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Application of Numerical Methods for Integrating Differential Equations of Crack Evolution Models to the Range of Constant Tension

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ABSTRACT

Background: A realistic approach to structures and components used in engineering should consider the existence of cracks. The presence of cracks in structures, or mechanical components, usually, is associated with the phenomenon of fatigue, and its propagation is strongly influenced by the state of stress in its vicinity. In this work approximated numerical solutions were obtained through the integration of the differential equations describing the crack propagation laws of types: Paris-Erdogan, Forman, Priddle and McEvily. These laws can be formulated with the use of numerical methods of Euler, implicit and explicit, and explicit fourth-order Runge-Kutta (RK4).
The equation proposed by Paris-Erdogan sets an initial value problem (IVP), defined by a nonlinear and autonomous ordinary differential equation (ODE) of first order. More generally, the equation can be used as a Cauchy problem that consists in determining the trajectories (functions) that satisfy the differential equation, passing through the point set in the initial condition, $a(N_0)=a_0$. The ODE is separable so direct integration can be used to obtain solutions for the equation. Nonetheless, generally, the definition of the function "geometric correction factor" prevents the determination of the explicit function "crack size". Thus, it is possible to obtain numerical solution to the ODE, only, through the use of numerical methods. Using a computational environment to assist the use of numerical method makes it possible to generate graphics to evaluate the evolution of a determined crack as the number of cycles increases. Small differences between the results of different methods are expected because each method works with a different degree of accuracy.

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Models of crack propagation:

The study of the evolution of a crack is based on the crack propagation rate. Regarding the behavior of the crack propagation rate, the diagram can be divided into three regions.

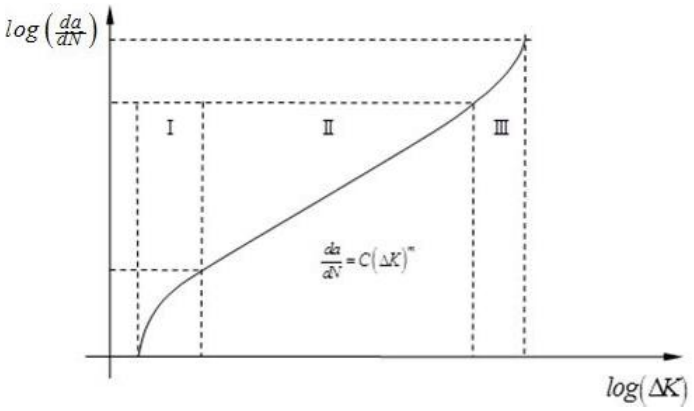

Fig. 1: Diagram $log\left(\frac{da}{dN}\right)\times log\left(\Delta K\right)$ Source: Ávila (2013).

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Region I: In this region small numerical values are observed for the stress intensity factor K and the crack behavior is associated with a limit (ΔK).

Region II: Many project methodologies based on "crack growth rate" are designed to be used in this region. Many functions have been adjusted for this region that is characterized by an approximately linear behavior. Among them stands out the equation of Paris and Erdogan (1963) that has been widely used.

Region III: This region has high crack growth rates. Generally, an instable behavior of the crack is observed in this region.

Paris-Erdogan Model:

The crack propagation model proposed by Paris-Erdogan describes crack growth in region II of figure 1. This model is based on experimental observations and heuristic guidelines. The evolution of a crack through this model is better represented when a probabilistic characterization is used. This is the proposed work of Ghonen & Dore (1987), which tested and analyzed results obtained from a sample of experiments for a finite plate containing a centered crack submitted to a stress load

Using the crack propagation model proposed by Paris-Erdogan, the IVP or Cauchy problem for crack growth can be formulated according to equation 1.

$$\begin{cases} \text{Determine } a \in C^1([N_0, N_1]; \Re^+), \text{ such that:} \\ \dfrac{da}{dN} = C_p \Delta K^{m_p}, \forall N \in (N_0, N_1); \\ a(N_0) = a_0; \end{cases} \quad (1)$$

C_p and m_p are parameters of the material, N is the number of cycles and ΔK is the variation of stress intensity factor.

