ISSN:1991-8178

Australian Journal of Basic and Applied Sciences

Journal home page: www.ajbasweb.com

M3Y-Type Interaction and the Binding Energy of Triton

[1]Frederick Gbaorun, [2]Joseph O. Fiase, [3]Terwase Aper, [4]Augustine A. Gundu

[1]Benue State University,Department of Physics, Faculty of Science, P.M.B. 102119, Makurdi, Nigeria.
[2]Benue State University,Department of Physics, Faculty of Science, P.M.B. 102119, Makurdi, Nigeria.
[3]Benue State University,Department of Physics, Faculty of Science, P.M.B. 102119, Makurdi, Nigeria.
[4]Federal University,Department of Physics, Faculty of Science, P.M.B. 0182, Kashere, Gombe State, Nigeria

ARTICLE INFO

Article history:
Received 12 December 2014
Received in revised form 26 December 2014
Accepted 28 January 2015
Available online 1 April 2015

Keywords:
Binding Energy, Three-Body Force, Triton

ABSTRACT

Calculation of the binding energy for few particle systems such as triton using as input two-body forces only shows under binding relative to their corresponding experimental values. In this paper, the role played by both two-body force, an M3Y-Type interaction that was derived from the lowest order constrained variational (LOCV) calculation, and the three-body force to the binding energy of a three-body system is investigated using the differential Faddeev equation in configuration space within the frame work of total angular momentum formalism. The binding energy of triton when only two-body force is used is – 7.002 MeV, whereas when three-body force is included, the binding energy goes to – 8.1889 MeV. Comparing the result of the present work with the experimental value of – 8.482 MeV, it is observed that two-body force greatly under binds triton, while three-body force brings the value closer to the experimental value.

© 2015 AENSI Publisher All rights reserved.

To Cite This Article: Frederick Gbaorun, Joseph O. Fiase, Terwase Aper, Augustine A. Gundu., M3Y-Type Interaction and the Binding Energy of Triton. ***Aust. J. Basic & Appl. Sci.,*** *9(11): 307-313, 2015*

INTRODUCTION

Calculation of the binding energy for few particle systems such as ^{3}H, ^{3}He and ^{4}He using as input two-body forces only shows an under binding relative to their corresponding experimental values. For example, the calculated binding energy of ^{3}He with various two-body potentials is found to be about 1.0 – 1.5 MeV less than the experimental value -8.482 MeV (Dithlas ,2006). Similarly, the binding for ^{4}He is about 2.0 – 4.0 MeV less compared to the experimental value of -28.30 MeV (Nogyn *et al*., 2000). This discrepancy between theory and experiment may be due to several sources (Dithlas, 2006), the main ones being the neglect of the: (i) off-energy shell effects in the two-body forces, (ii) relativistic effects, and (iii) three-body forces

Investigation on the role of relativistic effects, for example, shows that their inclusion in calculations increases the triton binding energy by about 0.25 MeV (Jackson and Tjon, 1983). The role played by the neglect of the other sources has been previously investigated (Muslim and Uede, 1982; Vittorio and Piotre, 2008; Voglis *et al.*,2007; Hajduk *et al*., 1981).The theory of two-body force is richly developed and there are various potentials representing the interaction of two-body systems. Dithlas (2006) investigated the role played by a two-body force and the three-body force. The present work is concerned with the investigation of the role played by both M3Y-type interaction, a two-body force derived from the lowest order constrained variational (LOCV) calculation (Fiase *et al.*, 2002), and three-body force, the Urbana Potential.

Within the non-relativistic frame work, a three-body force is expected to have an appreciable contribution relative to the other two sources mentioned, especially for systems with cluster structures. To this end the inclusion of these forces in the Hamiltonian could address the discrepancies in binding energy between experimental data and predictions based on two-body forces only.

