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Optimum Inspection Times of Step Stress Accelerated Life Tests with ProgressivelyType I Interval Censored

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ABSTRACT

This article presents k-levelstep stress accelerated life tests based on progressive type I interval censoringwhen the inspection times and the proportions of removed units in the experiment are pre-fixed. It is assumed that the lifetime at design stress has Weibull Poisson distribution. The scale parameter of Weibull Poisson failure time at constant levels is assumed to have an inverse power law of the stress levels. Two different inspection times areinvestigated using A-optimality and equal probability schemesto specify the optimal inspection interval times. Maximum likelihoodmethod is used to obtain the estimators of the model parameters.The,mean square errors of the MLEs of the parameters are examined to know the effect of different schemes. Finally, the numerical studies are performed to illustrate the proposed procedures.

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INTRODUCTION

In many life testing experiments, it is often more economical or practical to gauge observations into groups rather than to record their actual measurements. In such situations, an important question is then to determine appropriately the number of associated monitoring times before conducting the experiment in order to assess the parameters of interest with little reduction in efficiency as compared to the complete sample situation. For over four decades, there has been a growing literature on inference based on progressively censored data. Most of the work concentrated on point and interval estimation of the parameters, construction of reliability sampling plans, determination of optimal progressive censoring plans, and the planning of step-stress accelerated life tests. For more references in this field, Tai Lin *et al*. (2009) presented progressively type I interval censored data for lognormal distribution. Tai Lin *et al*. (2011) introduced planning life tests based on progressive type I interval censored for Weibull distribution. Attia and Assar (2012) proposed an approach to determine the number of test units, number of inspections and length of inspection interval of a life test under a pre-determined budget of experiment for progressive type I grouped data.

Accelerated life tests (ALTs) are used to estimate the lifetime of highly reliable products within a reasonable testing time. The test products are run at higher than usual levels of stress (which includes temperature, voltage, pressure, etc.) to obtain early failures. In ALT experiments, the researcher is often interested in the effects of varying stress levels on the lifetimes of experimental units.Nelson (1980) originally proposed the simple SSALT, in which only one change of stress occurs with a cumulative exposure (CE) model for Type-I and Type-II censored data. Miller and Nelson (1983) obtained the optimal simple SSALT plans for the case where test products have exponentially distributed lives and are observed continuously until all test products fail.Wu *et al*. (2006)discussed k step stress accelerated life tests under progressive type I censoring with grouped data assuming exponential life time distribution. Aly (2008) dealt with k step stress accelerated life tests under progressive type I censoring with grouped data. A log logistic life time distribution and a cumulative exposure model were considered. Wu *et al*. (2008) have discussed SSALT under type I progressive censoring with random removals for exponential grouped data.Balakrishnan and Han (2009) considered k step stress accelerated life testing with an equal duration time for progressively Type I censored data for exponential distribution.

The modeling and analysis of lifetimes is an important aspect of statistical work in a wide variety of scientific and technological fields. Several distributions have been proposed in the literature to model lifetime data by compounding some useful life distributions. Lu and Shi (2012) introduced a new compounding

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distribution named the Weibull Poisson (WP) distribution. The failure rate function of WP distribution has various shapes; it can be increasing, decreasing and upside down bathtub shaped or unimodal. The property of various shapes of the failure rate function encourages practitioners to adopt the WP for the inferences of life information and biological study.

This article dealswith the problem of designing an optimum k-level step stress accelerated life tests for Weibull Poisson distribution based on type I progressive interval censoring when the inspection times and the proportions of removed units in the experiment are pre-fixed.The scale parameter of the distribution is assumed to have an inverse power law function of stress level. A comparison of two inspections plans is made.

The layout of the article is as follows. The model and assumptions are described in details in Section 2. In Section 3 the maximum likelihood method is introduced to estimate the model parameters underprogressive type I interval censoring data.In Section 4 expected of the trace for the Fisher information matrix is introduced. In Section 5the optimal inspection interval time is presented using two different schemes. The simulation technique is presented in Section 6. Finally conclusion is presented in Section 7.

Assumptions and Test Procedure:

Let us consider the following k-level SSALT scheme with type I progressive interval censoring: Nunits are simultaneously placed on a life test at stress setting V_1, and run until time τ_1, at which point the number of failed units n_1 are counted and$r_1$surviving units are removed from the testand so on. The model assumptions for k-level SSALT based on type I progressive interval censoring will be described as follows:

i. There are multiple (k) levels of high stress,$V_j, j = 1,2, \dots, k$ in the experiment and V_0 is the design stress that is the stress level under normal use conditions, where, $V_0 < V_1 < \cdots < V_k$.

ii. The failure times $t_{ij}, i = 1,2, \dots, n_j$ at stress levels $V_j, j = 1,2, \dots, k$ are assumed to be WP distribution at any stress; withthe following probability density function (pdf) and cumulative distribution function(cdf)

$$f(t;\beta_j) = \frac{\alpha\beta_j\lambda t^{\alpha-1}}{1-e^{-\lambda}} e^{-\lambda-\beta_j t^{\alpha}+\lambda exp(-\beta_j t^{\alpha})}, \quad t > 0, \alpha, \beta_j, \lambda > 0, j = 1,2, \dots, k. \tag{1}$$

$$F(t;\beta_j) = \left(e^{\lambda exp^{-\beta_j t^{\alpha}}} - e^{\lambda}\right)\left(1-e^{\lambda}\right)^{-1}, \quad t > 0, \alpha, \beta_j, \lambda > 0, j = 1,2, \dots, k. \tag{2}$$

where,α is the shape parameter, β_j is the scale parameter.

iii. Let n_j be the number of units known to have failed in the time interval $[\tau_{j-1}, \tau_j]$.

iv. A random sample of N units are simultaneously placed on test at a stress V_1 and run until time τ_1, the number of failed units n_1 are recorded and q_1 fraction of remaining life units$r_1 = (N - n_1)q_1$ are removed from the test at random. Starting at inspection time τ_1, the $m_1 = (N - n_1 - r_1)$ surviving units are put on the test under a stress V_2, where $V_2 > V_1$, these units are run until time τ_2, the number of failures n_2 are recorded and q_2 fraction of remaining life units $r_2 = (N - n_1 - r_1 - n_2)q_2$ are removed from the test at random. Then, at thejth inspection time τ_j the test units are checked and the number of failed units n_j are recorded and q_j fraction of remaining live units, $r_j = \left(N - \sum_{h=1}^{j-1}(n_h + r_h) - n_j\right)q_j$ are removed from the test at random.

v. The lifetimes of the units at each stress level are identically independent distributed (i.i.d).

vi. The WP scale parameter$\beta_j, j = 1,2, \dots, k$ of the underlying lifetime distribution in (2) is assumed to have an inverse power function of stress levels i.e,$\beta_j = CS_j^p$, $j = 1,2, \dots, k, C, p > 0, S_j = \frac{V^*}{V_j}, V^* = \prod_{j=1}^{k} V_j^{b_j}, b_j = \frac{n_j}{\eta}$, $C$is the constant of proportionality and p is the power of the applied stress.

vii. The WP parameters λ and α are constant, i.e. independent of stress.

