_b^f + 3B_b^f\right)\theta_f^c\right\}- \\
&-\frac{1}{4(n-1)}\delta_b^c\left\{A_{ahg}^{dh}\omega^g + A_{ah}^{dhg}\omega_g + \left(A_f^d + 3B_f^d\right)\theta_a^f - \left(A_a^f + 3B_a^f\right)\theta_f^d\right\}= \\
&= -\frac{1}{2}\delta_{ab}^{cd}\left(c_g\omega^g + c^g\omega_g\right).
\end{aligned}
$$

The last equality can be rewritten as:

$$
\begin{aligned}
&-\left[B^{fdh}B_{abh} + \frac{1}{4(n-1)}\left\{\delta_b^d\left(A_a^f + 3B_a^f\right) + \delta_a^f\left(A_b^d + 3B_b^d\right) - \delta_a^d\left(A_b^f + 3B_b^f\right) - \delta_b^f\left(A_a^d + 3B_a^d\right)\right\}\right]\theta_f^c - \\
&-\left[B^{cfh}B_{abh} + \frac{1}{4(n-1)}\left\{\delta_a^c\left(A_b^f + 3B_b^f\right) + \delta_b^f\left(A_a^c + 3B_a^c\right) - \delta_b^c\left(A_a^f + 3B_a^f\right) - \delta_a^f\left(A_b^c + 3B_b^c\right)\right\}\right]\theta_f^d + \\
&+\left[B^{cdh}B_{fbh} + \frac{1}{4(n-1)}\left\{\delta_b^d\left(A_f^c + 3B_f^c\right) + \delta_f^c\left(A_b^d + 3B_b^d\right) - \delta_f^d\left(A_b^c + 3B_b^c\right) - \delta_b^c\left(A_f^d + 3B_f^d\right)\right\}\right]\theta_a^f + \\
&+\left[B^{cdh}B_{afh} + \frac{1}{4(n-1)}\left\{\delta_a^c\left(A_f^d + 3B_f^d\right) + \delta_f^d\left(A_a^c + 3B_a^c\right) - \delta_f^c\left(A_a^d + 3B_a^d\right) - \delta_a^d\left(A_f^c + 3B_f^c\right)\right\}\right]\theta_b^f + \\
&+\frac{1}{4(n-1)}\left\{\left[\delta_a^f\left(A_b^d + 3B_b^d\right) - \delta_b^f\left(A_a^d + 3B_a^d\right)\right]\theta_f^c + \left[\delta_b^f\left(A_a^c + 3B_a^c\right) - \delta_a^f\left(A_b^c + 3B_b^c\right)\right]\theta_f^d\right\}- \\
&-\frac{1}{4(n-1)}\left\{\left[\delta_f^c\left(A_b^d + 3B_b^d\right) - \delta_f^d\left(A_b^c + 3B_b^c\right)\right]\theta_a^f - \left[\delta_f^d\left(A_a^c + 3B_a^c\right) - \delta_f^c\left(A_a^d + 3B_a^d\right)\right]\theta_b^f\right\}+ \\
&+\frac{1}{4(n-1)}\left\{\left(\delta_a^c A_{bgh}^{dg} + \delta_b^d A_{agh}^{cg} - \delta_a^d A_{bgh}^{cg} - \delta_b^c A_{agh}^{dg}\right)\omega^h + \left(\delta_a^c A_{bg}^{dgh} + \delta_b^d A_{ag}^{cgh} - \delta_a^d A_{bg}^{cgh} - \delta_b^c A_{ag}^{dgh}\right)\omega_h\right\}= \\
&= -\frac{1}{2}\delta_{ab}^{cd}\left(c_h\omega^h + c^h\omega_h\right),
\end{aligned}
$$

which in view of (1) takes the form

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$$\frac{c}{2}\delta_{ab}^{fd}\theta_f^c+\frac{c}{2}\delta_{ab}^{cf}\theta_f^d-\frac{c}{2}\delta_{fb}^{cd}\theta_a^f-\frac{c}{2}\delta_{af}^{cd}\theta_b^f+\frac{1}{4(n-1)}\left\{\left(A_b^d+3B_b^d\right)\theta_a^c-\left(A_a^d+3B_a^d\right)\theta_b^c\right\}+$$