To do this, the question of solving the Schrödinger equation for three particles will be addressed. Early approaches were to use the Lippmann-Schwinger equation for the three-particle system. However, it is well known that the Lippmann-Schwinger equations for three-particles do not lead to a unique solution. This problem can be addressed by using the Faddeev formalism, in which the wave function is decomposed into a sum of the components describing the two-body subsystem. The resulting Faddeev equations are the most fundamental formulation of the non-relativistic quantum three-body problem. Originally, they were formulated in momentum space. The configuration

Corresponding Author: Frederick Gbaorun, Benue State University, Department of Physics, Faculty of Science, P.M.B. 102119, Makurdi, Nigeria. Phone: 2347038635405, e-mail address fredgbaorun@gmail.com

space equations were derived later by Fiedeley and Noyes (1968), and further formulated to completion by Merkuriev *et al*., (1976). Therefore, the Faddeev equations may be used in three different forms, namely

(i) as the integral equations in momentum space (Friar *et al.*, 1981)
(ii) as differential equations in configuration space (Friar *et al.*, 1981; Chen *et al.*, *1985*), or
(iii) as the combination of both configuration and momentum space(Harper *et al.*, 1972).

In their original form, Faddeev equations contain operators in six-dimension, and thus numerically impossible to solve. For a numerical solution to be realized, these equations are reduced to a more tractable form. In the total angular momentum representation considered here, the Faddeev equations are reduced to a set of coupled equations in three-dimension. For three identical particles, the coupled equations further reduced to a single three-dimensional equation defined on an infinite interval for numerical implementation, the equations are reduced to finite equations by using, for example,

(i) a transformation procedure, where the infinite interval is mapped into a finite interval (Baldo and Bulgio, 2001) without any approximation , or
(ii) the cut-off method.

In this work, the use of differential Faddeev equations in configuration space within the frame work of total angular momentum formalism is employed (Glickle, 1983)

In order to reduce the dimension of the three-dimensional Faddeev equation to a manageable size, the tensor trick technique (Glickle, 1983) is used. In this procedure, the Faddeev eigenvalue equations are transformed into an inverted eigenvalue matrix equations which can be stored as tensor product of simpler matrices, i.e. matrices with smaller dimensions or diagonal matrices. In this way, the convergence rate of the iterative techniques are improved. For further improvement on convergence rate the regularization polynomial accelerator technique are usually incorporated in calculations. The rest of this paper is organized as follows: In Section 2, we briefly described the two-body M3Y-type interaction used. Section 3 deals on the summary of the Faddeev equations used. Section 4 is the application to the binding energy of Triton, while conclusion is given in Section 5.

The M3Y-Type Interaction:

Bertsch *et al.* (1977) opened a wide door into the study of inelastic reaction from fundamental NN interaction since their pioneering G-matrix work. Their effective interactions which are expressed as sum of three Yukawa terms is popularly called Michigan three (3) Yukawa (M3Y) (Dao *et al.*, 1997; Zhang *et al*, 2000). One Yukawa terms was taken to be the one pion exchange potential (OPEP); the second was chosen to have a range of 0.4 fm from which roughly simulates multiple pion exchange, and the third range of 0.25 fm was chosen for computational convenience. Since then there has been several versions of this interaction (Bertsch and Das, 1988), and the most popular ones are the M3Y-interaction based on the G-matrix of Reid NN potential (Bertsch *et al*., 1977) (denoted here after as M3Y-Reid) and the M3Y-interaction based on the G-matrix of the paris NN potentials (Dao and Satchler, 2000) (denoted here after as M3Y-Paris). These are given respectively by equations (2.1) and (2.2) (Dao *et al.*, 1996; Misicu, 2007) below.

$$v_{00}^{D}(r) = 7999.0\frac{\exp(-4r)}{4r} - 2134.25\frac{\exp(-2.5r)}{2.5r}$$
$$v_{00}^{EX}(r) = 4631.38\frac{\exp(-4r)}{4r} - 1787.13\frac{\exp(-2.5r)}{2.5r} - 7.8474\frac{\exp(-0.7072r)}{0.7072r}, \quad (1)$$

$$v_{00}^{D}(r) = 11061.625\frac{\exp(-4r)}{4r} - 2537.5\frac{\exp(-2.5r)}{2.5r}$$
$$v_{00}^{EX}(r) = -1524.25\frac{\exp(-4r)}{4r} - 518.75\frac{\exp(-2.5r)}{2.5r} - 7.8474\frac{\exp(-0.7072r)}{0.7072r}, \quad (2)$$

where $v_{00}^{D}(r)$ and $v_{00}^{EX}(r)$ represent the direct and exchange term of the interactions respectively. The difference between the two interaction is mostly due to the difference found between the tensor-odd, triplet-odd and the triplet even components of the M3Y-Reid forces and those of the M3Y-paris forces (Bertsch and Das, 1988).