Therefore, at stress level $V_j j = 1,2, \dots, k$ and in the inspection interval $[\tau_{j-1}, \tau_j]$ and since failures are assumed to occur according to the CE model, then the cdf of the lifetime of a test unit under k-levelSSALT based on type I progressive interval censoring is given by:

$$G(t) = \begin{cases} F(t;\beta_1) & 0 \le t < \tau, \\ F(t-\tau_1+u_1;\beta_2) & \tau_1 \le t < \tau_2, \\ \vdots & \\ F(t-\tau_{k-1}+u_{k-1};\beta_k) & \tau_{k-1} \le t < \infty. \end{cases}$$

where$F_j(t-\tau_{j-1}+u_{j-1}) = (e^{\lambda exp\left(-CS_j^p\left(t-\tau_{j-1}+u_{j-1}\right)^{\alpha}\right)} - e^{\lambda})(1-e^{\lambda})^{-1}$ the cumulative distribution function of the failure at stresses $V_j, j = 1,2, \dots, k$, u_{j-1} is the solution of the equation $F(u_j;\beta_{j+1}) = F(\tau_j + u_{j-1};\beta_j)$.Therefore the general form of u_j is as follows:

$$u_j = \left(\frac{S_j}{S_{j+1}}\right)^{1/\alpha}(\tau_j + u_{j-1}), \; j = 1,2, \dots, k-1.$$

Note that $u_0 = 0$ and $\tau_0 = 0$ where τ_j is the inspection time.

Hence, the cumulative distribution function of a test unit under k-level SSALTis

$$G(t) = \begin{cases} (e^{\lambda exp^{\left(-CS_1^p t^\alpha\right)}} - e^\lambda)(1-e^\lambda)^{-1} & 0 \le t < \tau_1, \\ \left(e^{\lambda exp^{\left(-CS_2^p\left(t-\tau_1+\left(\frac{S_1}{S_2}\right)^{1/\alpha}\tau_1\right)^\alpha\right)}} - e^\lambda\right)\left(1-e^\lambda\right)^{-1} & \tau_1 \le t < \tau_2, \\ \vdots & \\ \left(e^{\lambda exp^{\left(-CS_k^p\left((t-\tau_{k-1})+\sum_{j=1}^{k-1}-CS_j^p\tau_j\right)^\alpha\right)}} - e^\lambda\right)\left(1-e^\lambda\right)^{-1} & \tau_{k-1} \le t < \infty. \end{cases}$$

Thus the associated pdfof a test unit is

$$g(t) = \begin{cases} \frac{\alpha CS_1^p \lambda t^{\alpha-1}}{1-e^{-\lambda}} e^{-\lambda - CS_1^p t^\alpha + \lambda exp^{(-CS_1^p t^\alpha)}} & 0 \le t < \tau_1, \\ \frac{\alpha CS_2^p \lambda [t-\tau_1+\left(\frac{S_1}{S_2}\right)^{1/\alpha}\tau_1]^{\alpha-1}}{1-e^{-\lambda}} e^{-\lambda - CS_2^p [t-\tau_1+\left(\frac{S_1}{S_2}\right)^{1/\alpha}\tau_1]^\alpha + \lambda exp^{(-CS_2^p [t-\tau_1+\left(\frac{S_1}{S_2}\right)^{1/\alpha}\tau_1]^\alpha)}} & \tau_1 \le t < \tau_2, \\ \vdots & \\ \frac{\alpha CS_k^p \lambda [(t-\tau_{k-1})+\sum_{j=1}^{k-1}-CS_j^p\tau_j]^{\alpha-1}}{1-e^{-\lambda}} e^{-\lambda - CS_k^p\left[(t-\tau_{k-1})+\sum_{j=1}^{k-1}-CS_j^p\tau_j\right]^\alpha + \lambda exp^{(-CS_k^p\left[(t-\tau_{k-1})+\sum_{j=1}^{k-1}-CS_j^p\tau_j\right]^\alpha)}} & \tau_{k-1} \le t < \infty. \end{cases}$$

Maximum Likelihood Estimation:

According to type-I progressive interval censoring, the test applied to N identical sample units. Let $t_{ij}, i = 1,2,\ldots,n_j, j = 1,2,\ldots,k$ be independent and identically distributed Weibull Poisson random variable, Aly (2008) and Wu *et al*. (2006) presented the likelihood function for k-level SSALT with type I progressive interval censored data which is considered to have the following form:

$$l \propto \prod_{j=1}^{k}\left[F(\tau_j) - F(\tau_{j-1})\right]^{n_j}\left[1-F(\tau_j)\right]^{r_j} \tag{3}$$

Assuming WP as the life time distribution, the likelihood function for three-parameter WP distribution for k-level SSALT with type I progressive interval censoring data takes the following form:

$$\begin{aligned} l \propto (1-e^\lambda)^{-n_1}(e^{\lambda e^{\left(-Cs_1^p\tau_1{}^\alpha\right)}} & - e^\lambda)^{n_1}\prod_{j=2}^{k}[(e^{\lambda e^{\left(\sum_{h=1}^{j}-Cs_h^p\tau_h{}^\alpha\right)}} - e^\lambda)\left(1-e^\lambda\right)^{-1} - (e^{\lambda e^{\left(\sum_{h=1}^{j-1}-Cs_h^p\tau_h{}^\alpha\right)}} \\ & - e^\lambda)(1-e^\lambda)^{-1}]^{n_j}\prod_{j=1}^{k}[1-((e^{\lambda e^{\left(\sum_{h=1}^{j}-Cs_h^p\tau_h{}^\alpha\right)}} - e^\lambda)\left(1-e^\lambda\right)^{-1})]^{r_j} \end{aligned}$$

