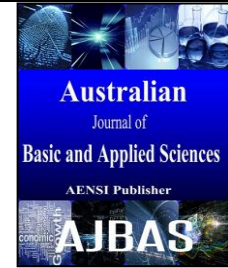




ISSN:1991-8178

Australian Journal of Basic and Applied Sciences

Journal home page: www.ajbasweb.com



Effect of Control Measures on the Dynamics Model of Cholera

¹Techinee Sathitdat, ²Prasit Thongjaem and ²Surapol Naowarat

^{1,2}Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University, Surat Thani, 84100, Thailand

ARTICLE INFO

Article history:

Received 16 April 2015

Accepted 12 June 2015

Available online 16 June 2015

Keywords:

Mathematical model, Cholera,
Stability analysis, Cholera
reproductive number,
Control measures

ABSTRACT

A mathematical model for the dynamics model of Cholera, caused by *Vibrio Cholerae* is proposed and analyzed. The effect of control measures (effectiveness of education program and infected human, cost of treatment) is taken into account. The model is analyzed using stability theory of ordinary differential equations. The disease free equilibrium and endemic equilibrium are determined and the stabilities of each equilibrium points are determined. Cholera reproductive number is found using the next generation matrix. The numerical results shown that when the value of the contact rate between the susceptible human and the infected human decreased, the number of infected human decreased and if the cost of treatment increased, the number of infected human decreased. Numerical simulations are also carried out to support the analytic results for the model.

© 2015 AENSI Publisher All rights reserved.

To Cite This Article: Techinee Sathitdat, Prasit Thongjaem and Surapol Naowarat. Effect of Control Measures on the Dynamics Model of Cholera. *Aust. J. Basic & Appl. Sci.*, 9(20): 625-631, 2015

INTRODUCTION

Cholera has become a worldwide health problem. It is an acute infection caused by the colonization and multiplication of *Vibrio cholerae* 01 or 0139 within human small intestine. It is a waterborne disease that causes severe diarrhea and vomiting which leads to dehydration of the body and can prove fatal unless treated quickly. Outbreaks usually result from contaminated food, poor sanitation and dirty drinking water (WHO, 2013).

Cholera was first mentioned in the India subcontinent in 1817. The disease spread through Asia continent in the 1960s, reached Africa in 1970 and Latin America in 1991 (Codeco, 2001; Wearing, 2005). Cholera epidemic was first presented formally in Snow's seminal work in 1855 (Codeco, 2001). In this study showed that cholera associated with contaminated water supply. In 1960, Cockburn and Cassanos (1960) published an article on epidemiology of endemic cholera in Asia. After that McCormack *et al.* (1969) investigated cholera in a rural community of Pakistan (McCormack, 1969). This study investigated *V. cholerae* in its aquatic habitat using a mathematical model to describe its nature in this habitat (Isere and Osemwenkhae, 2010). In 2014, Fatima and Krishnarajah proposed and analyzed a mathematical model for the control of cholera in Nigeria with modifications as compared

to previous cholera models. The dynamics of cholera is analyzed in this study using a system of four differential equations with two control measures τ and ω , which are; therapeutic treatment and sanitary measures, respectively. The foregoing articles lay credence to the fact that *V. cholerae* 01 was the main causative agent of cholera and much effort was directed towards its control.

Mathematical models have become important tool in analyzing the spread and control of infectious diseases. Ochoche (2013) proposed the mathematical model of cholera transmission using water treatment as a control strategy. The assumption of the model was the cholera is contacted only through the ingestion of contaminated water. Conditions of the existence of the disease free and endemic equilibrium point are derived and proved that the disease free equilibrium point is locally asymptotically stable under the given parameters. Fakai *et al.* (2013) proposed the modification model of the previous cholera models. Model analysis was applied on the Jacobian matrix assuming zero *V. cholerae* environments. The basic reproductive number (R_0) and the critical number (R_c) were obtained.