Fiase (Fiase, 2000; Fiase and Sharma 2002) derived an effective interaction which is suitable for calculations of inelastic scattering following the procedure prescribed by Bertsch *et al*. (1977) in which the two-body matrix-equations are fitted to those of the sum Yukawa functions with different ranges. The potentials is divided into three-parts, namely the central (*c*), spin-orbit (*ls*) and tensor (*t*) components and is given by Fiase *et al*., (2002):

$$V_c = \sum_k d_k \frac{\exp-(r_{ij}/R_k)}{(r_{ij}/R_k)},$$
$$V_{ls} = \sum_k d_k \frac{\exp-(r_{ij}/R_k)}{(r_{ij}/R_k)} \vec{L}\cdot\vec{S}$$
$$V_t = \sum_k d_k \frac{\exp-(r_{ij}/R_k)}{(r_{ij}/R_k)} r_{ij}^2 S_{ij} \quad (3)$$

where d_k are the strengths of the interaction which are determined by fitting the oscillator matrix elements of Eq. (3). The ranges $k \le 4$ are chosen to be 0.25, 0.4, 0.7 and 1.414 fm, which are theoretically motivated by the one boson exchanges. The results of the strengths of the effective interaction were separated into various angular momentum channels namely; the singlet-even (SE), singlet odd (SO), triplet even (TE), triplet odd (TO) and the spin-orbit and Tensor channels. The direct V^D and exchange V^{EX} potential in the SE, SO, TE and TO channels can be recast into spin-isospin formalism as

$$V^D = \frac{1}{16}\left(3V^{SE} + 3V^{TE} + V^{SO} + 9V^{TO}\right) \quad (4)$$

with

$$V^{EX} = V^D P^\sigma P^\tau \quad (5)$$

where P^σ and P^τ are projection operators in the spin and iso-spin channels. Hence

$$V^{EX} = \frac{1}{16}\left(3V^{SE} + 3V^{TE} - V^{SO} - 9V^{TO}\right). \quad (6)$$

Using the various ranges as listed in Table 5 of Fiase *et al*. (2002), the present M3Y-type interaction is obtained as

$$v^D(r) = 11012\frac{\exp(-4r)}{4r} - 2359\frac{\exp(-2.5r)}{2.5r}$$
$$v^{EX}(r) = 1039.25\frac{\exp(-4r)}{4r} - 1503.94\frac{\exp(-2.5r)}{2.5r} - 7.8474\frac{\exp(-0.7072r)}{0.7072r}. \quad (7)$$

Faddeev Equations:

The Hamiltonian for three particles interacting via two-body forces is defined as

$$H = -H_0 + \sum_{\alpha=1}^{3} V_\alpha^{(2N)}(x_\alpha), \quad (8)$$

where x_α is the Jacobi coordinate for the two-particle subsystem, $V_\alpha^{(2N)}(x_\alpha)$ is the central potential, and H_0 is the free three particle Hamiltonian given as follows:

$$H_0 \equiv -\nabla x = -\nabla_{x\alpha}^2 - \nabla_{y\alpha}^2, \quad (9)$$

Thus the Schrödinger equation for three-body system reads

$$\left[-\nabla_{x\alpha}^2 - \nabla_{y\alpha}^2 + \sum_{\alpha=1}^{3} V_\alpha^{(2N)}(x_\alpha)\right]\Psi_{3N}(x_\alpha, y_\alpha) = E_{3N}\Psi_{3N}(x_\alpha, y_\alpha), \quad (10)$$

where E_{3N} is the energy and $\Psi_{3N}(x_\alpha, y_\alpha)$ is the total wave function of the system. A direct numerical solution of equation (10) is cumbersome and difficult. In order to address this difficulty, Faddeev proposed the idea that the total wave function of the system be decomposed into three components as follows (Harper *et al*., 1972):

