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The Complementary Burr III Poisson Distribution

[1]Amal S. Hassan, [2]Abd-Elfattah, A.M. and [3]Asmaa H. Mokhtar

[1]*Institute of Statistical Studies&Research, Cairo University, Egypt.*
[2]*Institute of Statistical Studies& Research, Cairo University, Egypt.*
[3]*Modern Academy for Engineering & Technology, Department of Basic Sciences, Egypt.*

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ABSTRACT

In this paper, a new three-parameter lifetime distribution named as the complementary Burr III Poisson distribution is proposed. This distribution is obtained by mixing the distribution of the maximum of a random number of anindependent identically Burr III distributed random variables and zero truncated Poisson random variable. Some mathematical properties of new distribution such as quantile function, moments and moment generating function are derived. Furthermore, estimation by methods of maximum likelihood, moments and percentiles are discussed. An intensive simulation study is conducted for evaluating the performance of parameter estimation.

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INTRODUCTION

In recent years, compound distributions arise and applied in several areas, such as public health, economics, engineering, and industrial reliability. The compound models of distributions are well motivated for industrial applications and biological studies. Based on reliability problems, a new several compound models of distributions have been introduced when the lifetime can be expressed as the minimum or maximum of a sequence of independent and identical distributed (i.i.d.), random variable which represents the failure times of system components.

A lot of researchers have proposed a series of new compounding distributions by mixing the distribution of a minimum of a fixed number of independent for any continuous lifetime distributed random variables and discrete random variable. Among these, Adamidis and Loukas (1998), introduced an exponential-geometric distribution with decreasing failure rate by mixing an exponential and geometric distributions. Kus (2007) introduced a two-parameter exponential-Poisson, with decreasing failure rate, by mixing an exponential with Poisson distributions. Hemmati*et al*. (2011) introduced the Weibull-Poisson, which generalize the exponential Poisson distribution. Morais and Barreto-Souza (2011) defined the Weibull power series family which includes as sub-models the exponential power series distributions. The generalized exponential power series distributions were proposed by Mahmoudi and Jafari (2012) following the same approach of Morais and Barreto-Souza (2011) approach.

Recently, Silva *et al*. (2013) studied the extended Weibull power series family, which includes as special models the exponential power series and Weibull power series distributions. Silva and Cordeiro (2013) introduced a new family of Burr XII power series models. Abd-Elfattah*et al*. (2013) introduced Lomax-Poisson distribution with decreasing failure rate, the properties of the distribution are discussed. Estimations of the unknown parameters are derived by different methods. Asgharzadeh*et al*. (2014) introduced a general family of continuous lifetime distributions by compounding any continuous distribution and the Poisson–Lindley distribution. Several properties of this family are investigated. The parameters are estimated by the maximum likelihood method and the Fisher's information matrix is determined.

In the same way, several authors have proposed a new compounding distributions by mixing the distribution of a maximum of a fixed number of independent and identicallyfor continuous lifetime distributed random variables and discrete type. Rezaei and Tahmasbi (2012) introduced the exponential truncated Poisson with increasing failure rate, various properties are discussed and the estimation of the parameters are obtained via EM algorithm. Cancho*et al*. (2013) introduced a new

Corresponding Author: Amal S. Hassan, Institute of Statistical Studies&Research, Cairo University, Egypt.

lifetime distribution family, which is generalization of the complementary exponential geometric distribution. Leahu*et al*. (2013) introduced a compound family of distributions. This family is obtained as the maximum of sequence of iidrandom variables and the numbers of components are of power series random variable. Leahu*et al*. (2014) introduced two compound families, namely the max-Erlang power series distribution and min-Erlang power series distribution.

In this article, the complementary Burr III Poisson (CBIIIP) distribution is introduced. The rest of this article is organized as follows. CBIIIP distribution is defined by mixing the Burr III and zero truncated Poisson distributions, where the mixing procedure was previously proposed by Leahu*et al*. (2013). Some mathematical properties are derived,some distributions of order statistics are derived. Estimation of the model parameters is performed by maximum likelihood, moments and percentiles methods. A simulation study is carried out to illustrate theoretical results. Finally, conclusion are addressed.

The CBIIIP Model Distribution:

The aim in this section is to obtain the complementary Burr III Poisson distribution. This distribution is derived easily by substituting the probability density and cumulative distribution functions for Burr type III random variable in the class of lifetime distributions proposed by Leahu*et al*. (2013).

Burr (1942) introduced a system of twelve cumulative distribution functions for the primary purpose of fitting data. The Burr type III distribution properly approximates many familiar distributions such as normal, lognormal, gamma, Weibull, and exponential distributions. It plays an important role in reliability engineering, statistical quality control, and risk analysis models. The cumulative distribution function (cdf) of Burr III with two shape parameters c and k; denoted by Burr III (c, k); takes the following form:

$$\mathrm{G(y;\ c,k)} = (1+\mathrm{y^{-c}})^{-\mathrm{k}}, \mathrm{y,c,k} > 0 \qquad (1)$$

The corresponding probability density function (pdf) is

$$g(y;\ c,k) = cky^{-(c+1)}(1+y^{-c})^{-(k+1)}, y,c,k>0, \qquad (2)$$

Leahu*et al*. (2013) introduced a compound class of distributions. This class is obtained by mixing the maximum of a sequence of identically independent any lifetime distributed random variables with power series random variable. The class of Poisson and any proper lifetime distribution is also obtained as special case. Leahu*et al*. (2013) defined thepdf and the cdf of the complementary class of Poisson and any lifetime distributions as follows:

$$f(y;\lambda,\underline{\emptyset}) = \frac{\lambda g(y,\underline{\emptyset}) e^{\lambda G(y,\underline{\emptyset})}}{e^{\lambda}-1}, y>0, \lambda>0, \underline{\emptyset} = (\emptyset_1,\emptyset_2,\dots,\emptyset_l), \qquad (3)$$

and,

$$F(y;\lambda,\underline{\emptyset}) = \frac{e^{\lambda G(y,\underline{\emptyset})}-1}{e^{\lambda}-1}, y>0, \lambda>0, \underline{\emptyset} = (\emptyset_1,\emptyset_2,\dots,\emptyset_l). \qquad (4)$$

Therefore, the probability density function of the complementary Burr III Poisson distribution is obtained by substituting (1) and (2) in (3) as follows;

$$f(y;c,k,\lambda) = \frac{ck\lambda}{(e^{\lambda}-1)} y^{-(c+1)}(1+y^{-c})^{-(k+1)} e^{\lambda(1+y^{-c})^{-k}}, y>0, c,k,\lambda>0, \qquad (5)$$

where, $c,k$are shape parameters and λ is scale parameter of the Burr III- Poisson distribution.