Forman Model:

The model proposed by Forman (1967) is used to predict the crack growth rate on fatigue in regions II and III from the diagram $log\left(\frac{da}{dN}\right) \times log(\Delta K)$ of figure 1. This model includes the effect of crack instability at the beginning of accelerated fracture.

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_c - \Delta K}, \quad (2)$$

The parameter "n" is a constant of the material. This model is in accordance with the tests performed by Broek e Shijve (1963).

Priddle Model:

Among the most versatile models of crack propagation is the proposed by Priddle (1976). This model describes regions I, II and III from figure 1 and is given by the equation (3).

$$\frac{da}{dN} = C\left(\frac{\Delta K - \Delta K_{th}}{K_c - K_{máx}}\right)^m + C', \quad (3)$$

C and m are parameters of the material, ΔK_{th} is the initial value where the cracks in formation starts its propagation and $K_{máx}$ the maximum stress intensity factor.

McEvily Model:

McEvily and Groeger (1977) proposed a model to describe regions I, II and III from figure 1.

$$\frac{da}{dN} = \frac{A}{\sigma_y}(\Delta K - \Delta K_{th})^m \left(1 + \frac{\Delta K}{K_c - K_{máx}}\right), \quad (4)$$

Being A and m parameters of the material and σ_y is the yield stress of the material. The value of ΔK_{th} is function of stress ratio R, and of the environment.

Computational analysis:

The models of Paris-Erdogan, Forman, Priddle and McEvily were simulated to make comparisons between numerical methods. The integration required to obtain an evaluation of crack size is obtained from Euler, implicit and explicit, and fourth-order Runge-Kutta methods. The proposed problem considers an infinite plate with a centered crack. It evaluated the function "crack size" for amplitude of $\Delta\sigma = 20ksi$ for a range of 900000 cycles of stress load. The software used for analyze is the MATLAB.

Paris-Erdogan Model:

Initially, analyzing Paris-Erdogan model for a set of data, these values were applied in Paris-Erdogan equation to evaluate crack evolution.

Parameters for solution of Paris equation, (Barsom, Rolf (1999)):

$$\begin{cases} C_p = 10^{-9}; \\ m_p = 2; \\ b = 0{,}5in; \\ a_0 = 0{,}005in; \\ N_0 = 0; \\ N_1 = 900000cycles; \\ \Delta\sigma = 20ksi, \forall N \in [0, 900000]. \end{cases}$$

Applying Euler methods, figure 2 shows the graphic of the curve representing the development of the crack size as a function of the number of cycles.

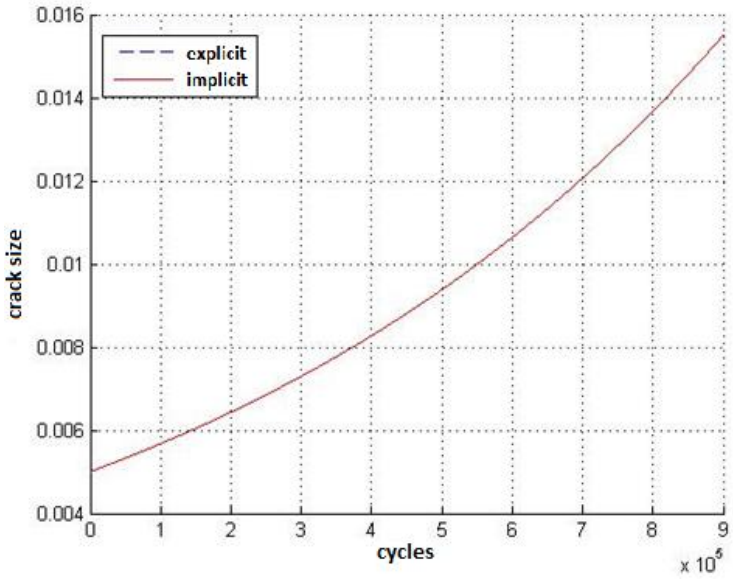

Fig. 2: Evolution of the crack size using Paris model and Euler method.

Completed 900000 load application cycles, the obtained value of the crack was 15,493338in for the implicit method and 15,493325in for the explicit method. The difference between values obtained at that point was $1.318630.10^{-5}$ *in*.