The objective of the study to determine the effects of education program on the dynamics transmission of cholera model. The structure of this paper is organized as follows. In section 2, we present a formulation model with the influence of education program. In section 3, we analyze the

Corresponding Author: Surapol Naowarat, Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University, Surat Thani, Thailand, 84100,
E-mail: su1963n@gmail.com, ph: +66-842462496.

model by using the stability differential equations theory, to determine both disease free and endemic equilibrium point, derive the basic reproductive number and investigate the stability of the model. In section 4, we simulate the numerical results of the model numerically, which support our analytic results. In section 5, we discuss the results. Finally, we summarized the conclusions of our study in section 6.

Model Formulation:

For this study, we formulated (SIB) model (Susceptible-Infected-Contaminated water) for the dynamics transmission of Cholera. There are two groups of population which consist of human population and concentration of the *Vibrios* environment. The diagram of the transmission of the Cholera disease as shown in Fig. 1.

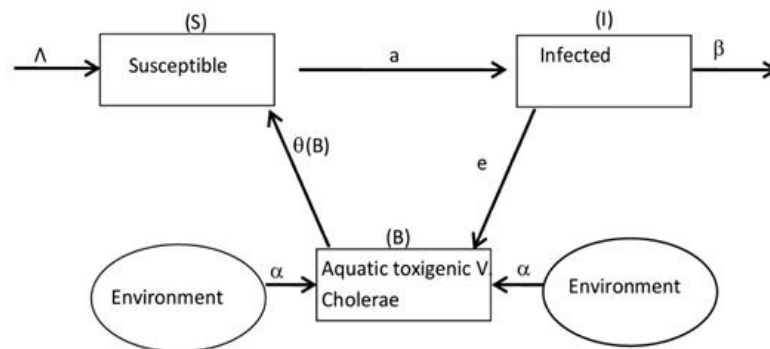


Fig. 1: Flow chart of the dynamical transmission of Cholera.

We defined:

$S(t)$ is the number of susceptible human at time t ,

$I(t)$ is the number of infected human at time t ,

$B(t)$ is the concentration of toxigenic *V. cholerae* in water.

The transmission model will be a system of ordinary differential equations given by,

$$\frac{dS}{dt} = \wedge - \frac{aB(1-u)S}{k+B} - \mu S \quad (4.1)$$

$$\frac{dI}{dt} = \frac{aB(1-u)S}{k+B} - (\mu+d+\beta+v)I \quad (4.2)$$

$$\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right) + eI - nB \quad (4.3)$$

where

\wedge is the recruitment rate of the susceptible human,

a is the per capita exposure rate to contaminated water,

u is the effectiveness of education program,

μ is the per capita natural death rate of human,

d is the per capita cholera related death rate,

k is the concentration of *V. cholerae* in water that yields 50% chance of catching cholera,

β is the rate at which infected human recover from cholera,

v is the rate increase cost of treatment by the drug,

r is the growth rate of *V. cholerae* in the aquatic environment,

K is the carrying capacity of *V. cholerae*,

n is the loss rate of *V. cholerae* in the aquatic environment,

e is the contribution of each infected human to concentration of *V. cholerae* in water,

α is the net mortality rate of *V. cholerae* in the aquatic environment.

Model Analysis

Equilibrium Points:

1. **Disease free equilibrium point** (E_0): there are no infected human and no *V. cholerae* in water, that is $I=0$. Substituting this in equations (1)-(3),

we are obtained: $E_0(S, I, B) = E_0\left(\frac{\wedge}{\mu}, 0, 0\right)$.

2. **Endemic equilibrium point** (E_1): In case B^* , we obtained

$$E_1 = (S^*, I^*, B^*)$$

$$S^* = \frac{\wedge}{\frac{aB^*}{k+B^*}(1-u) + \mu}, I^* = \frac{\wedge aB^*(1-u)}{aB^*(1-u)(\mu+d+\beta+v) + \mu(k+B^*)(\mu+d+\beta+v)},$$

$$B^* = \frac{Y + \sqrt{Y^2 - 4XZ}}{2X}$$

where

$$X = EC + GC, \quad Y = Er + Gr + CF - En - Gn, \quad Z = Fr - Fn - D$$

$$A = \mu + d + \beta + v, C = \frac{r}{k}, D = ea(1-u)(\mu + d + \beta + v), E = a(1-u)(\mu + d + \beta + v),$$

$$F = \mu k(\mu + d + \beta + v), G = \mu(\mu + d + \beta + v).$$

Cholera Reproductive number :