Note that: The complementary Burr III Poissondistribution with parameter vector$\underline{\theta}$, where$\underline{\theta} \equiv (c,k,\lambda)$, reduces to the Burr type III as $\lambda \to 0$.

$$\lim_{\boldsymbol{\lambda\to 0}} f(y;\underline{\theta}) = \lim_{\boldsymbol{\lambda\to 0}} \frac{cky^{-(c+1)}(1+y^{-c})^{-(k+1)}\left\{e^{\lambda(1+y^{-c})^{-k}} + \lambda e^{\lambda(1+y^{-c})^{-k}}(1+y^{-c})^{-k}\right\}}{e^{\lambda}}$$

$$= cky^{-(c+1)}(1+y^{-c})^{-(k+1)},$$

which is the pdf of Burr type III distribution with parameters c and k.

The distribution function of CBIIIP is given by substituting (2) in (4) as follows

$$F(y;\underline{\theta}) = \frac{e^{\lambda(1+y^{-c})^{-k}}-1}{(e^{\lambda}-1)}, y>0, c,k,\lambda>0, \qquad (6)$$

Figures (1) and (2) represent the pdf and the cdf of complementary Burr III Poisson distribution for selected values of $(c,k,\lambda) = (5,5,5),\ (7,\ 5,\ 5),(5,7,5),(7,9,5),(9,5,7)and(5,7,9)$ respectively.

The reliability and hazard rate functions of the complementary Burr III Poisson distribution are obtained respectively, as follows

$$R(y;\ \underline{\theta}) = \frac{e^{\lambda}-e^{\lambda(1+y^{-c})^{-k}}}{(e^{\lambda}-1)}, \qquad (7)$$

and

$$h(y;\ \underline{\theta}) = \frac{ck\lambda y^{-(c+1)}(1+y^{-c})^{-(k+1)}e^{\lambda(1+y^{-c})^{-k}}}{e^{\lambda}-e^{\lambda(1+y^{-c})^{-k}}}. \qquad (8)$$

Figures (3) and (4) illustrate reliability and hazard rate functions of complementary Burr III Poisson distribution for selected values of the parameters $(c,k,\lambda) = (5,5,5),\ (7,\ 5,$

5),(5, 7, 5), (7, 9, 5), (9, 5, 7)and(5, 7, 9) respectively.

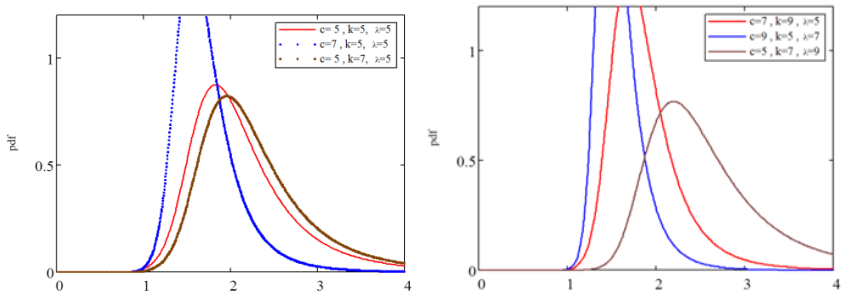

Fig. 1: Plots of the CBIIIP densities for some parameter values.

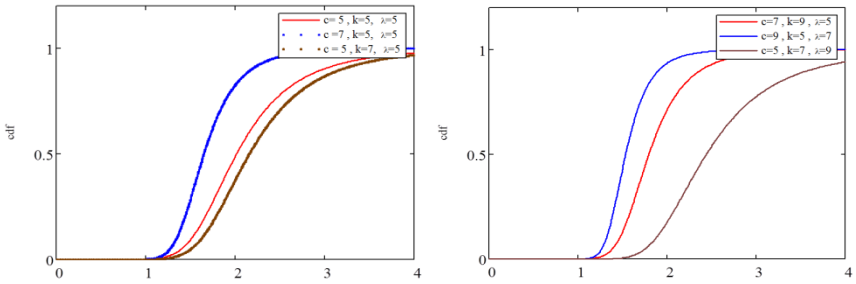

Fig. 2: Plots of CBIIIP distribution functions for some parameter values.

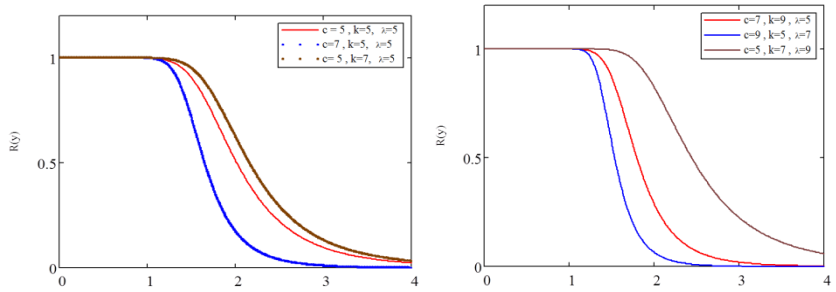

Fig. 3: Plots of CBIIIP reliability functions for some parameter values.

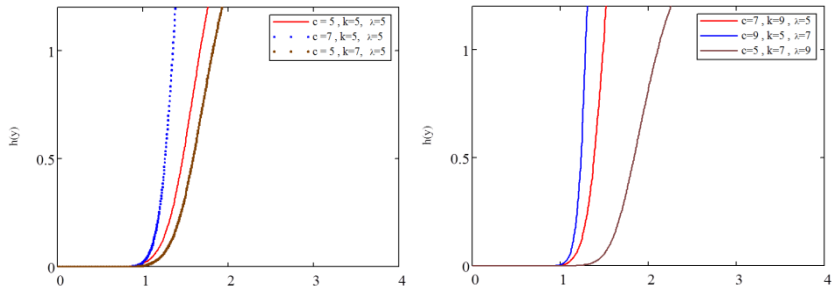

Fig. 4: Plots of the CBIIIP distribution hazard rates functions for some parameter values.