The maximum likelihood estimators (MLEs) of the parameters are obtained by maximizing the logarithm of the likelihood function expressed in the following form:

$$\begin{aligned} \ln l \propto & -n_1 \ln(1-e^\lambda) \\ & + n_1 \ln(Y_1 - e^\lambda) \\ & + \sum_{j=2}^{k} n_j \ln[(1-e^\lambda)^{-1}(Y_j - Y_{j-1})] + \sum_{j=1}^{k} r_j \ln\left[1-\left[(Y_j - e^\lambda)(1-e^\lambda)^{-1}\right]\right] \end{aligned} \tag{4}$$

where, $Y_1 = e^{\lambda e^{\left(-Cs_1^p\tau_1{}^\alpha\right)}}$, $Y_j = e^{\lambda e^{\left(\sum_{h=1}^{j}-Cs_h^p\tau_h{}^\alpha\right)}}$ and $Y_{j-1} = e^{\lambda e^{\left(\sum_{h=1}^{j-1}-Cs_h^p\tau_h{}^\alpha\right)}}$.The first derivatives of the log likelihood equation with respect to the parameters C, p, λ and α respectively will be as follows:

$$\frac{\partial \ln l}{\partial C} = \frac{-\lambda s_1^p {\tau_1}^\alpha e^{(-Cs_1^p {\tau_1}^\alpha)} n_1 Y_1}{Y_1 - e^\lambda} + \sum_{j=2}^{k} \frac{n_j (\lambda \sum_{h=1}^{j} -s_h^p {\tau_h}^\alpha e^{\left(\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha\right)} Y_j - \lambda \sum_{h=1}^{j-1} -s_h^p {\tau_h}^\alpha e^{\left(\sum_{h=1}^{j-1} -Cs_h^p {\tau_h}^\alpha\right)} Y_{j-1})}{Y_j - Y_{j-1}} + \sum_{j=1}^{k} \frac{-r_j (\lambda \sum_{h=1}^{j} -s_h^p {\tau_h}^\alpha e^{\left(\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha\right)} Y_j)(1 - e^\lambda)^{-1}}{1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]} \quad (5)$$

$$\frac{\partial \ln l}{\partial p} = \frac{-\lambda C s_1^p {\tau_1}^\alpha \ln s_1 \, e^{(-Cs_1^p {\tau_1}^\alpha)} n_1 Y_1}{Y_1 - e^\lambda} + \sum_{j=2}^{k} \frac{n_j [\lambda \sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha \ln s_h \, e^{\left(\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha\right)} Y_j - \lambda \sum_{h=1}^{j-1} -Cs_h^p {\tau_h}^\alpha \ln s_h \, e^{\left(\sum_{h=1}^{j-1} -Cs_h^p {\tau_h}^\alpha\right)} Y_{j-1}]}{Y_j - Y_{j-1}} - \sum_{j=1}^{k} \frac{[(\lambda \sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha \ln s_h \, e^{\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha} Y_j)(1 - e^\lambda)^{-1}] r_j}{1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]} \quad (6)$$

$$\frac{\partial \ln l}{\partial \lambda} = \frac{n_1 e^\lambda}{1 - e^\lambda} + \frac{n_1 (Y_1 e^{(-Cs_1^p {\tau_1}^\alpha)} - e^\lambda)}{Y_1 - e^\lambda} + \sum_{j=2}^{k} \frac{n_j [(1 - e^\lambda)^{-1} e^\lambda (Y_j - Y_{j-1}) + (Y_j e^{\left(\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha\right)} - Y_{j-1} e^{\left(\sum_{h=1}^{j-1} -Cs_h^p {\tau_h}^\alpha\right)})]}{Y_j - Y_{j-1}} + \sum_{j=1}^{k} - \frac{[(1 - e^\lambda)^{-1} (Y_j e^{\left(\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha\right)} - e^\lambda) + (Y_j - e^\lambda)(1 - e^\lambda)^{-2} e^\lambda] r_j}{1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]} \quad (7)$$

$$\frac{\partial \ln l}{\partial \alpha} = \frac{-Cs_1^p {\tau_1}^\alpha \lambda Y_1 n_1 \, e^{(-Cs_1^p {\tau_1}^\alpha)} \ln \tau_1}{Y_1 - e^\lambda} + \sum_{j=2}^{k} \frac{n_j [\lambda e^{\left(\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha\right)} \sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha \ln \tau_h \, Y_j - \lambda e^{\left(\sum_{h=1}^{j-1} -Cs_h^p {\tau_h}^\alpha\right)} \sum_{h=1}^{j-1} -Cs_h^p {\tau_h}^\alpha \ln \tau_h \, Y_{j-1}]}{(Y_j - Y_{j-1})} - \sum_{j=1}^{k} \frac{(1 - e^\lambda)^{-1} r_j \lambda e^{\left(\sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha\right)} \sum_{h=1}^{j} -Cs_h^p {\tau_h}^\alpha \ln \tau_h \, Y_j}{1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]} \quad (8)$$

After equating the Equations (5) to (8) with zero and solving it together, the likelihood estimates $\hat{C}, \hat{p}, \hat{\lambda}$and $\hat{\alpha}$ will be obtained. There is no closed solution to the non-linear equations, so an extensive numerical solution will be used. In addition, the estimates values $\hat{\beta}_j$ for each stress will be obtained by substituting the estimates values of $\hat{C}$ and $\hat{p}$ in the inverse power law relationship$\beta_j = CS_j^p$.

Expected of the Trace for the Fisher Information Matrix:

The expected Fisher-information matrix is derivedby taking expectations of the negative of the second derivatives of the log likelihood function. It can be given by the following matrix:

$$F = -E\begin{bmatrix} \frac{\partial^2 \ln l}{\partial C^2} & \frac{\partial^2 \ln l}{\partial C \partial p} & \frac{\partial^2 \ln l}{\partial C \partial \lambda} & \frac{\partial^2 \ln l}{\partial C \partial \alpha} \\ \frac{\partial^2 \ln l}{\partial C \partial p} & \frac{\partial^2 \ln l}{\partial p^2} & \frac{\partial^2 \ln l}{\partial p \partial \lambda} & \frac{\partial^2 \ln l}{\partial p \partial \alpha} \\ \frac{\partial^2 \ln l}{\partial C \partial \lambda} & \frac{\partial^2 \ln l}{\partial p \partial \lambda} & \frac{\partial^2 \ln l}{\partial \lambda^2} & \frac{\partial^2 \ln l}{\partial \lambda \partial \alpha} \\ \frac{\partial^2 \ln l}{\partial C \partial \alpha} & \frac{\partial^2 \ln l}{\partial p \partial \alpha} & \frac{\partial^2 \ln l}{\partial \lambda \partial \alpha} & \frac{\partial^2 \ln l}{\partial \alpha^2} \end{bmatrix} \downarrow (\hat{C}, \hat{p}, \hat{\lambda}, \hat{\alpha})$$

where,the elements of the trace is as follows:

$$\frac{\partial^2 \ln l}{\partial C^2} = \sum_{j=2}^{k} \frac{n_j}{Y_j - Y_{j-1}} (\lambda X_j e^{\Sigma_{h=1}^{j} -CS_h^p \tau_h{}^\alpha + \lambda e^{\Sigma_{h=1}^{j} -CS_h^p \tau_h{}^\alpha}} (X_j + \lambda Z_j X_j) - \lambda X_{j-1} e^{\Sigma_{h=1}^{j-1} -CS_h^p \tau_h{}^\alpha + \lambda e^{\Sigma_{h=1}^{j-1} -CS_h^p \tau_h{}^\alpha}} (X_{j-1} + \lambda Z_{j-1} X_{j-1}))$$
$$- \sum_{j=2}^{k} \frac{n_j}{(Y_j - Y_{j-1})^2} (\lambda X_j Z_j Y_j - \lambda X_{j-1} Z_{j-1} Y_{j-1})^2$$
$$+ \frac{1}{Y_1 - e^\lambda} (\lambda (X_1)^2 Z_1 n_1 Y_1 + (\lambda Z_1 X_1)^2 n_1 Y_1) + \frac{(\lambda Z_1 X_1 Y_1)^2 n_1}{(Y_1 - e^\lambda)^2}$$
$$+ \sum_{j=1}^{k} \frac{-r_j (1 - e^\lambda)^{-1}}{1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]} (\lambda (X_j)^2 Z_j Y_j + (\lambda X_j Z_j)^2 Y_j)$$
$$+ \sum_{j=1}^{k} \frac{-r_j (1 - e^\lambda)^{-2}}{[1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]]^2} (\lambda X_j Z_j Y_j)^2 \quad (9)$$

where$X_1 = -S_1^p \tau_1{}^\alpha, X_j = \sum_{h=1}^{j} -S_h^p \tau_h{}^\alpha, X_{j-1} = \sum_{h=1}^{j-1} -S_h^p \tau_h{}^\alpha, Z_1 = e^{-CS_1^p \tau_1{}^\alpha}, Z_j = e^{\Sigma_{h=1}^{j} -CS_h^p \tau_h{}^\alpha}$ and $Z_{j-1} = e^{\Sigma_{h=1}^{j-1} -CS_h^p \tau_h{}^\alpha}$.

$$\frac{\partial^2 \ln l}{\partial p^2} = \frac{1}{Y_1 - e^\lambda} (\lambda C X_1 (\ln S_1)^2 Z_1 n_1 Y_1 + \lambda (C X_1 \ln S_1)^2 Z_1 n_1 Y_1 + (\lambda \ln S_1 \, C X_1 Z_1)^2 n_1 Y_1)$$
$$- \frac{(\lambda \ln S_1 \, C X_1 Z_1 Y_1)^2 n_1}{(Y_1 - e^\lambda)^2}$$
$$+ \sum_{j=2}^{k} \frac{n_j}{Y_j - Y_{j-1}} \left(\lambda C X_j (\ln S_h)^2 Z_j Y_j + \lambda \left(C X_j \ln S_h\right)^2 Z_j Y_j + \left(\lambda C X_j \ln S_h \, Z_j\right)^2 Y_j \right.$$
$$\left. - \lambda C X_{j-1} (\ln S_h)^2 Z_{j-1} Y_{j-1} - \lambda \left(C X_{j-1} \ln S_h\right)^2 Z_{j-1} Y_{j-1} - \left(\lambda C X_{j-1} \ln S_h \, Z_{j-1}\right)^2 Y_{j-1}\right)$$
$$- \sum_{j=2}^{k} \frac{n_j}{(Y_j - Y_{j-1})^2} \left(\lambda C X_j \ln S_h \, Z_j Y_j - \lambda C X_{j-1} \ln S_h \, Z_{j-1} Y_{j-1}\right)^2$$
$$- \sum_{j=1}^{k} \frac{r_j (1 - e^\lambda)^{-1}}{1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]} \left(\lambda C X_j (\ln S_h)^2 Z_j Y_j + \lambda \left(C X_j \ln S_h\right)^2 Z_j Y_j + \left(\lambda C X_j \ln S_h \, Z_j\right)^2 Y_j\right)$$
$$- \sum_{j=1}^{k} \frac{\left(\lambda C X_j \ln S_h \, Z_j Y_j\right)^2 r_j (1 - e^\lambda)^{-2}}{[1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]]^2} \quad (10)$$

$$\frac{\partial^2 \ln l}{\partial \lambda^2} = \frac{(1 - e^\lambda)(n_1 e^\lambda) + n_1 e^{2\lambda}}{(1 - e^\lambda)^2}$$
$$+ \frac{n_1 (Y_1 - e^\lambda)(Y_1 (Z_1)^2 - e^\lambda) - n_1 (Y_1 Z_1 - e^\lambda)^2}{(Y_1 - e^\lambda)^2}$$
$$+ \sum_{j=2}^{k} \frac{n_j}{Y_j - Y_{j-1}} \left[(1 - e^\lambda)^{-2} e^{2\lambda} (Y_j - Y_{j-1}) + (1 - e^\lambda)^{-1} e^\lambda (Y_j - Y_{j-1}) + (1 - e^\lambda)^{-1} e^\lambda (Y_j Z_j - Y_{j-1} Z_{j-1}) \right.$$
$$\left. + Y_j (Z_j)^2 - Y_{j-1} (Z_{j-1})^2\right]$$
$$- \sum_{j=2}^{k} \frac{n_j}{(Y_j - Y_{j-1})^2} \left[(1 - e^\lambda)^{-1} e^\lambda (Y_j - Y_{j-1}) + Y_j Z_j - Y_{j-1} Z_{j-1}\right] (Y_j Z_j - Y_{j-1} Z_{j-1})$$
$$+ \sum_{j=1}^{k} \frac{-r_j}{1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]} \left[e^\lambda (Y_j Z_j - e^\lambda)(1 - e^\lambda)^{-2} + (Y_j \left(Z_j\right)^2 - e^\lambda)(1 - e^\lambda)^{-1} + 2e^{2\lambda} (Y_j \right.$$
$$\left. - e^\lambda)(1 - e^\lambda)^{-3} + e^\lambda (Y_j - e^\lambda)(1 - e^\lambda)^{-2} + e^\lambda (Y_j Z_j - e^\lambda)(1 - e^\lambda)^{-2}\right]$$
$$+ \sum_{j=1}^{k} \frac{-r_j}{[1 - [(Y_j - e^\lambda)(1 - e^\lambda)^{-1}]]^2} \left[(Y_j Z_j - e^\lambda)(1 - e^\lambda)^{-1} + e^\lambda (Y_j - e^\lambda)(1 - e^\lambda)^{-2}\right] \left[(Y_j Z_j - e^\lambda)(1 - e^\lambda)^{-1} \right.$$
$$\left. + e^\lambda \left(Y_j - e^\lambda\right)(1 - e^\lambda)^{-2}\right] \quad (11)$$

