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Multiscale Enhancement Filter Using Hessian Matrix: A Review

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ABSTRACT

Image segmentation plays an important role in many medical imaging applications by automating or facilitating the delineation of anatomical structures and other regions of interest. Vessel segmentation algorithms are the key segments of computerized radiological analytic frameworks. Though a lot of segmentation algorithms exist, there is no general unique solution to enhance and extract the vasculature. The 3D image data provided by the different modality can be interpreted using Maximum Intensity Projection (MIP).But the overlap of non-vascular structure and small blood vessels with low contrast are hardly visible in MIP. So, there is a need for vessel enhancement procedure as a preprocess step which will improve the small vessel delineation and reduce the organ over projection. This paper overviews most popular multiscale vessel enhancement technique which uses hessian matrix.

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INTRODUCTION

The blood vessel can be segmented directly based on gray value (Wilson D.L et .al,1999;Hassouna M.S et.al,2006;Gan R et.al 2005), prior knowledge (Faber S. C et.al,2003; Passat N et.al 2007), deformable model (Li C et.al,2011;Li C et.al 2010)and graphical linking technology (Cheng Z.J,2012). The effectiveness of the vessel segmentation algorithm and the results can be improved by applying the vessel enhancement procedures as a preprocessing step. The performance of the enhancement algorithm has been shown to massively impact the segmentation result. The clinicians use MIP images to clearly visualize the anatomical structures. But in vascular structure visualization, the small vessels with low contrast edges are not clearly visible in MIP. Additionally the overlapping of non-vascular structures also affects the vascular visualization. To alleviate these problems, enhancement algorithms can be first applied to the vascular images to suppress non-vascular structures and improve the delineation of small blood vessels. Segmentation of the vascular tree will facilitate volumetric display and will enable quantitative measurements of vascular morphology (Frangi R et.al 1988). Hessian matrix is one of the effective vessel enhancement methods, which uses local geometric feature of the image for enhancement.

Related Work:

In this paper, the classical hessian based vessel enhancement method and other multiscale filter methods which enhance the classical approach for better result and performance are discussed. Though the vessel enhancement methodologies are applicable for the 3D and 2D image data, the formula and data presented in this paper mostly consider 3D image data only.

Vessel enhancement Techniques:

A. Classical Hessian Multiscale filter:

Most of the Hessian-based vessel enhancement filters use eigenvalues extracted from the Hessian matrix to derive geometrical structures which can be regarded as tubular (Daniel Jimenez-Carretero et.al,2013). The filter is based on eigenvalue analysis of the scale space of the Hessian matrix. The eigenvalues and eigenvectors of Hessian matrix are closely related to vascular intensity and direction. For a 3D input image I, Hessian matrix is a 3 × 3 matrix composed of second-order partial derivatives of the input image I (Jiaoying Jin et.al,2013):

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$$\nabla^2 I = \begin{bmatrix} \frac{\partial^2 I}{\partial x^2} & \frac{\partial^2 I}{\partial x \partial y} & \frac{\partial^2 I}{\partial x \partial z} \\ \frac{\partial^2 I}{\partial y \partial x} & \frac{\partial^2 I}{\partial y^2} & \frac{\partial^2 I}{\partial y \partial z} \\ \frac{\partial^2 I}{\partial z \partial x} & \frac{\partial^2 I}{\partial z \partial y} & \frac{\partial^2 I}{\partial z^2} \end{bmatrix} \quad (1)$$

In practice, the second-order partial derivatives of input image I at a point(x,y,z) are defined as a convolution with derivatives of Gaussian filter at scale σ,(x, y, z, σ)(Jiaoying Jin,2013):

$$I(x,y,z,\sigma) = G(x,y,z,\sigma) \times I(x,y,z)$$

where

$$G(x,y,z,\sigma) = \frac{1}{\sqrt{(2\pi\sigma^2)^3}} \exp(-\frac{x^2+y^2+z^2}{2\sigma^2}) \quad (2)$$

Table 1: Possible structure orientations in 3D images depending on the eigenvalues of Hessian matrix.