The cholera reproductive number (R_0) (threshold condition in epidemiology) is the number of secondary infections induced by an infected individual introduced into the total susceptible

population (Anderson and May, 1991). By using the next generation method and used spectral radius (Van den Driessche and Watmough, 2002). Doing this, we rewritten the system in matrix form.

$$\frac{dx}{dt} = F(x) - V(x)$$

$$x = \begin{bmatrix} S \\ I \\ B \end{bmatrix}, F(x) = \begin{bmatrix} 0 \\ \frac{aBS(1-u)}{k+B} \\ 0 \end{bmatrix} \text{ and } V(x) = \begin{bmatrix} -\wedge + \frac{aBS(1-u)}{k+B} + \mu S \\ (\mu + d + \beta + v)I \\ -rB(1 - \frac{B}{k}) + nB - eI \end{bmatrix}.$$

Find the Jacobian of $F(x)$ and $V(x)$, denoted by $DF(x)=F$ and $DV(x)=V$, we obtained

$$F = \begin{bmatrix} 0 & 0 & 0 \\ \frac{aB(1-u)}{k+B} & 0 & \frac{(k+B)aS(1-u) - aBS(1-u)}{(k+B)^2} \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$V = \begin{bmatrix} \frac{aB(1-u)}{k+B} + \mu & 0 & \frac{(k+B)aS(1-u) - aBS(1-u)}{(k+B)^2} \\ 0 & \mu + d + \beta + v & 0 \\ 0 & e & -r + \frac{2Br}{k} + n \end{bmatrix}.$$

Find F and V at $E_0 = (\frac{\wedge}{\mu}, 0, 0)$

We obtained

$$F(E_0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & a(1-u)(\mu + d + \beta + v) \\ 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$V(E_0) = \begin{bmatrix} \mu & 0 & a(1-u)(\mu+d+\beta+v) \\ 0 & \mu+d+\beta+v & 0 \\ 0 & -e & n-r \end{bmatrix}.$$

Find $FV^{-1}(E_0)$

$$FV^{-1}(E_0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & a(1-u)(\mu+d+\beta+v) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\mu} & \frac{ea(1-u)(\mu+d+\beta+v)}{\mu(\mu+d+\beta+v)(n-r)} & -\frac{a(1-u)(\mu+d+\beta+v)}{\mu(n-r)} \\ 0 & \frac{1}{\mu+d+\beta+v} & 0 \\ 0 & -\frac{e}{(\mu+d+\beta+v)(n-r)} & \frac{1}{n-r} \end{bmatrix}$$

Find the spectral radius of FV^{-1} denoted by $\rho(FV^{-1})$

$$\rho(FV^{-1}) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & -\frac{a(1-u)(\mu+d+\beta+v)e}{(\mu+d+\beta+v)(n-r)} & \frac{a(1-u)(\mu+d+\beta+v)}{n-r} \\ 0 & 0 & 0 \end{vmatrix}.$$

We obtained the cholera basic reproductive number as shown.

$$R_0 = \frac{a \wedge (1-u)}{\mu k (\mu+d+\beta+v)(n-r)}.$$

Local stability:

The local stability of an equilibrium point is determined from the Jacobian matrix of the system of ordinary differential equation (1),(2), (3) evaluated at the equilibrium point. The Jacobian matrix at E_0 is

Theorem 1:

The disease free equilibrium of the system about the equilibrium E_0 , is local asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$

Proof. The Jacobian matrix of the model (Eq. 1-3) evaluated at $E_0 \left(\frac{\wedge}{\mu}, 0, 0 \right)$, is obtained

as

$$J_0 = \begin{bmatrix} -\mu & 0 & -\frac{a \wedge (1-u)}{\mu k} \\ 0 & -(\mu+d+\beta+v) & \frac{a \wedge (1-u)}{\mu k} \\ 0 & e & r-n \end{bmatrix}$$