$$\Psi_{3N}(x, y) = \sum_{\alpha=1}^{3} \phi_\alpha(x_\alpha, y_\alpha) \quad (11)$$

where $\phi_\alpha(x_\alpha, y_\alpha)$ describes the two-particle subsystem (β, α) in which particle α is a spectator. Substitution of equations (11) into (10) leads to a set of six-dimensional three coupled differential equations for the components $\phi_\alpha(x_\alpha, y_\alpha)$, namely

$$\left[H_0 + V_\alpha^{(2N)}(x_\alpha) - E_{3N}\right]\phi_\alpha(x_\alpha, y_\alpha) = -V_\alpha^{(2N)}(x_\alpha)\left[\phi_\beta(x_\beta, y_\beta) + \phi_\gamma(x_\gamma, y_\gamma)\right]. \quad (12)$$

These equations are the configuration space Faddeev equations when only two-body forces are present.

For a system of three identical particles the Faddeev components have the same functional form. In this case, the relationship between the components is

$$\phi_\beta(x_\beta, y_\beta) = P^+\phi_\alpha(x_\alpha, y_\alpha) \quad (13)$$
$$\phi_\gamma(x_\gamma, y_\gamma) = P^-\phi_\alpha(x_\alpha, y_\alpha), \quad (14)$$

where $P^\pm$ stands for cyclic and anti cyclic permutation operators acting on the coordinates. Substituting equations (13) and (14) into equation (12), and rearranging terms leads to the equation

$$\left[H_0 + V_\alpha^{(2N)}(x_\alpha) - E_{3N}\right]\phi_\alpha(x_\alpha, y_\alpha) = -V_\alpha^{(2N)}(x_\alpha)\left[P^+\phi_\alpha(x_\alpha, y_\alpha) + P^-\phi_\alpha(x_\alpha, y_\alpha)\right]$$
$$\Rightarrow \left[H_0 + \left(I + P^+ + P^-\right)V_\alpha^{(2N)}(x_\alpha) - E_{3N}\right]\phi_\alpha(x_\alpha, y_\alpha) = 0, \quad (15)$$

where I is the identity operator.

Faddeev Equation in Fixed Angular Momentum Representation:

For fixed angular momentum L the wave function $\phi_\alpha(x_\alpha, y_\alpha)$ can be expanded as

$$\phi_\alpha(x_\alpha, y_\alpha) = \sum_{L,m,n} \frac{\Phi^{Lmn}(x_\alpha, y_\alpha, z_\alpha)}{x_\alpha y_\alpha} D_{mn}^L(g), \quad (16)$$

where $\Phi^{Lmn}(x_\alpha, y_\alpha, z_\alpha)$ are the Eigen functions of the total angular momentum L, $D_{mn}^L(g)$ is the Wigner function, and g refers to the coordinate describing collective angular motion of the system. The intrinsic coordinate x_α and y_α are given by

$$x_\alpha = \left|x_\alpha\right|, \quad (17)$$
$$y_\alpha = \left|y_\alpha\right| \quad (18)$$

With z_α, the angle between x_α and y_α, given as

$$z_\alpha = \frac{(x_\alpha, y_\alpha)}{x_\alpha y_\alpha} = \cos(x_\alpha, y_\alpha) \quad (19)$$

For fixed total angular momentum *L*, the projection of the Faddeev components correspond to the free Hamiltonian (H_0^L) as follows

$$H_0^L = D^L(g^{-1})\, xy\, H_o \frac{1}{xy} D^L(g) \quad (20)$$

where $D^L(g)$ correspond to the matrix constructed from the Wigner function. In this work we consider *L* = 0 and substituting equation (16) into equation (15) and using the orthogonality properties of the Wigner function we obtain three-dimensional Faddeev equations

$$\left[H_0^0 + V^{2N}(x)\left(I + P^+ + P^-\right) - E_{3N}\right]\Phi^0(x,y,z) = 0, \quad (21)$$

where

$$P^+\Phi^0(x,y,z) \equiv xy\left(\frac{\Phi^0\left(x^+, y^+, z^+\right)}{x^+ y^+}\right), \quad (22)$$

$$P^-\Phi^0(x,y,z) \equiv xy\left(\frac{\phi^0\left(x^-, y^-, z^-\right)}{x^- y^-}\right), \quad (23)$$