Orientation pattern	3D image		
	λ_1	λ_2	λ_3
Noisy, no preferred direction	L	L	L
Plate-like structure (bright)	L	L	H−
Plate-like structure (dark)	L	L	H+
Tubular structure (bright)	L	H−	H−
Tubular structure (dark)	L	H+	H+
Blob-like structure (bright)	H−	H−	H−
Blob-like structure (dark)	H+	H+	H+

L: Low, H+: high positive, H−: high negative

Based on the eigenvalues of ∇^2I, the dissimilarity measures R_B,R_A and S are defined as

$$R_A = \frac{|\lambda_2|}{|\lambda_3|}$$

$$R_B = \frac{|\lambda_1|}{\sqrt{|\lambda_2 \lambda_3|}} \quad (3)$$

$$S = \sqrt{{\lambda_1}^2 + {\lambda_2}^2 + {\lambda_3}^2}$$

The first ratio R_B accounts for the deviation for a blob-like structure, but it cannot distinguish between a line-like and a plate-like pattern. The second ratio R_B is applied for distinguishing between plate-like and line-like structures. The third measure S is applied for distinguishing background (Daniel Jimenez-Carretero et.al, 2013).

Frangi *et al*. interpreted the eigenvalues of Hessian matrix to distinguish plate-like, blob-like and tubular structures. Let the eigen values of Hessian matrix be λ_1, λ_2, λ_3. Table 1 (Jiaoying Jin et.al,2013) summarizes the relations between λ_i and orientations of different structures in the image. Assuming a bright blood image, the vesselness function can be defined as follows

$$v_0(\sigma) = \begin{cases} 0, & if\ \lambda_2 > 0\ \ or\ \ \lambda_3 > 0, \\ \left(1 - \exp\left(-\frac{R_A^2}{2\alpha^2}\right)\right) \exp\left(-\frac{R_B^2}{2\beta^2}\right) & \\ \times \left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right), & otherwise \end{cases} \quad (4)$$

where α, β, and c are thresholds which control the sensitivity of the filter to the measures R_A, R_B, and S.

The filter is applied at multiple scales, and the maximum response V_η is selected to be a final estimate of vesselness.

$$v_\eta = \max_{\sigma_{\min} \le \sigma \le \sigma_{\max}} \{v_o(\sigma)\}, \quad (5)$$

The following section describes the various multi scale vessel enhancement filters which use hessian matrix to measure the vesselness.

B. Improved Hessian Multiscale vesselness filters:

1) Hessian combined with Directional Filter Banks(DFB):

The classical Hessian based method suffers from well-known drawbacks due to the assumption of Gaussian profile of vessel cross-section and localized characteristic such as intensity inhomogeneity inside vessel, junction suppression. In order to improve the performance in vessel junctions and lower the sensitive to noise, (Phan T.H et.al,2009) presents a Hessian-based method combined DFB. Additionally in (Xuang Xu et.al,2013), measurement is used as weight coefficient to retain the sharp vessels. The vessel similarity measure is computed by using vessel intensity feature and vessel profile curvature feature of directional images. The advantage of vessel similarity measure is not only improving the contrast in the regions of vessel edges but also suppress noise in the background with low vessel similarity. The vessel similarity measure W_i (Xuang Xu et.al,2013) is defined as the product of vessel intensity feature V_{i1} and profile curvature feature V_{i2}.

$$W_i(x,y) = V_{i1}(x,y).V_{i2}(x,y) \quad (6)$$

The vessel profile curvature feature is also included to discriminate vascular structures from dark background region. The vessel profile curvature feature is sensitive to the vascular structures with the same direction of corresponded directional images (Xuang Xu et.al,2013). Since the value of profile

curvature is zero at other regions of directional images, it could be easily used to filter out most background in the image. The final enhanced image *R* (Xuang Xu et.al,2013) is defined as

$$R(x,y)=\sum_{i=1}^{D} W_i(x,y).F_i(x,y) \tag{7}$$

where $W_i(x,y)$ is vessel similarity, D is number of directional images and $F_i(x,y)$is the Frangi enhanced directional image.

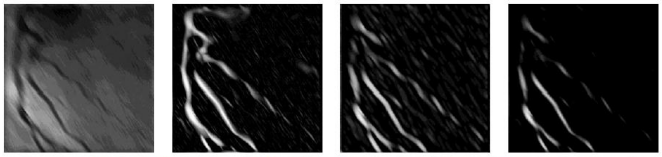

Fig. 1 Vessel shape correction in directional image. From left to right: directional image, enhanced directional image using Frangi's method, vessel similarity and enhanced directional image combined with vessel similarity.

As shown in Fig.1(Syed Hassan and Jungwon Yoon,2013), by combining Frangi's enhanced directional image with corresponding vessel similarity, most of the background noises have been suppressed. It is also clear that the edges of vessels get clearer compared with original enhanced directional image.