$$\frac{\partial^2 \ln l}{\partial \alpha^2} = \frac{1}{Y_1 - e^{\lambda}}(\lambda C X_1 n_1 Y_1 Z_1 (\ln \tau_1)^2 + n_1 Y_1 (\lambda C X_1 Z_1 \ln \tau_1)^2 + (C X_1 \ln \tau_1)^2 \lambda n_1 Y_1 Z_1) - \frac{n_1 (C X_1 \lambda Z_1 \ln \tau_1 Y_1)^2}{(Y_1 - e^{\lambda})^2}$$
$$+ \sum_{j=2}^{k} \frac{n_j}{Y_j - Y_{j-1}} [\lambda Z_j (C X_j \ln \tau_h)^2 Y_j + (\lambda Z_j C X_j \ln \tau_h)^2 Y_j + \lambda Z_j C X_j (\ln \tau_h)^2 Y_j - \lambda Z_{j-1} (C X_{j-1} \ln \tau_h)^2 Y_{j-1}$$
$$- (\lambda Z_{j-1} C X_{j-1} \ln \tau_h)^2 Y_{j-1} - \lambda Z_{j-1} C X_{j-1} (\ln \tau_h)^2 Y_{j-1}]$$
$$- \sum_{j=2}^{k} \frac{n_j}{(Y_j - Y_{j-1})^2} [\lambda Z_j C X_j \ln \tau_h Y_j - \lambda Z_{j-1} C X_{j-1} \ln \tau_h Y_{j-1}]^2$$
$$- \sum_{j=1}^{k} \frac{r_j (1 - e^{\lambda})^{-1}}{1 - [(Y_j - e^{\lambda})(1 - e^{\lambda})^{-1}]} \left(\lambda Z_j (C X_j \ln \tau_h)^2 Y_j + (\lambda Z_j C X_j \ln \tau_h)^2 Y_j + \lambda Z_j C X_j (\ln \tau_h)^2 Y_j\right)$$
$$+ \sum_{j=1}^{k} \frac{-(1 - e^{\lambda})^{-2} r_j}{[1 - [(Y_j - e^{\lambda})(1 - e^{\lambda})^{-1}]]^2} (\lambda Z_j C X_j \ln \tau_h Y_j)^2 \quad (12)$$

Let $\gamma_1 = E(n_1)$ and $\phi_1 = E(r_1|n_1)$.For $j = 2, .., k$, based on the multinomial setting (see Tai Lin *et al*. (2009)), then:

$$\gamma_1 = N\left[(e^{\lambda e^{-C s_1^p \tau_1^{\alpha}}} - e^{\lambda})(1 - e^{\lambda})^{-1}\right], \phi_1 = (N - \gamma_1) q_1,$$
$$\gamma_j = (N - \sum_{h=1}^{j-1} (\gamma_h + \phi_h)\left[\frac{F(\tau_j) - F(\tau_{j-1})}{1 - F(\tau_{j-1})}\right]$$
$$= (N - \sum_{h=1}^{j-1} (\gamma_h + \phi_h)) \frac{(1 - e^{\lambda})^{-1}(Y_j - Y_{j-1})}{1 - (Y_{j-1} - e^{\lambda})(1 - e^{\lambda})^{-1}}, \qquad j = 2, \ldots., k,$$
$$\text{And} \phi_j = (N - \textstyle\sum_{h=1}^{j-1} (\gamma_h + \phi_h) - \gamma_j) q_j, \qquad j = 2, \ldots., k. \quad (13)$$

Therefore the elements of the trace for the expected Fisher information matrix can be given as follows:

$$-\left[E\left(\frac{\partial^2 \ln l}{\partial C^2}\right) + E\left(\frac{\partial^2 \ln l}{\partial p^2}\right) + E\left(\frac{\partial^2 \ln l}{\partial \lambda^2}\right) + E\left(\frac{\partial^2 \ln l}{\partial \alpha^2}\right)\right] = -\left[I_{CC} + I_{pp} + I_{\lambda\lambda} + I_{\alpha\alpha}\right], \quad (14)$$

where$I_{CC} = E\left(\frac{\partial^2 \ln l}{\partial C^2}\right), I_{pp} = E\left(\frac{\partial^2 \ln l}{\partial p^2}\right), I_{\lambda\lambda} = E\left(\frac{\partial^2 \ln l}{\partial \lambda^2}\right), I_{\alpha\alpha} = E\left(\frac{\partial^2 \ln l}{\partial \alpha^2}\right)$ are derived by substitutingn_1(number of failures at first step) with its expectationγ_1, r_1 (number of removed units at first step) with its expectation ϕ_1 , n_j(number of failures at jth step)with its expectation γ_j and r_j(number of removed units atjthstep) with its expectation ϕ_j. Note that when $q_1 = q_2 = \cdots = q_{k-1} = 0$and $q_k = 1$, the progressive type I interval censored reduces to type I interval censored.

Optimum Inspection Times Schemes:

The main objective of this section is to determine the optimal inspection times which leads to the most accurate estimate. The two different schemes for obtaining the inspection time $\tau_1, \tau_2 \ldots \ldots, \tau_k$ will be introduced.