Using Runge-Kutta method to 900000 cycles the crack size was 15,493336*in*. Figure 3 shows a graphic representing the crack size, using the data provided.

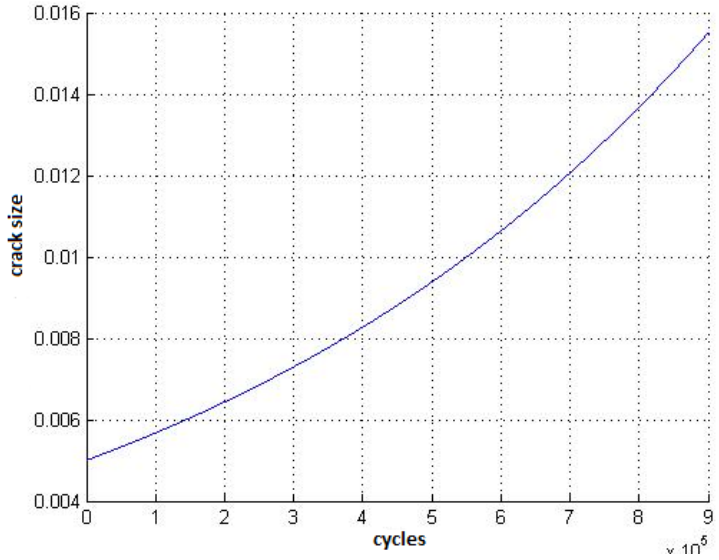

Fig. 3: Evolution of the crack size using Paris model and RK4 method.

Collecting values of different points from crack growth graphics it was possible to organize these data as observed in chart 1.

Chart 1 was created stipulating values for N, between 10000 cycles intervals. In this chart it is observed the following information: the values obtained for the crack size using Euler method, the difference between them, the value obtained using RK4 method and the differences between RK4 method and Euler methods, implicit and explicit.

Chart 1: Values and differences between methods using Paris model.

N (number of cycles)	Implicit Euler $(a.10^{-3}\ in)$	Explicit Euler $(a.10^{-3}\ in)$	difference Implicit-Explicit $(a.10^{-3}\ in)$	Runge-Kutta-4 $(a.10^{-3}\ in)$	difference RK4-Implicit $(a.10^{-3}\ in)$	difference RK4-explicit $(a.10^{-3}\ in)$
100000	5,6694968	5,6694963	0,0000004	5,6694968	0,0000000	0,0000004
200000	6,4286476	6,4286458	0,0000018	6,4286468	-0,0000008	0,0000010
300000	7,2894488	7,2894459	0,0000029	7,2894476	-0,0000012	0,0000017
400000	8,2655119	8,2655078	0,0000041	8,2655104	-0,0000015	0,0000026
500000	9,3722705	9,3722650	0,0000055	9,3722687	-0,0000018	0,0000037
600000	10,6272248	10,6272177	0,0000071	10,6272227	-0,0000020	0,0000050
700000	12,0502184	12,0502095	0,0000089	12,0502162	-0,0000022	0,0000067
800000	13,6637518	13,6637409	0,0000109	13,6637495	-0,0000023	0,0000086
900000	15,4933384	15,4933252	0,0000132	15,4933362	-0,0000022	0,0000110

Forman Model:

Using Forman model, analyzes for crack growth were made for different values of loading rate R, which represents the minimum stress value divided by the maximum stress. Thus two sets of data were obtained, one for each value of R.

Parameters for solution of Forman equation, (Forman (1967)):

$$\begin{cases} C_F = 5.10^{-13}; \\ m_F = 3; \\ R = 0{,}1 \text{ e } 0{,}9; \\ K_c = 68000 lb/in^{3/2}; \\ b = 0{,}5 in; \\ a_0 = 0{,}005 in; \\ N_0 = 0; \\ N_1 = 900000 cycles; \\ \Delta\sigma = 20 ksi, \forall N \in [0, 900000]. \end{cases}$$

Initially was used the R value of 0.1 to analyze the behavior of the crack. The value obtained for the crack was 5,000000116*in* when it reaches 900000 load cycles.