and $x^\pm(x,y,z)$, $y^\pm(x,y,z)$ and $z^\pm(x,y,z)$, are the permuted Jacobi coordinates, with the superfluous subscript α dropped. Explicit expressions for these coordinates are

$$x^\pm(x,y,z) = \left(\frac{1}{4}x^2 + \frac{3}{4}y^2 \mp \frac{\sqrt{3}}{2}xyz\right)^{1/2}, \quad (24)$$

$$y^\pm(x,y,z) = \left(\frac{3}{4}x^2 + \frac{1}{4}y^2 \mp \frac{\sqrt{3}}{2}xyz\right)^{1/2}, \quad (25)$$

$$z^\pm(x,y,z) = \pm\frac{\frac{\sqrt{3}}{2}x^2 \mp \frac{\sqrt{3}}{4}y^2 - \frac{1}{2}xyz}{x^\pm(x,y,z)\, y^\pm(x,y,z)}. \quad (26)$$

Assuming that there is only one bound state in each two-body subsystem, the Faddeev component Φ^0 has the following boundary conditions (which are further discussed in a little more detail in the next section):

$$\Phi^0(x,y,z) \approx \varphi_2(x) e^{-k_y y} + A(x,y,z)\frac{e^{-k_3 K z\left(x^2+y^2\right)^{1/2}}}{\left(x^2+y^2\right)^{1/4}}, \quad (27)$$

where $\varphi_2(x)$ represents the two-particle bound state wave function, while k_y and k_z are the energy dependent wave numbers, and $A(x,y,z)$ is the scattering amplitude. The first term in equation (27) corresponds to the virtual decay of the system into a particle and a two-particle bound state while the second term corresponds to the virtual decay into three single particles. The letter term decreases much faster that the first one. For this reason, we only consider the first term.

Application: Binding Energy of the Triton:

In order to obtain a numerical solution to equation (21), the equation is first transformed into eigenvalue equation. To this end we discretize the domain in each of the three coordinates, and expand the Faddeev component in terms of a set of basis functions B_i in each direction as follows:

$$\Phi^0(x,y,z) = \sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{n=1}^{N} C_{lmn} B_l(x) B_m(y) B_n(z), \quad (28)$$

where the subsystems L, M and N stand for the number of basis functions in the three variables (x, y, z), respectively, while C_{lmn} are the expansion coefficients. Substituting equation (28) into equation (21), followed by orthogonal collocation method (Osman, 1970), reduces the Faddeev equations (in the presence of two-body forces only) to a system of linear algebraic equations for the coefficients C_{lmn}, namely

$$\left[\hat{H}_0^0 + \hat{V}^{(2N)}\left(\hat{I} + P^+ + P^-\right) - E_{3N}\hat{I}\right]C = 0, \quad (29)$$

where $\hat{I}$ stands for the unit matrix, and C is a vector of the expansion coefficients C_{lmn}. To obtain the Eigen solution (iteratively), we treat E_{3N} as a parameter, E_0 say, and rewrite equation (29) as an eigenvalue equation

$$-\left(\hat{H}_1 - E_0\hat{I}\right)^{-1}\hat{H}_2 C = \lambda(E_0)C, \quad (30)$$

where

$$\hat{H}_1 = \hat{H}_0^0 + \hat{V}^{(2N)}, \quad (31)$$

$$\hat{H}_2 = \hat{V}^{(2N)}\left(P^+ + P^-\right) \quad (32)$$

λ is an eigenvalue corresponding to the energy E_0. Clearly $\lambda(E_0)$ in Eq. (30) corresponds to a physical solution of the equation.