The A Optimality Scheme:

For specified values of the number of monitoring times k and type I progressive interval censoring plan $(q_1, q_2, \ldots, q_k)$, the inspection times $\tau_1, \tau_2 \ldots \ldots, \tau_k$ can be obtained by maximizing the trace of the Fisher information matrix (A Optimality criterion) for given values of the parameters $C, p, \lambda$and α. That is the value of τ (assuming equal inspection times setting$\tau_j = j\tau_1$) are determined by maximize the trace of the expected Fisher information matrix as follows:

$$\max_{\tau_1} \left[I_{CC} + I_{pp} + I_{\lambda\lambda} + I_{\alpha\alpha}\right]. \quad (15)$$

The Equal Probability (EP) Inspection Scheme:

Tai Lin *et al*. (2011) presented EP Inspection schemeto find the optimum inspection times.When the values of the parameters C, p, λ and α are given so chosen that the expected numbers of failures within each inspected interval is the same. That is, the EP inspection times $\tau_1, \tau_2 \ldots \ldots, \tau_k$ (assuming not equal inspection times) are the solutions of the equations

$$\gamma_1 = \gamma_2 = \cdots = \gamma_k = \phi_k, \quad (16)$$

whereγ_j and ϕ_j , $j = 1, \ldots, k$ are given in (13).

Simulation Study:

In order to obtain the optimum inspection times and the MLEs, sample sizeof $N = 200$ is generated from three-parameter WP distribution of k-level SSALT data. Optimum inspection times for each stress level are calculated. In addition, the MSE for MLEs are calculated to compare the two optimality schemes (A optimality scheme and EP scheme) for the two selected progressively type-I grouped censoring plan (i) $q_1, q_{2,} \dots, q_{k-1}, q_k = 1$,(ii)$q_1 = q_2 = \cdots = q_{k-1} = 0, q_k = 1$.The simulation procedures are described through the following steps:

i. For a given values of the parameters $C = 0.5, \lambda = 1,\ p = (1.25,1.5,2\ )$ and $\alpha = (1.25,1.5,2)$ and selected values of stresses$V_1 = 0.75$,$V_2 = 1$ and $V_3 = 2$ calculate $\beta_j = CS_j^p$ for each stress level where $(k = 2,3)$.
ii. Generate random samples of size $N = 200$ from uniform $(0,1)$ distribution and obtain the order statistics $(U_{1:N} \dots, U_{N:N})$.
iii. For a given value of the first inspection time $\tau_1 = 0.5$, record the number of failures n_1 such that $U_{n_1:N} \le (e^{\lambda exp^{\left(-CS_1^p {\tau_1}^\alpha\right)}} - e^\lambda)(1 - e^\lambda)^{-1} < U_{n_1+1:N}$.
iv. Following Tai Lin *et al*. (2011).forthe first censoring scheme and for k=3, select $q_1 = 0.1, q_2 = 0.2 = q_3 = 1$. Then for a given value of $q_1 = 0.1$ the fraction of remaining life units $r_1 = (N - n_1)q_1$ are removed from the test at random.
v. For a given value of the second inspection time $\tau_2 = 1$, record the number of failures n_2 such that $U_{n_2:N-n_1} \le (e^{\lambda exp^{(-CS_2^p(\tau_2-\tau_1+(\frac{-S_1^p}{-S_2^p})^{1/\alpha}.\tau_1)^\alpha)}} - e^\lambda)(1 - e^\lambda)^{-1} < U_{n_2+1:N-n_1}$.
vi. For a given value of $q_2 = 0.2$,the fraction of remaining life units $r_2 = (N - n_1 - r_1 - n_2)q_2$ are removed from the test at random.
vii. For a given value of the third inspection time $\tau_3 = 1.25$, record the number of failures n_3 such that $U_{n_3:N-n_1-n_2} \le (e^{\lambda exp^{(-CS_3^p((\tau_3-\tau_2)+(\frac{-S_2^p}{-S_3^p})^{1/\alpha}.(\tau_2+(\frac{-S_1^p}{-S_2^p})^{1/\alpha}.\tau_1))^\alpha)}} - e^\lambda)(1 - e^\lambda)^{-1} < U_{n_3+1:N-n_1-n_2}$.
viii. For a given value of$q_3 = 1$, the fraction of remaining life units $r_3 = (N - n_1 - r_1 - r_2 - n_2 - n_3)q_3$are removed from the test at random.
ix. Based on$n_j, r_j, \tau_j, j = 1,2,3$ and for given values of the parameters $C, p, \lambda$and α, the optimum inspection times are obtained using the two different inspection schemes.
x. Compute MLEs for the parameters by using the values of the optimum inspection times from different schemes.
xi. The MSE for MLEs of the unknown parameters are obtained.
xii. The pervious steps are repeated for the second censoring plan with$q_1 = q_2 = 0$ and $q_3 = 1$.
xiii. The previous steps are repeated for 100 replications.

Simulation results based on k-level SSALT for WP distribution under progressive type I interval censored data with $k = (2,3)$are summarized in Tables from 1 to 5. First, inspection times are computed in Table 1 for the two censored plan, Tables 2-3 present the MSE of the MLEs for the unknown parameters and Tables 4-5 show the number of failures and removed units for the two selected progressively type-I grouped censoring plan.

Table 1: The optimal inspection times Using A inspection scheme and EP inspection scheme for some selected valuesof the parameters and two censoring plans

Parameter		A Inspection Scheme			EP Inspection Scheme		
p	α	k=1	k=2	k=3	k=1	k=2	k=3
$q_1 = 0.1, q_2 = 0.2, q_3 = 1$							
1.25	1.2	0.119	0.238		0.499	1.450	
	5	0.306	0.612	0.918	0.231	0.664	2.580
	1.5	0.149	0.297		0.546	1.326	
		0.324	0.648	0.972	0.272	0.655	2.031
	2	0.187	0.375		0.612	1.191	
		0.332	0.664	0.996	0.337	0.653	1.525
1.5	1.2	0.121	0.243		0.49	1.562	
	5	0.317	0.633	0.950	0.214	0.707	3.155
	1.5	0.147	0.294		0.535	1.405	
		0.328	0.655	0.983	0.247	0.669	2.327
	2	0.189	0.378		0.597	1.233	
		0.331	0.662	0.993	0.425	0.896	2.283
2	1.2	0.125	0.250		0.474	1.823	
	5	0.290	0.58	0.993	0.185	0.807	4.755
	1.5	0.150	0.299		0.515	1.583	
		0.336	0.672	1.008	0.213	0.725	3.178