It is clear from Figure (4) that the hazard function takes different forms which are increasing and decreasing according to different values of the parameters.

Ouantiles, Moments and Moment Generating Function:

This section is devoted to study statistical properties of complementary Burr III Poisson distribution, specifically, quantile function, median, moments and moment generating function.

The quantile function, is used in theoretical aspects and statistical applications, thequantile function, say $Q(u)$ of y is the inverse of its cumulative distribution function and takes the following form

$$y = Q(u) = \left[\left[\frac{1}{\lambda}\ln\left[u\left(e^{\lambda}-1\right)+1\right]\right]^{\frac{-1}{k}} - 1\right]^{\frac{-1}{c}}, (9)$$

where u is a uniform random variable on the unit interval (0,1).In particular the median of the complementary Burr III Poisson distribution, say m, obtained by substituting, $u = \frac{1}{2}$ in (9) as follows

$$m = \left[\left[\frac{1}{\lambda}\left[\ln\left[\frac{e^{\lambda}+1}{2}\right]\right]\right]^{\frac{-1}{k}} - 1\right]^{\frac{-1}{c}}.$$

The rth raw moment of the of complementary Burr III Poisson distribution about the origin is derived as follows

$$\grave{\mu}_r = E(Y^r) = \frac{ck\lambda}{(e^{\lambda}-1)}\int_0^{\infty} y^{r-c-1}(1+y^{-c})^{-(k+1)}e^{\lambda(1+y^{-c})^{-k}}\,dy,$$

$$= \frac{ck\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\int_0^{\infty} y^{r-c-1}(1+y^{-c})^{-k-kS-1}\,dy.$$

$$\grave{\mu}_r = \frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{r}{c},\ \frac{r}{c}+k+kS\right), r < c \text{and }, r = 1,2,3,\ldots \quad (10)$$

In particular, setting $r = 1$ in (10), the mean of Y reduces to

$$E(Y) = \frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\ \frac{1}{c}+k+kS\right), \quad (11)$$

Also, the variance of complementary Burr III Poisson distribution takes the following form

$$V(Y) = \frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{2}{c},\ \frac{2}{c}+k+kS\right) - \left[\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\ \frac{1}{c}+k+kS\right)\right]^2,$$

Furthermore, the skewness and kurtosis for complementary Burr III Poisson distribution take the following forms$\alpha_3 =$

$$\frac{\frac{k\lambda}{(e^{\lambda}-1)}\left[\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{3}{c},\frac{3}{c}+k+kS\right)-3\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{2}{c},\frac{2}{c}+k+kS\right)\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\frac{1}{c}+k+kS\right)+2\frac{k^2\lambda^2}{(e^{\lambda}-1)^2}\left[\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\frac{1}{c}+k+kS\right)\right]^3\right]}{\left[\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{2}{c},\frac{2}{c}+k+kS\right)-\left[\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\frac{1}{c}+k+kS\right)\right]^2\right]^{\frac{3}{2}}},$$

$$\alpha_4 = \frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{4}{c},\frac{4}{c}+k+kS\right) - 4\frac{k^2\lambda^2}{(e^{\lambda}-1)^2}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\ \frac{1}{c}+k+kS\right)\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{3}{c},\ \frac{3}{c}+k+kS\right)+$$

$$6\frac{k^3\lambda^3}{(e^{\lambda}-1)^3}\left(\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\ \frac{1}{c}+k+kS\right)\right)^2\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{2}{c},\ \frac{2}{c}+k+kS\right) - 3\left(\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\right.\right.$$

$$\left.\left.\frac{1}{c}+k+kS\right)\right)^4\left[\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{2}{c},\ \frac{2}{c}+k+kS\right)-\left[\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\ \frac{1}{c}+k+kS\right)\right]^2\right]^{-2}.$$

It is easy to show that

$$M_y(t) = \sum_{0}^{\infty}\frac{t^r}{r!}\grave{\mu}_r,$$

where$\grave{\mu}_r$ is the rth raw moments. Therefore, the moment generating function of Y takes the following form

$$M_y(t) = \sum_{r=0}^{\infty}\frac{t^r}{r!}\frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{r}{c},\ \frac{r}{c}+k+kS\right). \quad (12)$$

Some Distributions of Order Statistics:

In this section, expressions for the pdf of the order statistics of the complementary Burr III Poisson distribution are derived. In particular, the distribution of smallest and largest order statistics are obtained. In addition, the joint probability density function of $Y_{(i)}$ and $Y_{(j)}$ is obtained.

Let $Y_1 < Y_2 < \cdots < Y_n$ be a random sample with probability density (3) and$Y_{(1)} < Y_{(2)} < \cdots < Y_{(n)}$ be the corresponding order statistics.

The density function of the ith order statistics, say $f_n\left(y_{(i)};\ \underline{\theta}\right)$, where$\underline{\theta} \equiv (c,k,\lambda)$, can be obtained as the follows

$$f_n\left(y_{(i)};\ \underline{\theta}\right) = \frac{n!\ ck\lambda y_{(i)}^{-(c+1)}\left(1+y_{(i)}^{-c}\right)^{-(k+1)}}{(i-1)!\ (n-i)!\,(e^{\lambda}-1)^n}e^{\lambda\left(1+y_{(i)}^{-c}\right)^{-k}}\left[e^{\lambda\left(1+y_{(i)}^{-c}\right)^{-k}}-1\right]^{i-1}\left[e^{\lambda}-e^{\lambda\left(1+y_{(i)}^{-c}\right)^{-k}}\right]^{n-i},$$

$$, 0 < y_{(i)} < \infty \quad (13)$$

The distribution of the smallest order statistic is given by substitute $i = 1$ in (13)

$$f_n(y_{(1)};\underline{\theta}) = \frac{nck\lambda\, y_{(1)}^{-(c+1)}}{(e^{\lambda}-1)^n}\left(1+y_{(1)}^{-c}\right)^{-(k+1)}\sum_{j=0}^{n-1}\binom{n-1}{j}(-1)^j\, e^{\lambda(n-j-1)}e^{\lambda\left(1+y_{(1)}^{-c}\right)^{-k(j+1)}}, 0<y_{(1)}<\infty \quad (14)$$

