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Optimal Time History Analysis of 2d Trusses Using Free-Scaled Wavelet Functions

Seyed Hossein Mahdavi and Hashim Abdul Razak

StrucHMRS Group, Department of Civil Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia

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ABSTRACT

This paper presents an improved approach where the wavelet functions are developed for particularly two-dimensional (2D) trusses. In the proposed scheme, a clear-cut formulation in a fluent manner has been derived to numerically solve the second-ordered differential equation of motion governing 2D trusses using two different types of basis functions including, complicated Chebyshev and simple Haar wavelet functions. For this aim, a simple step-by-step algorithm is implemented to calculate corresponding dynamic quantities of 2D trusses. In addition, validity and effectiveness of the proposed approach are demonstrated by one example, compared with some of the existent numerical integration methods such as family of Newmark-β, Wilson-θ and central difference method. Finally, it is shown that the time history analysis of 2D trusses is optimally achieved by lesser computational time and high accuracy of responses, using a long-time increment.

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INTRODUCTION

Numerical and step-by-step time integration methods are widely utilized in the computational analysis and evaluation of dynamic systems. Precise and efficient numerical integration schemes have been the focus of significant interest to solve Multi-Degrees-of-Freedom (MDOF) problems of structural dynamics. As a result, because of complexity of not only nonlinear but also large-scaled structures, time integration methods are only options to investigate about responses of the system. In general, there are two basic categories of time integration approaches. First, direct integration schemes whereby the quantities of the dynamic system are being calculated through a direct space vector in the step-by-step solution of the equation of motion. Second, indirect integration schemes involving all corresponding equations being numerically transformed into a new space vector, e.g. from the time domain to the frequency domain. Afterwards, step-by-step dynamic analysis is accordingly performed and fulfilled on the current space vector (Bathe, K.J., 1996). Theoretically, it has been perceived that direct methods are most applicable for structural dynamics, where the response is computed in set of short time increments through the accurate approximation of external excitations. Consequently, it requires high amounts of storage capacity to gain a proper time history analysis of large-scaled structures. What is more, responses calculated by direct algorithms are not numerically optimal over the wide range of natural frequencies i.e. large-scaled and 2D trusses (Chang, S.Y., 2010).

Mathematically, researchers have proposed wavelet procedure to examine many types of differential equations. However, It has been observed that solution of time dependent equation is being constrained only in a unit time step (Yuanlu, L., 2010; Babolian, B., F. Fatahzadeh, 2010). Generally, wavelet functions are characterized into the two main categories. The first being 2D wavelet whereby a definite basis function of wavelet is being shifted for all scaled functions. The other category is three-dimensional (3D) wavelet involving used of an improved wavelet basis function being shifted on each new scale of the mother wavelet (Mahdavi, S.H. and H.A. Razak, 2013).

In this paper, a straightforward and comprehensive formulation is improved for MDOF systems by representing the basis function of Haar and Chebyshev wavelet. Subsequently, to confirm validity of the analytical schemes in this study, case study involving a 2D Howe truss under an impact loading is presented and discussed. The characteristics of Haar wavelet and Chebyshev wavelet functions are cited to some relevant references (Yuanlu, L., 2010; Lepik, U., 2005).

Corresponding Author: Seyed Hossein Mahdavi, University of Malaya, Department of Civil Engineering, Box.50603. Kuala Lumpur. Malaysia;
Tel: 0060122977526; E-mail: s.h.mahdavii@gmail.com

Implementation Of Wavelet Approach On Mdof Systems:

The dynamic equilibrium for a MDOF system is expressed as (Bathe 1996):

$$M^*.\ddot{U}_t + C^*.\dot{U}_t + K^*.U_t = F_t. \quad (1)$$

where, M^*, $C^*$and K^* denote the mass, damping and stiffness matrix of a MDOF system, respectively; vector F_t is the external loading; $\ddot{U}_t$, $\dot{U}_t$ and U_t are acceleration, velocity and displacement matrix associated to degrees of freedom ($\ddot{u}_t$, lower cases denote relative quantity on each degree of freedom) on discrete set of segmentation method's (SM) point of local time. For example, assumption of $2M$ and d, order of the wavelet and degree of freedom, with respect to the MDOF system, dimension of acceleration, velocity and displacement matrix are defined by $d\times 2M$ (the factor of transition is assumed by 2). Consequently, the characteristic matrix of system are obtained by $d\times d$ matrices of M^*, $C^*$and K^*. Firstly, the proposed scheme is implemented on only an axial degree of freedom. Term of acceleration is numerically approximated by corresponding wavelet coefficients (C^T) and wavelet functions (substituting $\psi(t)$=$h(t)$, for Haar wavelet (Lepik, U., 2005) or $\psi(t)$ for Chebyshev wavelet [4] as follows:

$$\ddot{u}(t) = C^T \times \psi(t). \quad (2)$$

Multiplying by operation of integration matrix of wavelet (P), the first order of derivation (velocity) is numerically approximated by (Mahdavi, S.H., S. Shojaee, 2013):

$$\dot{u}(t) = C^T \times P \times \psi(t) + v_n. \quad (3)$$

Accordingly, displacements are also numerically expanded as in following:

$$u(t) = C^T \times P^2 \times \psi(t) + u_n. \quad (4$$

where, in Equations 3 and 4, v_n and u_n denote initial condition for each global time of Δt=t_{i+1}−t_i , respectively. To solve above algebraic system, the unity is being extended by Chebyshev wavelet as:

$$1 \cong I^* \times \psi(t) \cong \left(\sqrt{\pi}/2\right) \times [1,0,0,\dots,1,0,0,\dots] \times \psi(t). \quad (5)$$

Equivalently, for Haar wavelet it is also determined as:

$$1 \cong I^* \times h(t) \cong [1,0,0,\dots,0,0,0,\dots] \times h(t). \quad (6)$$

Thereby, initial conditions of velocity and displacement are numerically developed as:

$$v_n = N_1^T \times \psi(t). \quad (7)$$

$$u_n = N_2^T \times \psi(t). \quad (8)$$

where, N_1^T and N_2^T are $2M \times 1$-dimensional vectors that are therefore determined by Chebyshev wavelet or any particular wavelet function as:

$$N_1^T \cong v_{n(0)} \times \left(\sqrt{\pi}/2\right) \times [1,0,0,\dots,1,0,0,\dots]^T. \quad (9)$$

$$N_2^T \cong u_{n(0)} \times \left(\sqrt{\pi}/2\right) \times [1,0,0,\dots,1,0,0,\dots]^T. \quad (10)$$

Substituting Equations. 9 and 10 into Equations 3 and 4, velocity and displacement are approximated as:

$$\dot{u}(t) = C^T \times P \times \psi(t) + N_1^T \times \psi(t). \quad (11)$$

$$u(t) = C^T \times P^2 \times \psi(t) + N_1^T \times P \times \psi(t) + N_2^T \times \psi(t). \quad (12)$$

Moreover, dynamic loading of $F(t)$ is approximated due to its frequency contents as follows:

$$F(t) = f^T \times \psi(t). \quad (13)$$

where, the corresponding time history force $F(t)$ is $2M$ vector, including, a set of discrete quantities related to the local times. Equally, the coefficient matrix of load is determined as:

$$F_1{}^T{}_{\times 2M} = F_{1\times 2M} / \phi_{(2M)\times(2M)}. \quad (14)$$

Secondly, Equation 1 is rearranged through the each degree of freedom and after simplification of each row, corresponding to each degree of freedom (d) denoted by j, yields:

$$\sum_{i=1}^{d} M_{ji}^* C_i^T + d_t.\sum_{i=1}^{d} C_{ji}^*(C_i^T P + N_{1-i}^T) + d_t^2.\sum_{i=1}^{d} K_{ji}^*(C_i^T P^2 + N_{1-i}^T P + N_{2-i}^T) = d_t^2.f_j^T \quad (15)$$

where, j refers to the degrees of freedom. Subsequently, Equation 15 is simplified as follow (j is also particular degree of freedom):

$$C_i^T\,(M_{ji}^* I + C_{ji}^* P + K_{ji}^* P^2) = f_j^T - \sum_{i=1}^{u}[C_{ji}^* N_{1-i}^T + K_{ji}^*(N_{1-i}^T P + N_{2-i}^T)]. \quad (16)$$

where, I denotes a $2M\times 2M$ identity matrix. Therefore, coefficients of wavelet are calculated for each degree of freedom by solving the above equation.