	2	0.191	0.382		0.572	1.327	
		0.331	0.662	0.993	0.34	0.683	1.552
$q_1 = 0, q_2 = 0, q_3 = 1$							
1.25	1.25	0.117	0.234		0.533	1.445	
		0.305	0.610	0.914	0.339	0.573	1.599
	1.5	0.146	0.292		0.575	1.321	
		0.320	0.639	0.959	0.388	0.600	1.413
	2	0.184	0.369		0.636	1.187	
		0.330	0.659	0.989	0.464	0.644	1.223
1.5	1.25	0.109	0.219		0.522	1.554	
		0.311	0.622	0.933	0.329	0.576	1.781
	1.5	0.144	0.289		0.562	1.397	
		0.321	0.642	0.963	0.374	0.597	1.530
	2	0.186	0.371		0.620	1.227	
		0.328	0.656	0.984	0.446	0.633	1.282
2	1.25	0.123	0.247		0.503	1.809	
		0.330	0.660	0.990	0.292	0.55	2.087
	1.5	0.148	0.296		0.540	1.569	
		0.323	0.645	0.968	0.331	0.562	1.706
	2	0.188	0.375		0.591	1.315	
		0.320	0.640	0.960	0.363	0.720	1.490

Note that, the first censored plan $q_1 = 0.1, q_2 = 0.2, q_3 = 1$ is progressive type I interval censored and the second censored plan is type I interval censored.

Table 2: The MSE for MLEs using the optimal inspection times of A inspection scheme for some selected values of the parameters and two censoring plans

Parameter		MSE							
p	α	$k = 2$				$k = 3$			
		$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{c}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{c}$
$q_1 = 0.1, q_2 = 0.2, q_3 = 1$									
1.25	1.25	0.027	0.496	0.389	0.012	0.001	0.055	0.0003	0.007
	1.5	0.030	0.370	0.520	0.009	0.006	0.349	0.0001	0.028
	2	0.07	0.113	0.049	0.028	0.033	0.146	0.079	0.062
1.5	1.25	0.0225	0.481	0.46	0.013	0.003	0.562	0.005	0.038
	1.5	0.046	0.417	0.527	0.009	0.001	0.266	0.0001	0.021
	2	0.078	0.151	0.304	0.019	0.059	0.104	0.053	0.011
2	1.25	0.230	0.451	0.009	0.086	0.0003	0.0005	0.0002	0.0001
	1.5	0.03	0.015	0.443	0.002	0.001	0.294	0.0007	0.017
	2	0.051	0.164	0.206	0.013	0.0005	0.0900	0.0005	0.015
$q_1 = 0, q_2 = 0, q_3 = 1$									
1.25	1.25	0.022	0.002	0.319	0.005	0.008	0.041	0.023	0.014
	1.5	0.011	0.002	0.448	0.005	0.05	0.117	0.046	0.034
	2	0.040	0.008	0.327	0.002	0.182	0.137	0.052	0.172
1.5	1.25	0.023	0.006	0.382	0.005	0.003	0.03	0.009	0.008
	1.5	0.008	0.002	0.468	0.004	0.028	0.097	0.032	0.026
	2	0.08	0.153	0.319	0.019	0.19	0.52	0.166	0.110
2	1.25	0.02	0.004	0.428	0.004	0.001	0.031	0.0004	0.006
	1.5	0.008	0.001	0.518	0.003	0.008	0.034	0.011	0.01
	2	0.046	0.151	0.212	0.013	0.114	0.338	0.123	0.071

Table 3: The MSE for MLEs using the optimal inspection times of EP inspection schemefor some selected values of the parameters and two censoring plans

Parameter		MSE							
p	α	$k = 2$				$k = 3$			
		$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{c}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{p}$	$\hat{c}$
$q_1 = 0.1, q_2 = 0.2, q_3 = 1$									
1.25	1.25	0.032	0.022	0.528	0.0007	0.079	0.204	0.289	0.023
	1.5	0.019	0.415	0.128	0.0009	0.108	0.006	0.041	0.002
	2	0.01	0.073	0.041	0.001	0.007	0.483	0.373	0.009
1.5	1.25	0.045	0.402	0.159	0.001	0.292	0.171	0.476	0.034
	1.5	0.015	0.530	0.127	0.001	0.197	0.106	0.258	0.034
	2	0.017	0.094	0.040	0.001	0.091	0.094	0.319	0.041
2	1.25	0.066	0.403	0.153	0.002	0.144	0.132	0.522	0.002
	1.5	0.024	0.400	0.138	0.002	0.177	0.165	0.260	0.033
	2	0.009	0.144	0.040	0.002	0.009	0.07	0.008	0.025
$q_1 = 0, q_2 = 0, q_3 = 1$									
1.25	1.25	0.003	0.0004	0.340	0.0005	0.077	0.006	0.012	0.019
	1.5	0.0100	0.0009	0.528	0.001	0.151	0.189	0.016	0.014
	2	0.035	0.017	0.516	0.002	0.069	0.182	0.007	0.012
1.5	1.25	0.011	0.0007	0.435	0.0006	0.172	0.370	0.019	0.021
	1.5	0.005	0.0006	0.506	0.001	0.162	0.291	0.016	0.019

	2	0.014	0.095	0.048	0.001	0.033	0.085	0.021	0.019
2	1.25	0.070	0.142	0.527	0.001	0.037	0.056	0.067	0.027
	1.5	0.018	0.017	0.436	0.001	0.007	0.019	0.007	0.030
	2	0.009	0.126	0.041	0.002	0.008	0.031	0.002	0.014

Table 4: Number of removed units (r_1, r_2, r_3) and failures (n_1, n_2, n_3) at each step of A inspection scheme for some selected values of the parameters and two censoring plans

Parameter		$k=2$				$k=3$					
p	α		r_2	n_1	n_2	r_1	r_2	r_3	n_1	n_2	n_3
$q_1=0.1, q_2=0.2, q_3=1$											
1.25	1.25	14	110	55	20	14	18	51	56	35	26
	1.5	15	113	47	24	15	18	44	49	40	34
	2	16	119	35	30	16	17	29	36	54	48
1.5	1.25	14	112	55	19	14	18	54	56	33	25
	1.5	15	114	47	23	15	18	46	49	40	32
	2	16	120	35	29	16	18	33	36	52	45
2	1.25	14	113	55	17	14	19	59	36	29	22
	1.5	15	116	47	22	15	19	51	49	36	29
	2	16	122	35	26	16	20	43	36	43	42
$q_1=0, q_2=0, q_3=1$											
1.25	1.25	0	135	35	30	0	0	73	54	37	36
	1.5	0	126	47	27	0	0	61	47	45	47
	2	0	132	35	33	0	0	41	35	58	66
1.5	1.25	0	124	55	21	0	0	76	54	36	34
	1.5	0	128	47	25	0	0	65	47	43	45
	2	0	134	34	32	0	0	47	35	54	64
2	1.25	0	126	55	19	0	0	84	55	31	30
	1.5	0	130	47	23	0	0	76	47	39	38
	2	0	135	35	30	0	0	50	35	51	64