Also, the distribution of the largest order statistic is obtained by substitute $i=n$ in (13)

$$f_n(y_{(n)};\underline{\theta}) = \frac{nck\lambda\, y_{(n)}^{-(c+1)}}{(e^{\lambda}-1)^n}\left(1+y_{(n)}^{-c}\right)^{-(k+1)}\sum_{j=0}^{n-1}\binom{n-1}{j}(-1)^{n+j-1}e^{\lambda\left(1+y_{(n)}^{-c}\right)^{-k(j+1)}}, 0<y_{(n)}<\infty \quad (15)$$

The joint distribution of two order statistics $y_{(i)}$ and $y_{(j)}$ is given by

$$f_n(y_{(i)},y_{(j)}) = \frac{n!\; c^2k^2\lambda^2\left(1-e^{-\lambda}\right)^{-n}}{(i-1)!\,(j-i-1)!\;(n-j)!}y_{(i)}^{-(c+1)}y_{(j)}^{-(c+1)}\left(1+y_{(i)}^{-c}\right)^{-(k+1)}\left(1+y_{(j)}^{-c}\right)^{-(k+1)}e^{\lambda\left(1+y_{(i)}^{-c}\right)^{-k}}$$
$$e^{\lambda\left(1+y_{(j)}^{-c}\right)^{-k}}\left[e^{\lambda\left(1+y_{(i)}^{-c}\right)^{-k}}-1\right]^{i-1}\left[e^{\lambda\left(1+y_{(j)}^{-c}\right)^{-k}}-e^{\lambda\left(1+y_{(i)}^{-c}\right)^{-k}}\right]^{j-i-1}\left[e^{\lambda}-e^{\lambda\left(1+y_{(j)}^{-c}\right)^{-k}}\right]^{n-j}$$
$$, 0<y_{(i)}<y_{(j)}<\infty \quad (16)$$

Parameter Estimation:

Some methods of estimation for the unknown parameters of complementary Burr III Poisson distribution will be studied as follows.

Method of Maximum Likelihood:

The maximum likelihood method principle is one of the most important methods in the estimation theory which consists of choosing, as an estimator of the parameter that maximizes the probability (likelihood) of the sample data. The likelihood function of the n random variables is the joint density of the n random variables.

Let $Y_1, Y_2, \dots, Y_n$ be a random sample of size n from a complementary Burr III Poisson distribution with parameters $\underline{\theta} \equiv (c,k,\lambda)$, the likelihood function based on n observed sample is given by

$$L(\underline{\theta}\,;\,y) = c^n k^n \lambda^n \left(e^{\lambda}-1\right)^{-n}\prod_{i=1}^{n} y_i^{-c-1}\prod_{i=1}^{n}(1+y_i^{-c})^{-(k+1)}e^{\lambda\sum_{i=1}^{n}\left(1+y_i^{-c}\right)^{-k}}.$$

The natural logarithm of the likelihood function $L \equiv L(\underline{\theta};y)$ is given by

$$\ln L = n\ln c + n\ln k + n\ln\lambda - n\ln\left(e^{\lambda}-1\right) - (c+1)\sum_{i=1}^{n}\ln y_i - (k+1)\sum_{i=1}^{n} ln(1+y_i^{-c}) + \lambda\sum_{i=1}^{n}(1+y_i^{-c})^{-k}.$$

The maximum likelihood estimates of c,k,λ, say $\hat{c}\,,\hat{k},\hat{\lambda}$, are obtained by setting the first partial derivatives of ln L to be zero with respective to c,k and λ. These simultaneous equations are as follows,

$$\frac{\partial \ln L}{\partial c} = \frac{n}{\hat{c}} - \sum_{i=1}^{n}\ln y_i + \left(\hat{k}+1\right)\sum_{i=1}^{n}\frac{\ln y_i}{\left(y_i^{\hat{c}}+1\right)} + \hat{\lambda}\hat{k}\sum_{i=1}^{n}\left(1+y_i^{-\hat{c}}\right)^{-\left(\hat{k}+1\right)}y_i^{-\hat{c}}\ln y_i = 0, \quad (17)$$

$$\frac{\partial \ln L}{\partial k} = \frac{n}{\hat{k}} - \sum_{i=1}^{n}\ln\left(1+y_i^{-\hat{c}}\right) - \hat{\lambda}\sum_{i=1}^{n}\left(1+y_i^{-\hat{c}}\right)^{-\hat{k}}\ln\left(1+y_i^{-\hat{c}}\right) = 0, \quad (18)$$

and,

$$\frac{\partial \ln L}{\partial \lambda} = \frac{n}{\hat{\lambda}} - \frac{n}{\left(1-e^{-\hat{\lambda}}\right)} + \sum_{i=1}^{n}\left(1+y_i^{-\hat{c}}\right)^{-\hat{k}} = 0. \quad (19)$$

It is not easy to solve the equations in $\hat{c}\,,\hat{k}$ and $\hat{\lambda}$, so numerical iteration method will be proposed to solve nonlinear Equations (17), (18) and (19).

