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Group Generated By A Centroidempotent and Counter Identity Matrix

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ABSTRACT

A centroidempotent matrix is introduced similar to the definitions of centrosymmetric and centrohermitian matrices in the literature. The eigen values of a centroidempotent matrix are determined. The basic properties of group generated by counter identity matrix and a centroidempotent matrix are derived. The Moore Penrose inverses of each element in the group are determined.

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INTRODUCTION

A centrosymmetric matrix (aij) of order n remain unchanged when the corresponding elements aij and a(n−i+1)(n−j+1) are interchanged. The ba- sic properties of such matrices were investigated by James R.Weaver in (Weaver, J.R., 1985) and spectral properties of centrosymmetric matrices were found in (Cantoni, A. and P. Butler, 1976). This work was further extended to the definitions of perhermitian, centrohermi- tian and k-hermitian matrices (Hill, R.D., 1990; Hill, R.D., 1990; Hill, R.D. and S.R. Waters, 1992). Let Cn×n denotes the set of all complex matrices of order n. A matrix A ∈ Cn×n is centrosymmetric if and only if J AJ = A. the identity matrix. R.D.Hill et al (1990) define a matrix 'A' is centrohermi- tian if and only if J A∗J = A, where A∗ denotes the conjugate transpose of the matrix A. A matrix A = (aij) ∈ Cn×n is called centroidempotent if t=1 a(n−i+1)t at(n−j+1) = aij for all i and j in {1, 2, .., n}. Equivalently, a matrix 'A' is centroidempotent if and only if J A2J = A.

Preliminaries:

Definition 2.1:

A matrix A is called square hermitian(skew square hermitian) if and only if $A^* = A^2$($A^* = -A^2$).

Definition 2.2:

An n×m matrix X that satisfies the following equations AXA = A, XAX = X, $(AX)^*$ = AX and $(XA)^*$ = XA is called a Moore Penrose inverse of a matrix A ∈ Cm×n. It can be prove that it is unique and denoted by A†.

Definition 2.3:

An n× n matrix X that satisfies the following equations AXA = A, XAX = X and AX = XA is called a group inverse of a matrix A ∈ Cn×n. It can be prove that it is unique and denoted by A^0.

Definition 2.4:

A matrix A is said to be range hermitian if range(A) = range(A^*). It is also known an Equiprojector matrix or simply EP matrix

Remark 2.5:

A is range hermitian if and only if A† = A^0

Remark 2.6:

If A is centroidempotent matrix then it can be proved that A∗ is also centroidempotent. The following relations are obtained from the definition of centroidempotent matrix, which would be useful in computational aspects.

$JA = A^2J$ or $JA^2 = AJ$
$JA^3 = A^3J$ or $JA^3J = A^3$
$A^3 = (JA)^2$ or $A^3 = (AJ)^2$

Theorem 2.1. If A is centroidempotent matrix then the following are true.

A^4 = A. (ii) A^3 is is idempotent (iii) JA and AJ are tripotent matrices.

Proof. $A^4 = A^2A^2 = JAJJAJ = JA^2J = A$.
$(A^3)^2 = A^4A^2 = AA^2 = A^3$.
$(JA)^3 = (JA)^2JA = A^3JA = JA^3A = JA$.

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Main Results:

Theorem 3.1. *If a matrix A is centroidempotent then the eigen values of A are* 0, 1, ω, ω^2 *where* $\omega = e^{3}$.

Proof. For a non zero vector $x, Ax = \lambda x$

$A^4x = \lambda^4 x \; Ax = \lambda^4 x$

$(\lambda^4 - \lambda)x = 0$

$\lambda = 0, 1, \omega, \omega^2$

Theorem 3.2. *Let* $n_i (i = 0, 1, 2)$ *denote the multiplicity of eigenvalues 0,1 and* (ω, ω^2) *respectively of a centroidempotent matrix A of order n. Then,*

$trA = n1 - n2$ *(ii)* $\det A = 0$ *or* 1 *(iii)* $rankA = 3n1 - 2\ trA$

Proof. It is clear that $n = n0 + n1 + 2n2$ $(i) trA = 0n0 + 1n1 + \omega n2 + \omega^2 n2 = n1 - n2$

If there is atleast one eigen value zero then $detA = 0$ otherwise $n0 = 0$.

$detA = 1^{n1}\ \omega^{n2}\ (\omega^2)^{n2} = (\omega^3)^{n2} = 1$

$(iii) rankA = n - n0 = n1 + 2n2 = 3n1 - 2trA$

Theorem 3.3. *If A is a centroidempotent matrix then*

$tt = \dot{} A, A^2, A^3, JA, AJ, JA^3\dot{}$

is a group generated by A and J under matrix multiplication.