Handling Convergence of Solution:

Three-body state calculations are highly demanding in terms of computer memory and numerical stability, and hence convergence toward the correct solution is generally a huge challenge. Thus, in order to make numeric tractable, innovative techniques are necessary. In this work, the underlying two-body potentials have strong repulsive core. Strong repulsion leads to a plethora of negative eigenvalues being generated which lie close to unity. This accumulation of eigenvalue around unity retards the convergence rate towards a physical solution of the iterative algorithm. In order to address this problem a modifying potential $\hat{V}_m$ is introduced so that the Hamiltonian reads (Dithlas, 2006).

$$\hat{H}_{negative} = \hat{H}_0 + 2\hat{V}_m + \hat{V}^{(2N)}\left[\left(P^+ + P^-\right)\right], \quad (33)$$

compared to the Hamiltonian in equation (3.38). Similar to the discussion leading to Eqs. (31) and (32), $\hat{H}_{negative}$ is decomposed into $\hat{H}_1$ and $\hat{H}_2$ as follows

$$\hat{H}_1 = \hat{H}_0 + \hat{V}^{(2N)} + \hat{V}_m \quad (34)$$

$$\hat{H}_2 = -\hat{V}^{(2N)}\left(P^+ + P^-\right) + \hat{V}_m \quad (35)$$

In other words, the problem of calculating the eigensonlutions of Eq. (21) now reduces to that of finding the discrete spectrum of the operator $\hat{\chi}$ (Dithlas, 2006)

$$\hat{\chi}(\in) = -\left(\hat{H}_0 + V^{(2N)} + \hat{V}_m - \in\right)^{-1}\left(\hat{V}_m - V^{(2N)}\hat{P}\right) \quad (36)$$

where $\in$ is an eigenvalue

In our procedure for calculating the binding energies, we first obtain the spectrum of the operator defined by Eq. (36) (ie. Using as input only two-body forces and Restarted Arnoldi algorithm with its necessary changes).

Several parameters must be adjusted to achieve optimality when implementing the procedure. The main ones are: first, the length parameter (hereafter denoted a_0) that defines the distance from the origin to the first nonzero value of the x-grid. This parameter must be properly chosen so that:

i) The correct BCS are properly satisfied, otherwise one gets pathologies associated with incorrect truncating of the space.
ii) The effects of repulsive core of the potential are encountered (together with the background potential V_m, of course) and thus make numeric stable.

Other parameters of importance are the number of collocation points associated with the x^-, y^-, and z^- grid, as well as the size of the Krylov subspace and the cutoff lengths ($x_{\max}$, $y_{\max}$) which determine the range of the potential. These must be chosen to obtain a balance between accuracy of the results and computational cost.

For the two-body input, we employed the radial part (direct part) of the M3Y-type interaction [8] as defined by Eq. (37) below with the parameter given in Table 1.

$$V^{(2\mu)}(r) = V_A \frac{\exp(\mu_A r)}{\mu_A r} - V_R \frac{\exp(-\mu_R r)}{\mu_R r} \quad (37)$$

This potential has been used to calculate the binding energy of nuclear matter and nuclear incompressibility correctly [29].

Table 1: Input parameters for the radial part of the M3Y-Type two-body interaction.

Ref.	$V_A[M_e V\ Fm]$	$\mu_A[Fm^{-1}]$	$V_R[M_e V\ Fm]$	$\mu_R[Fm^{-1}]$
Fiase et al., (2002)	11012	4	2359	2.5

With respect to the three-body forces the Urbana type potential taken from (Dithlas, 2006), given by

$$V^{(3N)}(r_{12}, r_{23}, r_{31}) = \sum_{cyc}\left[0.003T_\pi^2(r_{12})T_\pi^2(r_{23}) + 4.5T_\pi(r_{12})T_\pi(r_{23})\left\{\frac{1}{2}\left(3C_2^2 - 1\right)\right\}\right]. \quad (38)$$

where

$$T_\pi(r) = \exp(-0.7r)\left(1 + \frac{3}{0.7r} + \frac{3}{(0.7r)^2}\right)\left(1 - \exp(-2r^2)\right)^2,$$

and

$$C_2 = \frac{\left(r_{12}^2 + r_{23}^2 - r_{31}^2\right)}{2r_{12}r_{23}} = \cos(\theta_1)$$

is employed. The summation runs over the three cyclic permutation of particle 1, 2, and 3. The coordinate r_{ij} means the vector between particle i and j.