Method of Moments:

Let $Y_1, Y_2, \dots, Y_n$ be an independent and identicallydistributed random variables, each has the CBIIIP distribution, then the first three population moments are given respectively by using (10) as the following

$$\mu_1' = \frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{1}{c},\ \frac{1}{c}+k+kS\right),$$

$$\mu_2' = \frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{2}{c},\frac{2}{c}+k+kS\right),$$

and

$$\mu_3' = \frac{k\lambda}{(e^{\lambda}-1)}\sum_{S=0}^{\infty}\frac{\lambda^S}{S!}\beta\left(1-\frac{3}{c},\frac{3}{c}+k+kS\right).$$

Let $Y_1, Y_2, \dots, Y_n$ be a random sample of size n drawn from complementary Burr III Poisson distribution, the first three sample moments are given by

$$\acute{m}_1 = \frac{1}{n}\sum_{i=1}^{n} y_i,\ \acute{m}_2 = \frac{1}{n}\sum_{i=1}^{n} y_i^2 \text{ and } \acute{m}_3 = \frac{1}{n}\sum_{i=1}^{n} y_i^3.$$

The method of moments principle is to choose estimators say, $\overline{c}, \overline{k}$ and $\overline{\lambda}$ of parameters c,k and

λthat make the population moments equal the sample moments

$$\frac{\overline{k}\overline{\lambda}}{\left(e^{\overline{\lambda}}-1\right)}\sum_{S=0}^{\infty}\frac{\overline{\lambda}^{S}}{S!}\beta\left(1-\frac{1}{\overline{c}},\ \frac{1}{\overline{c}}+\overline{k}+\overline{k}S\right)=\frac{1}{n}\sum_{i=1}^{n}y_i,(20)$$

$$\frac{\overline{k}\overline{\lambda}}{\left(e^{\overline{\lambda}}-1\right)}\sum_{S=0}^{\infty}\frac{\overline{\lambda}^{S}}{S!}\beta\left(1-\frac{2}{\overline{c}},\ \frac{2}{\overline{c}}+\overline{k}+\overline{k}S\right)=\frac{1}{n}\sum_{i=1}^{n}y_i^{2},(21)$$

and,

$$\frac{\overline{k}\overline{\lambda}}{\left(e^{\overline{\lambda}}-1\right)}\sum_{S=0}^{\infty}\frac{\overline{\lambda}^{S}}{S!}\beta\left(1-\frac{3}{\overline{c}},\ \frac{3}{\overline{c}}+\overline{k}+\overline{k}S\right)=\frac{1}{n}\sum_{i=1}^{n}y_i^{3}.\ (22)$$

Equations (20) to (22) have no explicit solution, so numerical iterative technique is applied to obtain themoment estimators; $\overline{c},\overline{k}$ and $\overline{\lambda}$.

Method of Percentiles:

Let $Y_1,Y_2,..,Y_n$ is a random sample from the complementary Burr III Poisson distribution function $F(y;c,k,\lambda)$ and $Y_{(i)}$ denotes the i^{th} order statistic, i.e., $Y_{(1)}<Y_{(2)}<..<Y_{(n)}$. If $p_i$denotes some estimates of $F(Y_{(i)};c,k,\lambda)$, then the estimate of c,k and λ can be obtained by minimizing the following equation with respect to c,k andλ .

$\sum_{i=1}^{n}[\ln(p_i)-\ln[F(Y_{(i)};c,k,\lambda)]]^2$.(23)

It is possible to use several p_i as estimates of $F(Y;c,k,\lambda)$. For example $p_i=(i/(n+1))$ is the most used estimator of $F(Y_{(i)};c,k,\lambda)$, where $(i/(n+1))$ is an unbiased estimator of $F(Y_{(i)};c,k,\lambda)$. Percentiles estimators$\tilde{c}$,$\tilde{k}$ and $\tilde{\lambda}$ of the complementary Burr III Poisson distribution can be obtained by minimizing

$$\sum_{i=1}^{n}\left[\ln(p_i)-\ln\left[\frac{1}{(e^{\lambda}-1)}\left[e^{\lambda\left(1+y_{(i)}^{-c}\right)^{-k}}-1\right]\right]\right]^2,(24)$$

with respect to c,k and λ.

Simulation Study:

In this section a simulation study is performed to compare the different estimators proposed above. The maximum likelihood, moments and percentiles estimators of the model parameters c,k and λ for the complementary Burr III Poisson distribution are obtained. The performances of the different estimators will be compared through their biases and mean square errors (MSEs), for different sample sizes and for different parametric values. The simulation procedures are described through the following steps

Step (1): A random sample $Y_1,Y_2,\dots,Y_n$ of sizes n= 10 (10) 50 isselected; these random samples are generated from the complementary Burr III Poisson distribution.

Step (2):Different values of the parameters will be selected as, $c=(3.1,3.2)$, $k=(0.5,\ 0.8,\ 1.1,1.4)$ and $\lambda=1$.

Step (3): The maximum likelihood estimators $\hat{c}$, $\hat{k}$ and $\hat{\lambda}$ will be obtained by solving the nonlinear Equations (17), (18) and (19). The moments estimators;$\bar{c},\bar{k}$ and $\bar{\lambda}$ can be obtained by solving thenonlinear Equations (20), (21) and (22). Also the percentiles estimators $\tilde{c}$,$\tilde{k}$ and $\tilde{\lambda}$ can be obtained by minimizing Equation (24) with respect to c, k, andλ respectively.

Step (4): Steps from 1 to 3 will be repeated 1000 times for each sample size and for the selected sets of parameters. Then, biases and mean square errors of different estimators of unknown parameters are computed.

Simulation results are reported in Tables (1) and (2) and represented through some Figures from (5) to (9). From these tables, the following conclusions can be observed on the performance of different estimators.

1- The MSEs of the maximum likelihood estimators $\hat{c},\hat{k}$ and $\hat{\lambda}$ decrease as the sample sizes increase for different selected set of parameters (see for example Figure (5) for the set of parameters $(c=3.2,k=1.4,\lambda=1)$. The MSEs of the percentiles estimators;$\tilde{c},\tilde{k}$and $\tilde{\lambda}$decrease as the sample sizes increase for different selected set of parameters (see for example Figure (6) for the set$(c=3.1,k=1.4,\lambda=1)$.

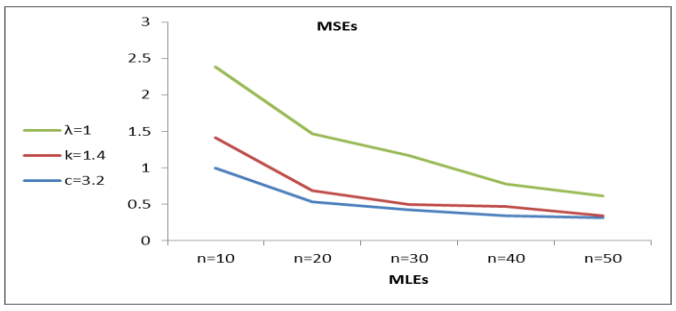

Fig. 5: MSEs for MLEs forthe set$(c=3.2,k=1.4,\lambda=1)$.