The eigenvalues of the J_0 are obtained by solving $\det(J_0 - \lambda I) = 0$. We obtained the characteristic equation,

$$(\lambda + \mu)(\lambda^2 - N_1\lambda - N_2) = 0$$

$$\text{Where } N_1 = (r-n) - (\mu + d + \beta + \nu), \quad N_2 = (\mu + d + \beta + \nu)(r-n) + \frac{ea \wedge (1-u)}{\mu k},$$

From the characteristic equation, We see that one eigenvalue is $\lambda = -\mu < 0$. The other two eigenvalues are solution of $\lambda^2 - N_1\lambda - N_2 = 0$. The roots of this equation will be negative if two coefficients satisfied with the Routh-Hurwitz criteria.

$$1) N_1 < 0 \quad \text{and} \quad 2) N_2 < 0.$$

Endemic Equilibrium Point:

To determine the stability of the endemic equilibrium point, E_1 by examining the eigenvalues of Jacobian matrix at E_1 , which is

Theorem 2:

The endemic equilibrium of the system about the equilibrium E_1 , is local asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$

Proof. The Jacobian matrix of the model (Eq. 1-3) evaluated at

$$J_1 = \begin{bmatrix} -\frac{aB^*}{k+B^*}(1-u) - \mu & 0 & \frac{(k+B^*)aS^*(1-u) - aB^*S^*(1-u)}{(k+B^*)^2} \\ \frac{aB^*}{k+B^*}(1-u) & -(\mu+d+\beta+\nu) & \frac{(k+B^*)aS^*(1-u) - aB^*S^*(1-u)}{(k+B^*)^2} \\ 0 & e & r - \frac{2rB^*}{k} - n \end{bmatrix}.$$

We obtained the characteristic equation:

$$\lambda^3 + Q_1\lambda^2 + Q_2\lambda + Q_3 = 0.$$

Where

$$Q_2 = P_1A - P_4A - P_1P_4 + eP_3, \quad Q_3 = P_3P_2e - P_1P_4A - eP_1P_3, \quad A = \mu + d + \beta + \nu, \quad P_1 = \frac{aB^*(1-u)}{k+B^*} - \mu,$$

$$P_2 = \frac{aB^*(1-u)}{k+B^*}, \quad P_3 = \frac{(k+B^*)aS^*(1-u) - aB^*S^*(1-u)}{(k+B^*)^2}, \quad P_4 = r - \frac{2B^*r}{k} - n, \quad Q_1 = A + P_1 - P_4.$$

The three eigenvalues of $\lambda^3 + Q_1\lambda^2 + Q_2\lambda + Q_3 = 0$ will be negative real part if they satisfy the Routh-Hurwitz criteria.

$$1) Q_1 > 0, \quad 2) Q_3 > 0, \quad 3) Q_1Q_2 > Q_3.$$

Numerical Results:

We solved the system of ordinary differential equations. The numerical results of the model obtained by simulation are presented as the follows in Fig. 2 and Fig. 3.

Table 1: Parameter values used in the numerical simulations

Parameter	Description	Value	Reference
\wedge	Recruitment rate into the susceptible human	1,000 day ⁻¹	Isere and Osemwenkhae (2014)
a	Exposure rate to contaminated water	0.5 day ⁻¹	
u	Per capita natural death rate of human	0.8 day ⁻¹	
μ	Per capita natural death rate of human	0.000559 day ⁻¹	
d	Per capita cholera related death rate	0.000156 day ⁻¹	

k	Concentration of <i>V. cholerae</i> in water that yields 50% chance of catching cholera	10,000,000
β	Rate at which infected human recover from cholera	0.2
ν	Rate of increase treatment	0.5 day ⁻¹
r	Growth rate of <i>V. cholerae</i> in the aquatic environment	0.2497 day ⁻¹
K	Carrying capacity of <i>V. cholerae</i>	1,000,000,000 cells/ml day ⁻¹
n	Loss rate of <i>V. cholerae</i> in the aquatic environment	0.4 day ⁻¹
e	Contribution of each infected human to concentration of <i>V. cholerae</i> in water	10 cells/ml day ⁻¹