Table5: Number of removed units (r_1, r_2, r_3) and failures (n_1, n_2, n_3) at each step of EP inspection scheme for some selected values of the parameters and two censoring plans

Parameter		$k=2$				$k=3$					
p	α		r_2	n_1	n_2	r_1	r_2	r_3	n_1	n_2	n_3
$q_1=0.1, q_2=0.2, q_3=1$											
1.25	1.25	14	110	55	21	14	18	51	56	35	26
	1.5	15	113	47	25	15	18	44	49	40	34
	2	16	119	35	31	16	17	29	36	54	48
1.5	1.25	14	111	55	20	14	18	53	56	33	25
	1.5	15	114	47	23	15	18	46	49	40	32
	2	16	120	35	29	16	18	33	36	52	45
2	1.25	14	113	55	18	14	19	59	36	29	22
	1.5	15	116	47	21	15	19	51	49	36	30
	2	16	122	35	27	16	20	43	36	43	42
$q_1=0, q_2=0, q_3=1$											
1.25	1.25	0	118	55	27	0	0	164	16	6	14
	1.5	0	126	47	27	0	0	161	15	6	18
	2	0	132	35	33	0	0	153	10	10	27
1.5	1.25	0	124	55	21	0	0	173	13	4	10
	1.5	0	127	47	26	0	0	171	10	7	12
	2	0	133	35	32	0	0	168	7	7	18
2	1.25	0	126	55	19	0	0	185	7	3	5
	1.5	0	130	47	23	0	0	185	5	3	7
	2	0	135	35	30	0	0	80	34	30	56

From these tables, the following conclusions can be observed:

i. In general, based on the two selected progressively type-I grouped censoring plan$[(q_1, q_2, \dots, q_{k-1}, q_k = 1), (q_1 = q_2 = \cdots = q_{k-1} = 0, q_k = 1)]$ the inspection timesusing A optimality inspection scheme have smaller inspectiontimes than EP inspection scheme.
ii. For the two inspection schemes, the censored plan $q_1 = 0, q_2 = 0, q_3 = 1$ has smaller values of the inspection time than the censored plan$q_1 = 0.1, q_2 = 0.2, q_3 = 1$for all different values of the parameters.
iii. For the first censored plan $q_1 = 0.1, q_2 = 0.2, q_3 = 1$ and for the two inspection schemes, the values ofthe MSE for the MLEs of the unknown parameters are smaller for $k = 3$ than in the case of $k = 2$.
iv. For the second censored plan $q_1 = 0, q_2 = 0, q_3 = 1$ and for the two inspection schemes, the values of the MSE for the MLEs of the unknown parametersarevery smellier for $k = 3$and $k = 2$except for $\hat{p}$ where its smaller in the case of $k = 3$ than in the case of $k = 2$.

v. For the first censored plan$q_1 = 0.1, q_2 = 0.2, q_3 = 1$ and when using A inspection scheme, $\hat{\lambda}$ and $\hat{c}$ have the smallest MSE for all different values of the parameters in the case of $k = 2$.

vi. For the first censored plan $q_1 = 0.1, q_2 = 0.2, q_3 = 1$ and when using A inspection scheme$\hat{\lambda}, \hat{p}$ and $\hat{c}$ have the smallest MSE for all different values of the parameters in the case of $k = 3$.

vii. For the second censored plan $q_1 = 0, q_2 = 0, q_3 = 1$ and when using A inspection scheme, $\hat{\lambda}, \hat{\alpha}$and $\hat{c}$ have the smallest MSE for all different values of the parameters in the case of $k = 2$.

viii. For the second censored plan $q_1 = 0, q_2 = 0, q_3 = 1$ and when using A inspection scheme,$\hat{\lambda}, \hat{p}$ and $\hat{c}$ have the smallest MSE for all different values of the parameters in the case of $k = 3$.

ix. For the two censored plan and when using EP inspection scheme,$\hat{\lambda}, \hat{\alpha}$ and $\hat{c}$ have the smallest MSE for all different values of the parameters in the case of $k = 2$ and $k = 3$.

x. For the two censored plan and when using EP inspection scheme, MSE for the MLEs of the unknown parameters have smaller values in the case of $k = 3$ than in the case of $k = 2$ for all different values of the parameters.

xi. As $k$increases,the number of failures $(n_1, n_2, n_3)$are decreased and number of removed units$(r_1, r_2, r_3)$are increased for all different values of the parameters and for the two censored plans.

xii. Number of failures (n_1, n_2, n_3) are very small using EP inspection scheme for the second censored plan $q_1 = 0, q_2 = 0, q_3 = 1$ in the case of $k = 3$compared to number of failures in the first censored plan $q_1 = 0.1, q_2 = 0.2, q_3 = 1$for all different values of the parameter except when $p = 2$,$\alpha = 2$.

Conclusion Remarks:

In this paper, theinspection times are computed using two inspection schemes and for two censored plans. The maximum likelihood method for estimating the unknown parameters with type I progressive interval censoring data isapplied to determine the effect of the two inspection schemes. The performance of the estimated parameters is evaluated using the mean square error criterion. MLEs of the unknown parameters are obtained using different values of the parameters. The failure timesdata for k-level SSALT are assumed to follow the three-parameter WP distribution at each stress level with scale parameter which has an inverse power law function of the stress. Number of failures and number of removed units are recorded for the two inspectiontime schemes.

In general, the inspection times of the experiment have longer time when using EP inspection scheme than A inspection scheme for the two censored plans. The MLEs have better performance as k (number of steps) increases for the two censored plans. In almost all cases, for the two inspection schemes, the MSE for the MLEs of $\hat{\lambda}, \hat{c}$ have the smallest values for all different values of the parameters. The MSE when using EP inspection scheme are slightly smaller than the A inspection scheme for the two censored plans. Number of failures and remaining units from the experiment for the two different optimality schemes are significantly bigger than the number of removed units for the two censored plans.

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