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Numerical Application:

Figure 1 shows a 2D Howe truss subjected to a 29.43 KN concentrated impact load, which vertically strikes node 1 shown in figure. All truss elements are chosen as I-shaped section of IPE160 and damping ratio is assumed proportionally 0.01 percentages of stiffness and further characteristics are shown in Figure 1. Furthermore, T_{min}=0.0252 sec has been computed as minimum period of the proposed truss, thus at least $\Delta t \leq 0.55 T_{min}$= 0.0138 sec should be utilized as time increment for common integration schemes, particularly, to gain stability of responses those calculated by central difference method (Bathe 1996). To obviously compare stability of responses Δt=0.1 sec and Δt=0.02 sec has been nominated for numerical approaches and Duhamel integration method, respectively.

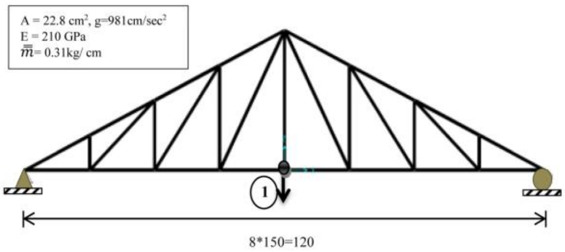

Fig. 1: A 2D truss subjected to an impact loading, Δ**t**=0.1 sec.

This example is evaluated by using five numerical schemes including, average acceleration of Newmark-β family (designated by A_Acc), central difference (CD), Wilson (θ=1.4), the proposed method using 16nd scale of Haar (designated by HA(2M16)) and 16th scale of Chebyshev wavelet (designated by CH(2M16)). The piecewise modal Duhamel integration is supposed as the exact solution and accuracy of results has been compared with those. Figure 2 shows the first 2 sec vertical time history displacements of node 1. It shows rejected results calculated by using CD method in which that conditional stability is almost neglected. However, accuracy of responses for those unconditional stable method such as Wilson or average acceleration method is not completely achieved.

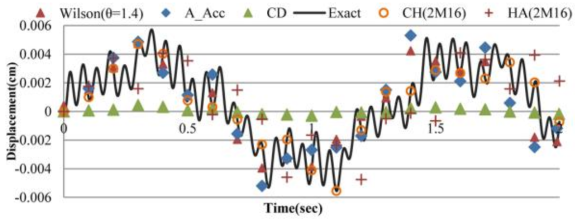

Fig. 2: The first 2 sec vertical time history displacement of node 1, shown in Figure 1.

Eventually, the total average error and computation time involved is plotted in Figure 3. It is shown that the results computed by Haar wavelet, significantly returned the second highest value of 27.28% (total average error). However, the computational time was the lowest at 0.603 sec compared with 1.309 sec for Chebyshev wavelet or 2.098 sec for exact results.

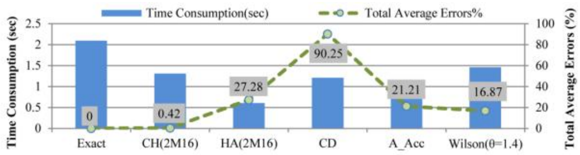

Fig. 3: Total average errors in displacement of node 1, shown in Figure 1 and relative computation time involved.

Conclusions:

The dynamic analysis of 2D trusses is optimally satisfied using an indirect time integration scheme with free-scaled wavelet functions. In addition, due to the inherent complexity of natural frequency, especially for large-scaled and 2D trusses, an adaptive wavelet scheme will be achieved by using various types of basis functions. Overall, it is practically shown that, the proposed method is an unconditional stable scheme, while its computational time is faster than other numerical approaches.

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REFERENCES

Bathe, K.J., 1996. Finite Element Procedures, New Jersey: Prentice-Hall, Englewood Cliffs.

Chang, S.Y., 2010. A new family of explicit methods for linear structural dynamics, Computers and Structures, 8: 755-772.

Mahdavi, S.H. and H.A. Razak, 2013. A wavelet-based approach for vibration analysis of framed structures, Applied Mathematics and computation, 220: 414-428.

Yuanlu, L., 2010. Solving a nonlinear fractional differential equations using Chebyshev wavelet, Communications in Nonlinear Science and Number Simulation, 15: 2284-2292.

Babolian, B., F. Fatahzadeh, 2010. Numerical solution of differential equations by using Chebyshev wavelet operational matrix of integration, Applied Mathematics and Computations, 188: 417-426.

Lepik, U., 2005. Numerical solution of differential equations using Haar wavelets, Mathematics and Computers in Simulation, 68: 127-143.

Mahdavi, S.H., S. Shojaee, 2013. Optimum time history analysis of SDOF systems using free scale of Haar wavelet, Structural Engineering and Mechanics, 45: 95-110.