$$+\frac{1}{4(n-1)}\left\{\left(A_a^c+3B_a^c\right)\theta_b^d-\left(A_b^c+3B_b^c\right)\theta_a^d\right\}-$$

$$-\frac{1}{4(n-1)}\left\{\left(A_b^d+3B_b^d\right)\theta_a^c-\left(A_b^c+3B_b^c\right)\theta_a^d-\left(A_a^c+3B_a^c\right)\theta_b^d+\left(A_a^d+3B_a^d\right)\theta_b^c\right\}+$$

$$+\frac{1}{4(n-1)}\left\{\left(\delta_a^c A_{bgh}^{dg}+\delta_b^d A_{agh}^{cg}-\delta_a^d A_{bgh}^{cg}-\delta_b^c A_{agh}^{dg}\right)\omega^h+\left(\delta_a^c A_{bg}^{dgh}+\delta_b^d A_{ag}^{cgh}-\delta_a^d A_{bg}^{cgh}-\delta_b^c A_{ag}^{dgh}\right)\omega_h\right\}=$$

$$=-\frac{1}{2}\delta_{ab}^{cd}\left(c_h\omega^h+c^h\omega_h\right),$$

and we have

$$1)\ \frac{1}{2(n-1)}\left(\delta_a^d A_{bgh}^{cg}+\delta_b^c A_{agh}^{dg}-\delta_a^c A_{bgh}^{dg}-\delta_b^d A_{agh}^{cg}\right)=\delta_{ab}^{cd}c_h;$$

$$2)\ \frac{1}{2(n-1)}\left(\delta_a^d A_{bg}^{cgh}+\delta_b^c A_{ag}^{dgh}-\delta_a^c A_{bg}^{dgh}-\delta_b^d A_{ag}^{cgh}\right)=\delta_{ab}^{cd}c^h. \quad (4)$$

This theorem proved.

Theorem 2. Local conharmonic constant type Nerely Kahler manifold is equivalent to its global conharmonic constant type.

The proof:

From (4) we have

$$1)\ \frac{1}{2(n-1)}\left(\delta_a^d A_{bgh}^{cg}+\delta_b^c A_{agh}^{dg}-\delta_a^c A_{bgh}^{dg}-\delta_b^d A_{agh}^{cg}\right)=\delta_{ab}^{cd}c_h;$$

$$2)\ \frac{1}{2(n-1)}\left(\delta_a^d A_{bg}^{cgh}+\delta_b^c A_{ag}^{dgh}-\delta_a^c A_{bg}^{dgh}-\delta_b^d A_{ag}^{cgh}\right)=\delta_{ab}^{cd}c^h.$$

Rolling recent last equality in the first by indices a and c, and then the indices b and d, we obtain:

$$1)\ c_h=\frac{1}{n(n-1)}A_{abh}^{ab};\quad 2)\ c^h=\frac{1}{n(n-1)}A_{ab}^{abh}. \quad (5)$$

Rolling equation (4), first in the indices a and d, and then the indices b and c, we get:

$$1)\ c_h=-\frac{1}{n(n-1)}A_{abh}^{ab};\quad 2)\ c^h=-\frac{1}{n(n-1)}A_{ab}^{abh}. \quad (6)$$

Comparing (5) and (6) we find that: $c_h=c^h=0$.

This theorem proved.

Theorem 3. Nerely Kahler manifold conharmonic constant type is a manifold of constant scalar curvature.

The proof:

In particular, from (6) it follows that. From $A_{abh}^{ab}=A_{ab}^{abh}=0$ this and from (2: 1) it follows that

$$dA=dA_a^a=dA_{ab}^{ab}=A_{abc}^{ab}\omega^c+A_{ab}^{abc}\omega_c+A_c^a\theta_a^c-A_a^c\theta_c^a=0.$$

From (2: 3), we have $dB=dB_a^a=B_c^a\theta_a^c-B_a^c\theta_c^a=0$. So from the last two equations, we have that $d\chi=2dA+6dB=0$, that $\chi=const$

Thus, we have the following theorem.

This theorem proved.

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