Proof. An easy proof may be given. Here A^3 acts as the identity element.

Corollary:

If A is non singular centroidempotent matrix then $A^2 = A^{-1}$ and $A^3 = I$.

Theorem 3.4. *If A is a centroidempotent matrix then* $A^{\circ} = A^2, (A^2)^{\circ} = A, (A^3)^{\circ} = A^3, (JA)^{\circ} = JA, (AJ)^{\circ} = AJ$ *and* $(JA^3)^{\circ} = JA^3$

Proof. Since $rank(A^2) = rank(A)$, group inverse exists. It is evident from theorem 2.1 that

$A^0 = A^2$. A similar proof may be given for the remaining.

Theorem 3.5. *If A is a centroidempotent matrix and* $A^{\dagger}$ *is the correspond-*

ing Moore Penrose inverse of A then (1) $(A^2)^{\dagger} = JA^{\dagger}J$

(2) $(A^3)^{\dagger} = A^{\dagger}A^2A^{\dagger}$

(3) $(JA)^{\dagger} = A^{\dagger}J$

(4) $(AJ)^{\dagger} = J A^{\dagger}$

(5) $(J A^3)^{\dagger} = A^{\dagger}A^2A^{\dagger}J$

Proof. The Moore Penrose inverse for various elements can be proved one by one.

(1) $A^2J\ A^{\dagger}J\ A^2 = J\ AA^{\dagger}AJ = J\ AJ = A^2$

$J\ A^{\dagger}J\ A^2\ J\ A^{\dagger}J = J\ A^{\dagger}AA^{\dagger}J = J\ A^{\dagger}J$

$(A^2J\ A^{\dagger}J)^* = (J\ AA^{\dagger}J)^* = J\ (AA^{\dagger})^*J = J\ AA^{\dagger}J = A^2J\ A^{\dagger}J$

$(J\ A^{\dagger}J\ A2)^* = (J\ A^{\dagger}AJ)^* = J\ (A^{\dagger}A)^*J = J\ A^{\dagger}AJ = J\ A^{\dagger}J\ A^2$

(2) $A^3A^{\dagger}A^2\ A^{\dagger}A^3 = A + AAA^2 = A^3$

$A\dagger A2\ A\dagger A3A\dagger A2A\dagger = A\dagger AAAAAA\dagger = A\dagger A2A\dagger$

$(A^3A^{\dagger}A^2A\dagger)^* = (A^2\ AAA\dagger)^* = (AA^{\dagger})^* = AA^{\dagger} = A^3AA^{\dagger} = A^3A^{\dagger}A^2A^{\dagger}$

$(A^{\dagger}A^2A^{\dagger}A^3)^* = (A^{\dagger}AAA^2)^* = (A^{\dagger}A)^* = A^{\dagger}A = A^{\dagger}AA^3 = A^{\dagger}A^2A^{\dagger}A^3$

(3) $J\ AA^{\dagger}J\ J\ A = J\ AA^{\dagger}A = J\ A\ A^{\dagger}J\ J\ AA^{\dagger}J = A^{\dagger}AA^{\dagger}J = A^{\dagger}J\ (J\ AA^{\dagger}J)^* = J\ (AA^{\dagger})^*J = J\ AA^{\dagger}J\ (A^{\dagger}J\ J\ A)^* = (A^{\dagger}A)^* = A^{\dagger}A = A^{\dagger}J\ J\ A$

(4) $AJ\ J\ A^{\dagger}AJ = AA^{\dagger}AJ = AJ\ J\ A\dagger AJ\ J\ A^{\dagger} = J\ A^{\dagger}AA^{\dagger} = J\ A^{\dagger}\ (AJ\ J\ A\dagger)^* = (AA^{\dagger})^* = AA^{\dagger} = AJ\ J\ A^{\dagger}$

$(J\ A^{\dagger}AJ)^* = J\ (A^{\dagger}A)^*J = J\ A^{\dagger}AJ$

(5) $(J\ A^3)^{\dagger} = (A^3)^{\dagger}J = A^{\dagger}A^2A^{\dagger}J$

Theorem 3.6. Let A be a centroidempotent matrix.A reduces to range hermitian when it is square hermitian.

Proof. If A is square hermitian then $A^2 = A^*$

$(A^3)* = (A^2A)^* = (A^*A)^* = A^*A = A^2\ A = A^3$ and hence A^3 is hermitian.

It can be verified that A† = A2.

But $A^0 = A^2 = A\dagger$ by remark 2.5.

Thus A is range hermitian.

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