It was not possible to observe measurable differences between methods for the stipulated accuracy. The modulus of the differences between the methods was null for the adopted accuracy as it is observed in chart 2.

For R value of 0.9 it was obtained a maximum crack size of 5,00000104*in* when it reaches 900000 load cycles. Again, it was not possible to observe measurable differences between methods. Chart 3, shows the obtained data.

The crack size obtained for 900000 cycles was 5,0000001155*in* for R equal to 0.1 and 5,0000010429*in* for R equal to 0.9. For an R of 0.9, crack growth was 9.03 times the value obtained to a value equal to 0.1. Thus we observe, for this example, an approximately proportional and linear character between R and crack growth.

Chart 2: Values and differences between methods using Forman model, R=0.1.

N (number of cycles)	Implicit Euler $(a.10^{-3}\ in)$	Explicit Euler $(a.10^{-3}\ in)$	difference Implicit-Explicit $(a.10^{-3}\ in)$	Runge-Kutta-4 $(a.10^{-3}\ in)$	difference RK4-Implicit $(a.10^{-3}\ in)$	difference RK4-explicit $(a.10^{-3}\ in)$
100000	5,00000001	5,00000001	0,00000000	5,00000001	0,00000000	0,00000000
200000	5,00000003	5,00000003	0,00000000	5,00000003	0,00000000	0,00000000
300000	5,00000004	5,00000004	0,00000000	5,00000004	0,00000000	0,00000000
400000	5,00000005	5,00000005	0,00000000	5,00000005	0,00000000	0,00000000
500000	5,00000006	5,00000006	0,00000000	5,00000006	0,00000000	0,00000000
600000	5,00000008	5,00000008	0,00000000	5,00000008	0,00000000	0,00000000
700000	5,00000009	5,00000009	0,00000000	5,00000009	0,00000000	0,00000000
800000	5,00000010	5,00000010	0,00000000	5,00000010	0,00000000	0,00000000
900000	5,00000012	5,00000012	0,00000000	5,00000012	0,00000000	0,00000000

Chart 3: Values and differences between methods using Forman model, R=0.9.

N (number of cycles)	Implicit Euler $(a.10^{-3}\ in)$	Explicit Euler $(a.10^{-3}\ in)$	difference Implicit-Explicit $(a.10^{-3}\ in)$	Runge-Kutta-4 $(a.10^{-3}\ in)$	difference RK4-Implicit $(a.10^{-3}\ in)$	difference RK4-explicit $(a.10^{-3}\ in)$
100000	5,00000012	5,00000012	0,00000000	5,0000001	0,00000000	0,00000000
200000	5,00000023	5,00000023	0,00000000	5,0000002	0,00000000	0,00000000
300000	5,00000035	5,00000035	0,00000000	5,0000003	0,00000000	0,00000000
400000	5,00000046	5,00000046	0,00000000	5,0000005	0,00000000	0,00000000
500000	5,00000058	5,00000058	0,00000000	5,0000006	0,00000000	0,00000000
600000	5,00000070	5,00000070	0,00000000	5,0000007	0,00000000	0,00000000
700000	5,00000081	5,00000081	0,00000000	5,0000008	0,00000000	0,00000000
800000	5,00000093	5,00000093	0,00000000	5,0000009	0,00000000	0,00000000
900000	5,00000104	5,00000104	0,00000000	5,0000010	0,00000000	0,00000000

Chart 4: Values and differences between methods using Priddle model.

N (number of cycles)	Implicit Euler *a(mm)*	Explicit Euler *a(mm)*	difference Implicit-Explicit *a(mm)*	Runge-Kutta-4 *a(mm)*	difference RK4-Implicit *a(mm)*	difference RK4-explicit *a(mm)*
100000	10,3904495	10,3904488	0,0000007	10,3904492	-0,0000003	0,0000004
200000	10,8547231	10,8547216	0,0000015	10,8547225	-0,0000006	0,0000009
300000	11,4139521	11,4139496	0,0000026	11,4139512	-0,0000009	0,0000017
400000	12,0974011	12,0973971	0,0000040	12,0973999	-0,0000012	0,0000028
500000	12,9464326	12,9464267	0,0000059	12,9464313	-0,0000013	0,0000046
600000	14,0208447	14,0208361	0,0000085	14,0208435	-0,0000012	0,0000074
700000	15,4092528	15,4092405	0,0000123	15,4092523	-0,0000005	0,0000118
800000	17,2465708	17,2465531	0,0000176	17,2465720	0,0000012	0,0000189
900000	19,7443020	19,7442763	0,0000257	19,7443070	0,0000050	0,0000307

Chart 5: Values and differences between methods using McEvily model.