The first step, in accordance with the procedure pointed above, is to determine the a_0 which gives the first estimate of the result. The optimal value is obtained in conjunction with other parameters, such as the number of basis functions in the different Jacobi coordinates. Table (2) shows the triton binding energy as a function of a_0 using the M3Y-type interaction as a two-body force only, and by using the same two-body force with the Urbana potential added as three-body force

Table 2: Values of the first non-zero x-grid from the origin, a_0 and their corresponding binding energies for triton ($E_B^{(3)}$) when two- and three-body forces are used.

a_0	$E_B^{(3)}$ (MeV)	$E_B^{(3)} + Urbana$ (MeV)
0.025	7.00196	8.18921
0.035	7.00184	8.18919
0.045	**7.00152**	8.18910
0.055	7.00158	**8.18891**
0.065	7.00164	8.18917
0.075	7.00182	8.18923
0.085	7.00212	8.18929
0.095		

With two-body forces included, $a_0 = 0.045\ fm$, whereas when the three-body force (the Urbana potential taken from (Dithlas, 2006)) is included, $a_0 = 0.055\ fm$. The results of the present work for the binding energy of triton, together with the literature results, are given in Table 3.

Table 3: Binding energy of the triton using two- and three-body forces.

Model	Schellingerhout (1995) (MeV)	Lekala (2005) (MeV)	Dithlas (2006) (MeV)	Present (MeV)
Two-body force only	-7.736	-7.73661	-7.728	-7.002
Two-+ three-body force	-	-	-8.555	-8.1889

From Table 2, as it is summarized in Table 3, the result of the binding energy of triton when only two-body force is used is -7.002 MeV, whereas when three-body force is included, the binding energy goes to -8.1889 MeV. Comparing the result of the present work with the experimental value of -8.482 MeV, and other workers (Dithlas, 2006; Schellingerhout, 1995; Lekala, 2005) it is observed that two-body force greatly under binds the system, Triton, while three-body force brings the value closer to the experimental value.

Conclusion:

In this study, we have examined the role played by both a two-body force and a three-body force to the binding energy of a tri-nucleon system using the differential Faddeev equations in configuration space within the frame work of total angular momentum formalism. In the investigation, the two-body force used is an M3Y-type nucleon-nucleon interaction derived from the lowest order constrained variational method. It was found that the two-body force under binds Triton with an energy of -7.002 MeV. When a three-body force was included, the binding energy goes to -8.1889 MeV. The experimental value of the binding energy of triton is -8.482 MeV. This shows that the inclusion of three-body force in the determination of the binding energy of a three-body systems brings the results closer to experimental data.

REFERENCES

Baldo, M. and G.F. Bulgio, 2001. Microscopic theory of nuclear equation of state and neutron star structure in: D. Blaschke, N. Glendenning, A. Sedrakian (Eds)., 578: of Lecture notes in Physics, Springer, Heidelberg.

Berstch, G., W.Z.T. Boryso, H. Mcmanus and W.G. Love, 1977. Interaction for inelastic scattering derived from realistic potentials. NUcl. Phys. A., 284: 399-403.

Bertsch, G.F. and S. Das Gupta, 1988. Phase transformation in nuclear matter. Phys. Rep., 160: 189-194.

Chen, C.R., G.L. Payne, J.L. Friar and B.F. Gibson, 1985. Calculations of neutron-deuteron scattering. Phys. Rev. C., 31: 226-231.

Dao, T. Khoa and G.R. Satchler, 2000. Generalized folding model for inelastic nucleus-nucleus scattering using realistic density nucleon-nucleon interaction. Nucl. Phys. A., 668: 3-9.

Dao, T. Khoa, V.W. Oertzen and A.A. Ogloblin, 1996. Study of equation of state for asymmetry nuclear matter and interaction potential between neutron-rich nuclei using the density-dependent M3Y interaction. Nucl.Phys. A, 602: 98-103.

Dao, T., Khoa, G.R. Satchler and W. Oertzen von, 1997. Effective interaction for multistep processes. Phys. Rev. C., 53: 2-9.

Dithlas, F.M., 2006. The role of three-body force on binding energy of triton., M. Sc. Thesis, University of South Africa, Pretoria:

Faddeev, L.D., 1961. Few-body problems in physics. Sov. Phys. JETP, 2: 1014-1018.

Fiase, J.O. and F. Gbaorun, 2011. Binding energy and compression modulus of infinite nuclear mater derived from variational calculations. JNAMP, 91: 615-619.