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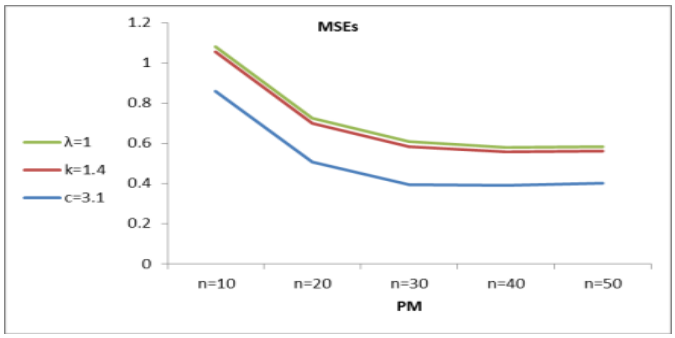

Fig. 6: MSEs for percentiles estimators for the set $(c = 3.1, k = 1.4, \lambda = 1)$.

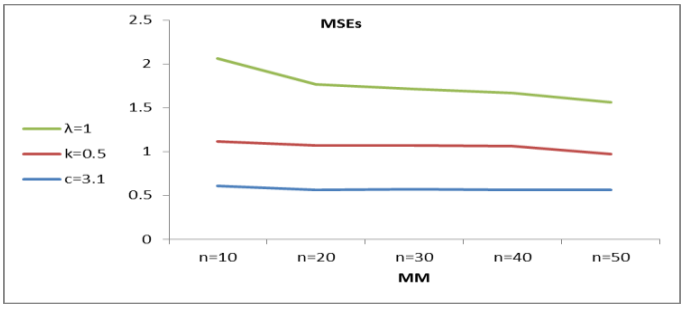

Fig. 7: MSEs for moment estimators for the set $(c = 3.1, k = 0.5, \lambda = 1)$.

2- The MSEs of the most moment estimators $\overline{c}, \overline{k}$, and $\overline{\lambda}$ decrease as the sample sizes increase for different selected set of parameters (see for example Figure (7) for the set$(c = 3.1, k = 0.5, \lambda = 1)$.

3- The MSEs for the percentiles estimators,$\tilde{c}, \tilde{k}$ and$\tilde{\lambda}$ take the smallest value among the corresponding MSEs for the other methods in almost all the cases.

4– The MSEs for the percentiles estimator, $\tilde{\lambda}$ is approximately constant in almost all the selected set of parameters.

5- For the values $c = 3.1$and$\lambda = 1$, the biases of c in the maximum likelihood method decrease as the value of k increases. Also,the biases of k increase as the values of k increase for different set of parameters,in almost all the cases.

6- The biases of c is approximately constant in the moments method for different set of parameters in almost all the cases.

7-For the values of $c = 3.1, 3.2$and $\lambda = 1$, the biases of λ increases as the value of k increases in the percentiles method in different sets of parameters in almost all the cases.

8- As it seems from Figures (8 a) and (8 b)**,** the MSEs of the MLEs of$c$ take the smallest values corresponding to the other estimators $\tilde{c}$ and $\bar{c}$ for the same sample size and for the four set of parameters. Also, from Figure (8 a) the MSEs of MLEs of c for the set of parameters $(c = 3.1, k = 1.4, \lambda = 1)$ have the smallest values corresponding to the other set of parameters for the same sample size. The MSEs of MLEs of c for the set of parameters $(c = 3.2, k = 0.5, \lambda = 1)$ have the smallest values corresponding to the other set of parameters for the same sample size (see Figure (8 b)).

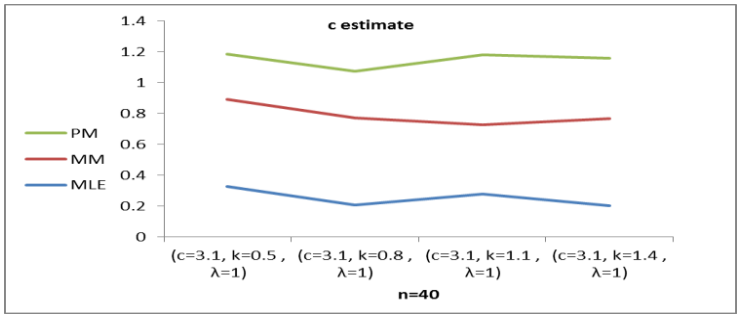

Fig. 8a:MSEs of the estimate c for the different method of estimation and different set of parameters.

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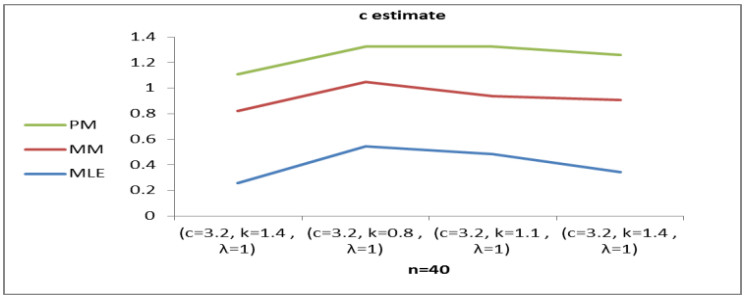

Fig. 8b:MSEs of the estimate c for the different method of estimation and different set of parameters.

9- As it seems from Figures (9 a) and (9 b)**,** the MSEs of the MLEs of$k$ take the smallest values corresponding to the other estimators $\tilde{k}$ and $\bar{k}$ for the same sample size. Also the MSEs of MLEs of k for the set of parameters $(c = 3.1, k = 1.1, \lambda = 1)$ have the smallest values corresponding to the other set of parameters for the same sample size (see Figure (9 a). The MSEs of MLEs of k for the set of parameters $(c = 3.2, k = 1.4, \lambda = 1)$ have the smallest values corresponding to the other set of parameters for the same sample size (see Figure (9 b)).