N (number of cycles)	Implicit Euler *a(mm)*	Explicit Euler *a(mm)*	difference Implicit-Explicit *a(mm)*	Runge-Kutta-4 *a(mm)*	difference RK4-Implicit *a(mm)*	difference RK4-explicit *a(mm)*
100000	10,0239123	10,0239123	0,0000000007	10,0239123	-0,0000000004	0,0000000004
200000	10,0478960	10,0478960	0,0000000014	10,0478960	-0,0000000007	0,0000000007
300000	10,0719513	10,0719513	0,0000000021	10,0719513	-0,0000000011	0,0000000011
400000	10,0960783	10,0960783	0,0000000029	10,0960783	-0,0000000014	0,0000000014
500000	10,1202773	10,1202773	0,0000000036	10,1202773	-0,0000000018	0,0000000018
600000	10,1445485	10,1445485	0,0000000043	10,1445485	-0,0000000021	0,0000000022
700000	10,1688923	10,1688923	0,0000000050	10,1688923	-0,0000000025	0,0000000025
800000	10,1933088	10,1933088	0,0000000058	10,1933088	-0,0000000028	0,0000000029
900000	10,2177984	10,2177984	0,0000000065	10,2177984	-0,0000000032	0,0000000033

Priddle Model:

For the simulations of the Priddle model it was used the data obtained in the same article where Priddle demonstrates his model for an EN3A steel.

Parameters for solution of Priddle equation, (Priddle, 1976):
Material: EN3A Steel.

$$\begin{cases} C = 2{,}2.10^{-7}; \\ C' = 2.10^{-10}; \\ a_0 = 10mm; \\ N_0 = 0; \\ N_1 = 900000cycles; \\ Kc = 70MN.m^{-\frac{3}{2}}; \\ \Delta K_0 = 7MN.m^{-\frac{3}{2}}; \\ K_{máx} = 2{,}3MN.m^{-\frac{3}{2}}. \end{cases}$$

Analyzing chart 4, when the load application reaches 900000 cycles it was obtained a crack size value of 19,744307mm for RK4 method.

McEvily Model:

For the McEvily model, the following set of data for an aluminum alloy was used to obtain crack size values.

Parameters for solution of McEvily equation, (WANG et al, (2008)):
Material: 6013 aluminum alloy.

$$\begin{cases} R = 0{,}5; \\ \sigma_y = 448MPa \\ \Delta K_{th} = 2{,}20MPa\sqrt{m}; \\ K_{máx} = 2{,}50MPa\sqrt{m}; \\ K_c = 61{,}67MPa\sqrt{m}; \\ a_0 = 10mm; \\ m_m = 2; \\ \Delta\sigma = 20ksi. \end{cases}$$

Values for crack size were very near as it is observed for *N*=900000 in chart 5.

Conclusions:

As was proposed in this work, various relationships between numerical methods and crack propagation models were observed. The use of numerical methods facilitates and complements initial value problems as the proposed by Paris, used as reference for other crack growth models.

The order of a method measures how quickly this method converges to the analytical solution. The inaccuracy of each method decreases when the number of steps increases. Another way to obtain more accurate values is to use methods of higher order as the Runge-Kutta.

Both, methods of Euler and RK4 method, have shown great efficacy for the integration of differential equations found in crack propagation models, and the method of highest order among the analyzed is the fourth-order Runge-Kutta. The results obtained in implicit Euler method were the nearest to those obtained using RK4 in all studied models.

We have available several crack growth prediction models. A relevant question for engineering is the inherent cost of each type of analysis. A more complex model involving many parameters, may require additional information, usually obtained experimentally, promoting rise of costs during analyze. Moreover, depending on the required reliability of results, a simpler model may not provide the desired accuracy.

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