Stability of the disease free state:

From the values of parameters in **Table 1**, we obtained the eigenvalues and basic reproductive number are:

$$\lambda_1 = -0.000559, \lambda_2 = -0.268742, \lambda_3 = -0.0525225, R_0 = 0.414679 < 1$$

Since all of eigenvalues are to be negative and the basic reproductive is to be less than one, the disease free equilibrium point will be local asymptotically stable, as shown in Fig. 2

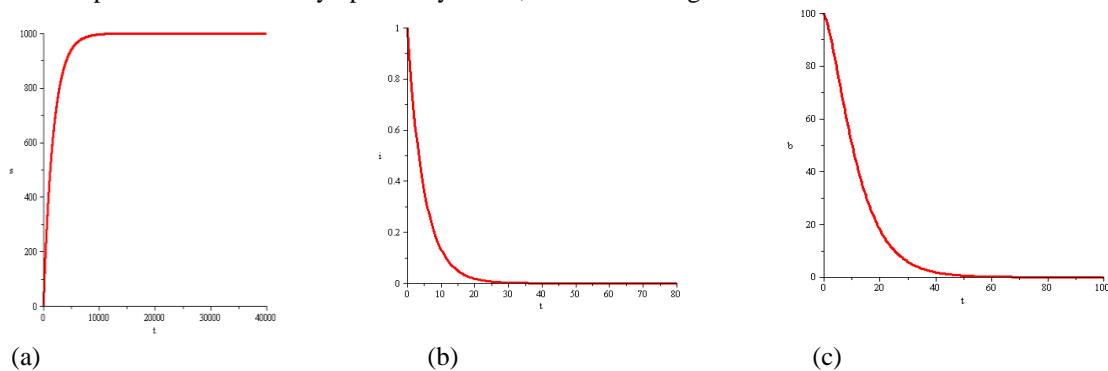


Fig. 2: Time series of (a) Susceptible human (s), (b) Infected human (I), (c) concentration of toxigenic

V. cholerae in water (B). The values of parameters are in the text. We see that the solutions converge to the disease free state $E_1 = (11.9334, 2.20302, 10212.40)$, where

$$\wedge = 0.559, a = 0.5, k = 1000000, \mu = 0.000559, d = 0.000156, \\ \beta = 0.2, r = 0.24745, n = 0.367, e = 10, u = 0.8, v = 0.5.$$

Stability of the endemic state:

We changed the values of $u = 0.8$ and $v = 0.5$ keep the values of the other values of parameters to be those given in Table 1, we obtained the eigenvalues and basic reproductive number are:

$$\lambda_1 = -0.248017, \lambda_2 = -0.0506116, \lambda_3 = -0.0233024, R_0 = 7.9533 > 1.$$

Since all of eigenvalues are to be negative and the cholera reproductive is to be greater than one, the endemic equilibrium point will be local asymptotically stable, as shown in Fig. 3

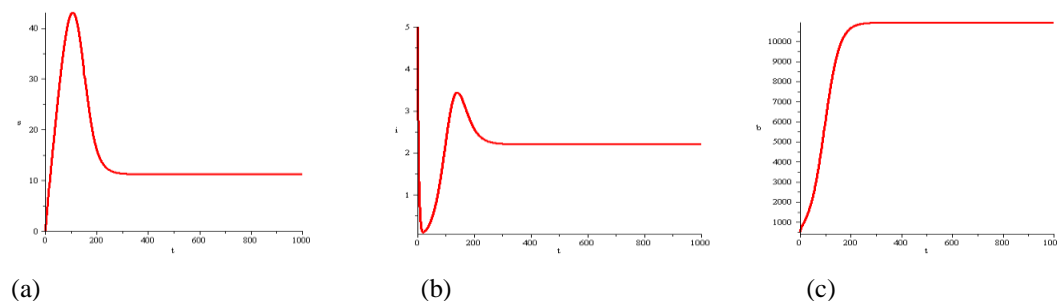


Fig. 3: Time series of (a) Susceptible human (S), (b) Infected human (I), (c) concentration of toxigenic

V. cholerae in water (B). The values of parameters are in the text.,

$$\text{where } \wedge = 0.559, a = 0.5, k = 1000000, \mu = 0.000559, d = 0.000156, \\ \beta = 0.2, r = 0.24745, n = 0.367, e = 10, u = 0.1, v = 0.01.$$

We see that the solutions converge to the disease free state $E_1(0.040496, 0.000005, 0.000474, 290000)$.