Fiase, J.O. and L.K. Sharma, 2002. Missing ideas: self- consistent problem in th derivation of one- and two-body effective intraction for shell-model calculations. JNAMP, 6: 93-98.

Fiase, J.O., 2000. Mass dependence of single-particle energies for shell-model calculations. Phys. Rev. C., 63: 0373003-0373009.

Fiase, J.O., K.R.S. Devan and A. Hosaka, 2002. Mass dependence of M3Y-typeinteraction. Phys. Rev. C, 66: 014004-014012.

Friar, J.L., B.F. Gibson and G.L. Payne, 1981. Configuration space Faddeev calculations. Phys. Lett. A, 301: 309-314.

Friar, J.L., B.F. Gibson, E.L. Tomusiak and G.L. Payne, 1981. Configuration space Faddeev calculations, IV. Tri-nucleon charge density. Phys. Rev. C., 24(2): 665-671.

Glickle, W., 1983. The Quantum Mechanical few-body problems. Berlin-Heidelberg- NewYork: Springer: 243-246.

Hajduk, C., P.U. Sauer, H. Arenhovel, D. Dreschel and M.M.I. Gianini, 1981. Three-nucleon forces and three-nucleon systems. Phys. Rev. A., 352: 413-419.

Harper, E.P., Y.A. Kim and A. Tubis, 1972. Faddeev equation for realistic three-nucleon systems. II Bound state wave function. Phys. Rev. C., 6(1): 126-131.

Jackson, A.D. and J.A. Tjon, 1983. The role of three-body force in few body systems. Phys. Lett. B., 32: 9- 16.

Lekala, M.L., 2005. Numerical Calculations for few-body systems. Ph.D. Thesis, University of South Africa, Pretoria.

Merkurier, S.P., C. Gignoux and A. Laverne, 1976. Quantum scattering theory for several particle systems. Ann. Of Phys., 99: 30-37.

Misicu, S., 2007. Nuclear matter equation of state for the practitioner. Romanian Report in Physics, 59(4): 1127-1152.

Muslim Kim, Y.E. and T. Uede, 1982. Relativistic three-particle scattering equation. Phys. Lett. B., 115: 273-282.

Nogyn, A., H. Kamada and W. Glocide, 2000. Nuclear three- and four-body bound states. Phys. Rev. C., 2: 0004023- 0004029.

Noyes, H.P. and H. Fiedeldey, 1968. Three-particles scattering in Quantum Mechanics. Proceedings of Texas A and M conference, New York, 195-201.

Osman, A., 1970. Short range correlation in the three-body problem and the Faddeev equations. Nucl. Phys. A., 153: 524-530.

References should be cited in the text as: Hafez (2011) or (Hafez, 2011), Tajik and Nazifi (2011) or (Tajik and Nazifi, 2011), Bai et al. (2012) or (Bai et al., 2012).

Schellingerhou, N.W., 1995. Factorizability in the numerical few –body problems. Ph.D Thasis, University of Groningen, Netherlands.

Tajik, J. and S. Nazifi, 2011. A Study of Correlation of Serum Leptin with Trace Elements in Water Buffalo (Bubalus bubalis). Australian Journal of Basic and Applied Sciences, 31: 231-234.

The list of references at the end of manuscript must be arranged alphabetically and each reference in the list should appear in the following form:

Tomovska, J., S. Presilski, N. Gjorgievski, N. Tomovska, M.S. Qureshi and N.P. Bozinovska, 2013. Development of a spectrophotometric method for monitoring angiotensin-converting enzyme in dairy products. Pak Vet J., 33(1): 14-18.

Vittorio, S. and B. Piotre, 2008. Nuclear matter with three-body force from self-consistent spectra calculations. ACTA PHYSICA POLONICA B, 39(6): 1405-1410.

Voglis, C., I.E. Lagaris, M.L. Lekala, G.J. Rampho and S.A. Sofianos, 2007. Globa minimization in few-body systems. Nucl. Phys. A., (790): 655-660.

Zhang, Gao-Long, L, Hao and L.C. Xiao-yum, 2000. Properties of nulcei using equation of state. Phys. Rev. C., 26: 1621-1627.