2) Hessian - Enhancing Airways:

The complete automatic method to extract lung vessels using hessian-based filter was proposed by Daniel Jimenez-Carretero et.al ,2013.The classical Hessian model was modified to avoid false positives in areas that present locally tubular-like features such as the airways. If the scale range in the filter covers the possible radius of both the vessels and airways, the classical filter shows nonnegative response for airways also. This is the case if the airways are partially tubes with the radius of vessels or the airways may be in contact with the vessels and cannot be distinguishable from them. So, while segmenting lung vessels, the vesselness function need to be executed twice, one for the vessel extraction and other for airway extraction. But the condition need to be checked is different for airways and the vesselness function for airways is given below

$$v_0(\sigma)=\begin{cases} 0, & \text{if } \lambda_2<0 \text{ or } \lambda_3<0, \\ \left(1-\exp\left(-\frac{R_A^{\,2}}{2\alpha^2}\right)\right)\exp\left(-\frac{R_B^{\,2}}{2\beta^2}\right) & \\ \times\left(1-\exp\left(-\frac{S^2}{2c^2}\right)\right), & \text{otherwise} \end{cases} \tag{8}$$

3) Smoothed vesselness Filter:

The Frangi's vesselness function $V_0(\sigma)$is not smooth at the origin, and therefore, it cannot be used in the diffusion process. Since the amount and orientation of diffusion depend on the local vessel likeliness, there is a need to get the smooth vessel using filter process.

Hence, Rashindra Manniesing *et al*, 2006 proposed smoothed version of Frangi's vesselness function which performs better than most and can be presented as below:

$$v_0(\sigma)=\begin{cases} 0, & \text{if } \lambda_2>0 \text{ or } \lambda_3>0, \\ \left(1-\exp\left(-\frac{R_A^{\,2}}{2\alpha^2}\right)\right)\exp\left(-\frac{R_B^{\,2}}{2\beta^2}\right) & \\ \times\left(1-\exp\left(-\frac{S^2}{2c^2}\right)\right)\exp\left(-\frac{2c^2}{|\lambda_2|\lambda_3^{\,2}}\right), & \text{otherwise} \end{cases} \tag{9}$$

The common denominator for the exponential term in Frangi's vesselness function (Eqn 4) is $|\lambda_2|\lambda_3^2$. So, the smoothed version of vesselness function (Daniel Jimenez-Carretero et.al,2013) is obtained by multiplied with

$$\exp\left(-\frac{2c^2}{|\lambda_2|\lambda_3^{\,2}}\right) \tag{10}$$

where ***c*** is a small constant. It resembles a Gaussian function with its argument inverted, and it is controlled by the standard deviation **c**. This constant **c** should be chosen very small to only have influence around the origin (Rashindra Manniesing et.al,2006). As in classical function, the maximum response across selected scale range is selected as the final estimate of vesselness.

4) Fast computation of Hessian based filter:

The computation of Eigenvalues of Hessian Matrix corresponding to image element requires substantial computational effort. Hence, the computation cost is high and become more challenging in the applications which use Hessian-based enhancement filters. Using a property of the characteristic polynomial coefficients of a matrix and the well-known Routh–Hurwitz criterion in control engineering, it is shown that under certain conditions, the response of a Hessian-based enhancement filter to an image element can be obtained without having to compute Hessian eigenvalues. The computational cost can thus be reduced (Shih-Feng Yang et.al,2014).

To reduce the computational costs of Hessian-based enhancement filters for tubular-like structures in 3D images, Orlowski and Orkisz ,2009 proposed a method by performing two conditional tests. If the condition in the tests is not satisfied, then there is no need to compute the eigenvalues and the response of the Hessian-based enhancement filter to the voxel

can be determined to be zero .The first test examine the sign of the Hessian trace and the second test verify the sign of the Hessian determinant and whether the Hessian matrix is negative-definite. Thus Orlowski and Orkisz,2009 method reduces the computational cost.