Discussion:

In our finding, there exist a disease free equilibrium state. We showed that if $R_0 < 1$, then the disease free equilibrium point is local asymptotically stable, that is the disease will died out from the community. We also showed that an endemic equilibrium state exist when $R_0 > 1$, then the endemic equilibrium point is local asymptotically stable, that is the disease will occur in the community. We obtained the cholera reproductive number through by using the spectral radius of the next generation matrix. The cholera reproductive number is R_0 where $R_0 = \frac{a \wedge (1-u)}{\mu k(\mu+d+\beta+v)(n-r)}$. Our numerical simulations shown that the value of $R_0 = 7.9533 > 1$ if $u=0.1, v=0.01$ and $R_0 = 0.414679 < 1$ if $u=0.8, v=0.5$ (Isere, Osemwenkhae and Okuonghae, 2014). The cholera reproductive number will increase if the effectiveness of education program and the cost of treatment by the drug increase.

Conclusion:

We concluded that the education program and the cost of treatment by the drug can be used for an alternate way to control cholera.

ACKOWLEGEMENT

The author is grateful to the Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University for providing the facilities to carry out the research. Finally, Surapol Naowarat would like to thanks Dr. Phananoi Rotchu for good suggestion about the manuscript.

REFERENCES

- Anderson, R.M. and R.M. May, 1991. *Infectious Diseases in Human: Dynamics and Control*. Oxford University Press, Oxford. ISBN: 019854040.
- CDC., 2014. <http://www.cdc.gov/cholera/index.html>.
- Codeco, T.C., 2001. Endemic and epidemic dynamics of cholera: The role of the aquatic reservoir. *BMC infectious Diseases*. 1:1, <http://www.biomedcentral.com>
- Fakai, S.A., M.O. Ibbahim and A. Danbaba, 2013. Deterministic Mathematical Model of Cholera, Predicting Chances of Its Outbreak. *IJSTR* 2(4): 19-23.
- Fakai, Ibrahim and Siddiqui, 2014. A Deterministic Mathematical Model on Cholera Dynamics and Some Control Strategies. *IJSET.*, 3(8): 1115-1118.
- Fatima, S., I. Krishnarajah, M. Jaffar and M. Adam, 2014. A Mathematical Model for the Control of Cholera in Nigeria. *Res. J. Environ. Earth Sci.*, 6(6): 321-325.
- Fakai, S.A., M.O. Ibrahim and A. Danbaba, 2014. Deterministic Mathematical Model of Cholera, Predicting Chances of its Outbreak. *IJSTR*, 2: 2277-8616.
- Isere, A. and J. Osemwenkhae, 2010. *Vibrio Cholerae* in Nigeria : a logistic investigation of its stability in its habitat. *ANS*, 10(1): 71-76.
- Isere, A., J. Osemwenkhae and D. Okuonghae, 2014. Optimal control model for the outbreak of cholera in Nigeria. *Afr. J. Math. Comput. Sci. Res.*, 7(2): 24-30.
- Ochoche, J.M., 2013. A Mathematical Model for the Transmission Dynamics of Cholera with Control Strategy. *IJST.*, 2(11): 797-803.
- Tian, J. and J. Wang, 2011. Global stability for cholera epidemic models. *Math. Biosci.*, 232: 31-41.
- Van den Driessche, P and J. Watmough, 2002. Reproductive numbers and sub-threshold endemic equilibria for compartment models of disease transmission. *Math. Biosci.*, 180: 29-48.
- WHO, 2013. Media center: Cholera. Retrieved from; <http://www.who.int/media/centre/factsheets/fs107/en>.
- Wearing, H.J., 2005. Appropriate models for management of infectious disease. *PLoS medicine*. <http://www.plosmedicine.org>.