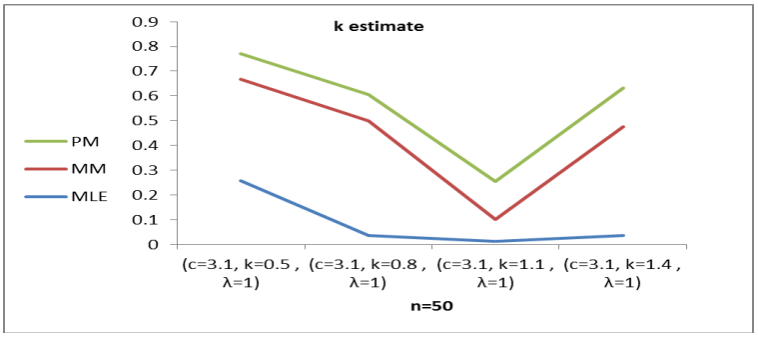

Fig. 9a:MSEs of the estimate k for the different method of estimation and different set of parameters

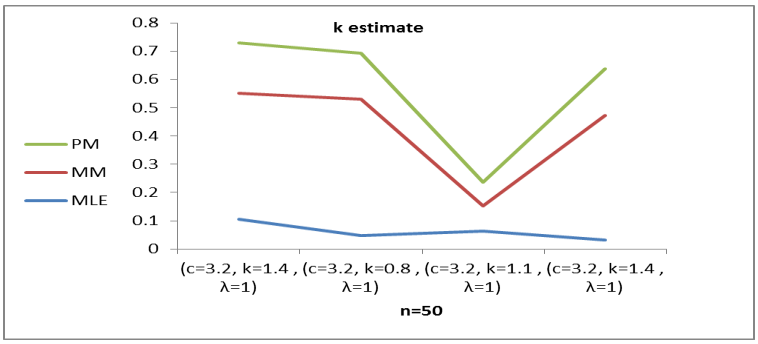

Fig. 9b:MSEs of the estimate k for the different method of estimation and different set of parameters

Conclusions:

In this paper, a new distribution called the complementary Burr III Poisson distribution with three-parameter isproposed. The plots of pdf, cdf, reliability and hazard rate functions are presented to show the flexibility of the new distribution. Some mathematical properties such as the rth raw moment and moment generating function are derived. Estimation problem of model parameters for CBIIIP distribution based on maximum likelihood, moments, and percentiles methods are provided. The biases and MSEs are calculated, to compare different estimators, via simulation technique.

In general, it is observed that the biases and MSEs of all estimators decrease as sample size increases in almost all the cases. Furthermore the MSEs for the percentiles estimators $\tilde{c}, \tilde{k}$ and $\tilde{\lambda}$havethe smallestvalue corresponding to the MSEs ofthe other estimators in almost all the cases.

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Table 1: Results of simulation study of biases and MSEs of estimates for different values of parameters $(c, k\lambda)$ for the complementary Burr III Poisson distribution**.**

		Set of parameters												
Sample Size	Method	Properties	(c= 3.1 , k= 0.5, λ = 1)			(c= 3.1 , k= 0.8 , λ = 1)			(c= 3.1 , k= 1.1 , λ = 1)			(c= 3.1 , k= 1.4 , λ = 1)		
n=10	ML	Bias	0.280	-0.446	0.533	-0.246	-0.237	-0.372	0.104	0.237	-0.629	-0.113	0.496	-0.498
		MSE	0.779	0.890	0.872	0.954	0.743	0.833	0.754	0.188	0.815	0.908	0.368	0.597
	MM	Bias	-0.746	-0.495	-0.309	-0.753	-0.646	-0.238	-0.673	-0.224	-0.824	-0.755	-0.554	-0.163
		MSE	0.613	0.507	0.942	0.654	0.486	0.998	0.461	0.097	0.865	0.646	0.414	0.760
	PM	Bias	-0.380	0.270	-0.181	-0.395	0.308	-0.161	-0.489	0.355	-0.162	-0.554	0.324	-0.160
		MSE	0.383	0.181	0.064	0.582	0.181	0.270	0.723	0.156	0.027	0.859	0.195	0.026
n=20	ML	Bias	0.294	-0.833	0.899	0.363	-0.657	0.908	0.029	-0.183	0.712	-0.079	0.107	0.699
		MSE	0.675	0.854	0.817	0.650	0.448	0.803	0.418	0.066	0.793	0.430	0.068	0.551
	MM	Bias	-0.756	-0.495	-0.276	-0.745	-0.606	-0.266	-0.668	-0.238	-0.760	-0.748	-0.589	-0.092
		MSE	0.567	0.506	0.692	0.568	0.473	0.985	0.456	0.097	0.736	0.563	0.426	0.746
	PM	Bias	-0.393	0.352	-0.162	-0.426	0.367	-0.157	-0.460	0.380	-0.152	-0.489	0.377	-0.150
		MSE	0.269	0.177	0.037	0.378	0.174	0.028	0.492	0.159	0.029	0.506	0.194	0.025
n=30	ML	Bias	0.144	-0.888	0.984	0.247	-0.859	0.893	-0.250	-0.026	0.900	-0.162	-0.487	0.801
		MSE	0.464	0.790	0.607	0.414	0.203	0.756	0.324	0.027	0.561	0.278	0.258	0.547
	MM	Bias	-0.747	-0.495	-0.244	-0.745	-0.606	-0.228	-0.662	-0.248	-0.709	-0.753	-0.517	-0.247
		MSE	0.571	0.497	0.644	0.568	0.472	0.960	0.450	0.094	0.647	0.646	0.441	0.757
	PM	Bias	-0.455	0.397	-0.158	-0.400	0.398	-0.157	-0.494	0.397	-0.152	-0.433	0.414	-0.133
		MSE	0.282	0.165	0.028	0.344	0.162	0.026	0.431	0.167	0.026	0.394	0.188	0.026
n=40	ML	Bias	0.071	-0.923	0.436	0.075	-0.215	0.686	0.288	-0.103	0.888	-0.221	-0.183	0.687
		MSE	0.217	0.699	0.607	0.209	0.049	0.474	0.276	0.016	0.517	0.204	0.044	0.475
	MM	Bias	-0.751	-0.682	-0.195	-0.757	-0.598	-0.253	-0.569	-0.241	-0.700	-0.740	-0.522	-0.218
		MSE	0.564	0.499	0.604	0.562	0.476	0.903	0.449	0.089	0.637	0.563	0.428	0.755
	PM	Bias	-0.478	0.415	-0.158	-0.456	0.415	-0.197	-0.544	-0.386	-0.156	-0.477	0.424	-0.144
		MSE	0.291	0.132	0.029	0.302	0.141	0.027	0.454	0.158	0.033	0.391	0.166	0.023
n=50	ML	Bias	0.005	-0.943	0.663	0.037	-0.247	0.149	0.236	-0.153	-0.530	-0.259	-0.854	0.270
		MSE	0.226	0.162	0.331	0.181	0.063	0.756	0.207	0.013	0.284	0.200	0.035	0.076
	MM	Bias	-0.747	-0.677	-0.215	-0.746	-0.591	-0.263	-0.656	-0.250	-0.682	-0.740	-0.518	-0.250
		MSE	0.563	0.412	0.586	0.562	0.464	0.557	0.437	0.089	0.597	0.606	0.440	0.754
	PM	Bias	-0.463	0.418	-0.155	-0.467	0.424	-0.148	-0.543	-0.381	-0.150	-0.508	0.418	-0.127
		MSE	0.272	0.173	0.027	0.282	0.106	0.025	0.450	0.151	0.028	0.403	0.158	0.023