The computation of eigenvalues using the analytical method developed by kopp J,2008 is more efficient and faster than the iterative algorithms. Shih-Feng Yang et.al,2014 proposed a model to reduce the computational cost of Hessian-based enhancement filters for blob-, tubular-, and plane-like structures in 2D and3D images. In this model, conditions are developed by using a property of the characteristic polynomial coefficients of the Hessian matrix corresponding to the image element and the well-known Routh–Hurwitz criterion in control engineering. The characteristic polynomial for 3 * 3 Hessian matrix and the condition need to be checked for 3D tubular-like structures are given below (Shih-Feng Yang et.al,2014)

Characteristic polynomial equation is,

$$P_{H_3}(\lambda) = \lambda^3 + \beta_1\lambda^2 + \beta_2\lambda + \beta_3 \quad (11)$$

where

$$\begin{aligned}\beta_1 &= -(f_{xx} + f_{yy} + f_{zz})\\ \beta_2 &= f_{xx}f_{yy} + f_{xx}f_{zz} + f_{yy}f_{zz} - f_{xy}f_{yx} - f_{xz}f_{zx} - f_{yz}f_{zy}\\ \beta_3 &= f_{xx}f_{yz}f_{zy} + f_{xy}f_{yx}f_{zz} + f_{xz}f_{yy}f_{zx} - f_{xx}f_{yy}f_{zz} - f_{xy}f_{yz}f_{zx} - f_{xz}f_{yx}f_{zy}\end{aligned} \quad (12)$$

where f_{xx}, f_{xy}, f_{zy}, etc. are the second order partial derivatives of the input image I at (x,y,z). This method (. Shih-Feng Yang et.al,2014) propose different conditional tests for each structure depending on the image space (2D/3D).For example the conditional test for tubular structure in 3D image space is as follows

Theorem 1:

The vesselness condition in Eqn. (4) is not satisfied if any of the following condition holds

(i) $\beta 1 <= 0$

(ii) $\beta 2 <= 0$ and $\beta 3 = 0$

(iii) $\beta 1 > 0$, $\beta_3 > 0$ and $\beta_1\beta_2 < \beta 3$

So, there is no need to compute the eigenvalues if any of the above mentioned condition holds true in tubular vesselness condition.

The experimental results for this algorithm for different structures are given in Table2 below

Table 2: Experimental results of Fast computation Method.

Fast Computation Hessian method	percentage reductions of the number of computations		percentage reductions of processing time	
	2D	3D	2D	3D
Blob like structures	90%	97%	31%	68%
Tubular like structures	65%	75%	8%	55%
Plane like structures	–	12%	–	3%

5) Hessian combined with Gray Scale value:

Classical Hessian method does not consider the global grayscale information to compute the vesselness. It only considers the local geometric feature. It increases the difficulty in cerebrovascular segmentation, where some pseudo vascular structures or some isolate noise points, such as the nasal soft tissues that have the similar shape with the vessels will also be isolated. To overcome this difficulty, an improved Hessian multiscale filter is proposed in Jinzhu Yang et.al ,2014. Here, the gray level factor F is added with the vesselness function which is shown in the Eq. (13).

$$F = \frac{1}{2}\left(1 + \frac{2}{\pi}\tan^{-1}\left(\frac{(I_{(x,y,z)} - H_u)}{P_{\max}} \times \gamma\right)\right) \quad (13)$$

where $I_{(x,y,z)}$ is the gray value of pixel point (x,y,z) , $P_{\max}$ is the maximum gray value , H_u is the global adaptive threshold value and γ is the smoothness factor. Adaptive threshold is an automatic search of threshold. First, the image is segmented into foreground and background based on average image gray value. Subsequently, a new threshold T is calculated by averaging of these two parts. The process is repeated until there is no change in threshold value. The improved Hessian enhancement filtering defined in this paper (Jinzhu Yang et.al ,2014) is represented as below

$$v_0(\sigma) = \begin{cases} 0, & if\ \lambda_2 > 0\ \ or\ \ \lambda_3 > 0, \\ \left(1 - \exp\left(-\frac{R_A^2}{2\alpha^2}\right)\right)\exp\left(-\frac{R_B^2}{2\beta^2}\right) & \\ \times\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right).F, & otherwise \end{cases} \quad (14)$$

This method effectively reduce the pseudo vessel structures and isolated noise points.

Conclusion:

This paper reviews about several Hessian based multiple enhancement filter techniques which overcome the difficulties in classical approach. The fast computation technique using a property of the characteristic polynomial coefficients of a matrix and the well-known Routh–Hurwitz criterion in control engineering can be applied to other techniques (Jinzhu Yang et.al ,2014; Xuang Xu et.al 2013) to significantly improve the processing speed of algorithm. But while deriving the polynomial coefficients, the value of gray scale factor should be concerned. The extension to the current work would be to implement the proposed hybrid approach which

incorporates the smoothed vesselness filter on directional images.

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