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Table 2: Results of simulation study of biases and MSEs of estimates for different values of parameters $(c, k\lambda)$ for the complementary Burr III Poisson distribution.

		Set of parameters												
Sample Size	Method	Properties	(c= 3.2 , k= 0.5 , λ = 1)			(c= 3.2 , k= 0.8 , λ = 1)			(c= 3.2 , k= 1.1 , λ = 1)			(c= 3.2 , k= 1.4 , λ = 1)		
n=10	ML	Bias	0.244	-0.234	0.798	-0.705	0.152	-0.612	0.031	0.435	-0.798	-0.326	0.520	-0.795
		MSE	0.991	0.061	0.646	0.978	0.067	0.389	0.974	0.285	0.845	0.996	0.420	0.967
	MM	Bias	-0.745	-0.675	-0.273	-0.749	-0.656	-0.202	-0.677	-0.238	-0.733	-0.760	-0.562	-0.149
		MSE	0.560	0.528	0.755	0.565	0.489	0.601	0.465	0.104	0.708	0.660	0.455	0.991
	PM	Bias	-0.359	0.287	-0.163	-0.306	0.307	-0.162	-0.463	0.320	-0.161	-0.439	-0.342	-0.161
		MSE	0.369	0.198	0.028	0.570	0.106	0.027	0.781	0.184	0.028	0.802	0.194	0.027
n=20	ML	Bias	-0.350	-0.028	-0.561	-0.350	-0.055	0.799	-0.321	-0.056	-0.746	-0.287	-0.135	-0.584
		MSE	0.948	0.016	0.324	0.732	0.021	0.366	0.765	0.066	0.859	0.530	0.154	0.783
	MM	Bias	-0.751	-0.691	-0.219	-0.750	-0.646	-0.149	-0.671	-0.243	-0.691	-0.751	-0.590	-0.111
		MSE	0.566	0.508	0.558	0.566	0.459	0.552	0.465	0.099	0.621	0.604	0.459	0.972
	PM	Bias	-0.415	0.358	-0.157	-0.442	0.370	-0.157	-0.487	0.369	-0.154	0.548	0.381	-0.148
		MSE	0.348	0.161	0.028	0.405	0.142	0.027	0.544	0.180	0.028	0.548	0.185	0.027
n=30	ML	Bias	0.024	-0.287	0.792	-0.417	-0.162	0.695	-0.331	-0.165	-0.646	-0.350	-0.270	-0.581
		MSE	0.353	0.084	0.631	0.539	0.020	0.489	0.603	0.033	0.624	0.420	0.079	0.673
	MM	Bias	-0.753	-0.687	-0.197	-0.753	-0.654	-0.094	-0.668	-0.250	-0.636	-0.748	-0.566	-0.148
		MSE	0.568	0.496	0.531	0.560	0.458	0.500	0.452	0.095	0.538	0.594	0.455	0.927
	PM	Bias	-0.447	0.391	-0.156	-0.462	0.422	-0.146	-0.466	0.402	-0.147	-0.424	0.408	-0.146
		MSE	0.286	0.174	0.035	0.301	0.180	0.026	0.391	0.176	0.027	0.395	0.181	0.029
n=40	ML	Bias	-0.111	-0.343	0.863	-0.603	-0.250	0.263	-0.311	-0.222	-0.659	-0.351	-0.344	-0.482
		MSE	0.256	0.118	0.778	0.257	0.064	0.172	0.387	0.069	0.618	0.344	0.122	0.315
	MM	Bias	-0.752	-0.686	-0.194	-0.752	-0.642	-0.135	-0.663	-0.251	-0.632	-0.739	-0.534	-0.208
		MSE	0.568	0.494	0.505	0.503	0.455	0.558	0.451	0.092	0.526	0.563	0.449	0.916
	PM	Bias	-0.469	0.412	-0.156	-0.434	0.417	-0.156	-0.493	0.410	-0.145	-0.470	0.418	-0.148
		MSE	0.286	0.159	0.029	0.279	0.175	0.025	0.389	0.171	0.026	0.352	0.172	0.032
n=50	ML	Bias	-0.037	-0.323	0.880	-0.507	-0.212	0.885	-0.294	-0.260	-0.669	-0.361	-0.389	-0.495
		MSE	0.232	0.106	0.747	0.254	0.048	0.105	0.344	0.063	0.604	0.310	0.032	0.271
	MM	Bias	-0.756	-0.682	-0.194	-0.749	-0.658	-0.120	-0.667	-0.252	-0.634	-0.746	-0.531	-0.188
		MSE	0.542	0.444	0.405	0.566	0.482	0.572	0.450	0.089	0.527	0.506	0.440	0.906
	PM	Bias	-0.460	0.420	-0.158	-0.451	0.402	-0.155	-0.480	0.423	-0.144	-0.496	0.426	-0.138
		MSE	0.269	0.180	0.027	0.251	0.164	0.026	0.350	0.084	0.026	0.355	0.166	0.025

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