



AENSI Journals

Australian Journal of Basic and Applied Sciences

ISSN:1991-8178

Journal home page: www.ajbasweb.com



Model Reference Adaptive Control of a Spherical Tank

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ARTICLE INFO

Article history:

Received 19 August 2014

Received in revised form

19 September 2014

Accepted 29 September 2014

Available online 3 December 2014

Keywords:

Modelling of Spherical Tank, Model Reference Adaptive Control, MIT Rule.

ABSTRACT

Background: The objective is to track the reference model and to develop a control scheme which forces the plant dynamics to follow the dynamics of the reference model. The controller so designed forces the error between the plant and the reference states to zero as time tends to infinity and ensuring that the plant output following the reference model output faithfully even in the presence of unmodelled dynamics or disturbances in the actual plant. The servomechanism is also discussed where the system has to track the reference signal in the presence of disturbances for various feedforward Gains. In this paper the spherical tank is taken as the model of the actual plant and using MIT Rule and Lyapunov Approach where the system is adapted.

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To Cite This Article: B. Senthil Kumar, Dr. K. Suresh Manic and P. Deepa., Model Reference Adaptive Control of a Spherical Tank. *Aust. J. Basic & Appl. Sci.*, 8(18): 350-355, 2014

INTRODUCTION

A nonlinear spherical tank level process, whose parameters vary with respect to the process variable, is considered. To facilitate statistical evaluation a mathematical modelling was discussed earlier for the benchmark processes taken for the study. It was a theoretical approach as dynamics observed based on laws of conservation of mass. Nevertheless, it has not been sufficient to determine appropriate model by first principle method discussed in the previous chapter. Hence an empirical modelling is discussed here which led to the graphical representation of open loop responses along with Multi-model approach

In spite of the complexity of the system it turns out that its gross behaviour is well captured by set of linear models at specific operating points. The transfer function models are leading to the pathway for obtaining the closed loop responses of the system. Specifically, theoretical and practical framework of the empirical modelling of the spherical tank system has been taken (Deepa.P, *et al.*, 2006). The empirical modelling issues are dealt here with the graphical description of the open loop schemes. In This paper the Multimode ling approach is taken for the spherical tank and its initial height is considered. Although nonlinear tank problems has been widely addressed in classical system dynamics, when the control process is nonlinear it is not feasible to obtain satisfactory closed loop responses. Designing the spherical tank system as a whole system over the entire operating region becomes a challenging task. In order to explain the physical insight of the whole system behaviour, linearization is an approximation of the actual model of the system is taken for this paper. In the controlling part, here adaptive control is used where the uncertainties are at large example on modelling the parameters like time constant of the system and the control element acting on the system are not modelled exactly. Mainly they are called modelled uncertainties and also external disturbances acting on the system. The parametric uncertainties and their issues are mainly dealt using heuristics method (Latha, K *et al.*, 2013; Rajinikanth and Latha, 2012; Suresh Manic *et al.*, 2014). Design of PID Controllers using double feedback loops for SISO system using setpoint filters were widely discussed (vijayan and Rames 2012a; Vijayan and Panda 2012). Modelling Uncertainties and bounded external disturbances were done using sliding mode control techniques (Senthil Kumar and Suresh Manic, 2014). The Paper is organized as Modelling Issues which where taken from the real time system setup and various values where recorded which deals with the first part, the second part about Model reference adaptive System approach where lyapunov and MIT rule where taken into consideration, The third part gives the simulation results and finally about conclusion.

Modelling issues:

Spherical tanks are used in many process industries as storage of cryogenic liquids, fuels and other liquids also as surge tanks (Padmasree and Chidambaram, 2006; Wayne Bequette, 2003). In such tanks, in order to maintain the level constant, a level sensor and a controller is needed. If level comes down to a low mark or goes up to a high mark the process gets disturbed or overflows/spills out. The behaviour of liquid height can be known by proper mathematical model of liquid level in spherical tank. (The spherical tank system exhibits the property of nonlinearity and it is considered as the real time model for our work. The real time spherical tank system is used to obtain effective mathematical model by establishing a relationship between the process variables and basic physical laws governing the process. In process industries, step response based methods are most commonly used for system identification. The modelling part of nonlinear system behaviour is illustrated by linearizing the system into many piecewise segments. The Schematic sketch of spherical tank liquid level system is shown in Figure



Fig. 1: Non – Linear Process Loop (Spherical Tank).

Nonlinear Process Loop Spherical Tank System:

The transfer function obtained from the open loop response by process reaction curve method for the operating region of 0-20 cm at the operating point at 14 cm is written as in Equation (1). The parameters mentioned in the transfer functions of all the regions are K (Cm/litres per minute), τ (minutes).

$$G(s) = \frac{2.452}{0.5125s + 1} \quad (1)$$

The first order system is considered and taken without delay. The parameters are taken from the following papers Deepa *et al.*, 2012)

Model Reference Adaptive System:

This may be regarded as an adaptive servo system in which the desired performance is expressed using reference model, which gives the desired performance to a command signal. The reference model need not be an actual hardware, but a model simulated on a computer. In the block diagram the system has an ordinary feedback loop composed of the process and the controller and another loop that consists of the reference model and the adjustable mechanism that changes the controller parameters. The parameters of the controller are changed on the basis of the error that is the difference between the outputs of the model and the actual Plant (Landau, J, 1974; Broussard and O'Brain, 1980).

To obtain the closed loop system described by the model

Adaptation of the feedforward gain

$$e = y - y_m \quad (2)$$

$$e = kG(s) \theta u_c - k_0 G(s) u_c \quad (3)$$

$$\frac{\partial e}{\partial \theta} = kG(s) u_c = \frac{k}{k_0} y_m$$

And the adaptation law given by MIT Rule is given by

$$\frac{d\theta}{dt} = -\gamma' \frac{k}{k_0} y_m e = -\gamma y_m e \quad (4)$$

The mechanism for adjusting the control parameter can be obtained in two ways (i) by using gradient method and (II) by applying Lyapunov stability theory. In this paper the first is considered and it is popularly called as MIT Rule.

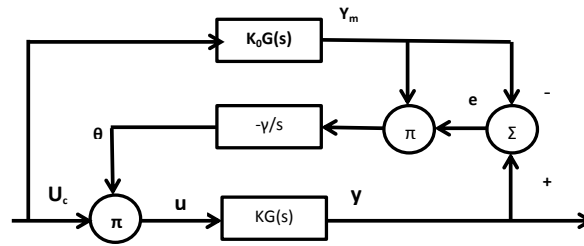


Fig. 2: Adaptation of the Feedforward gain.

A MIT Rule:

To present rule let us consider a closed loop system in which the controller has one parameter the desired closed loop response is specified by a model whose output is y_m (Landau, 1979 [4]). Let error e be the error between the output of the actual system y and y_m . One possibility is to adjust the parameter in such a way that the loss function

$$J(\theta) = \frac{1}{2} e^2 \tag{5}$$

is minimized. To make J small the parameter is changed in the direction of the negative gradient of J .

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \frac{\partial J}{\partial e} \frac{\partial e}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \tag{6}$$

This is called as MIT Rule. The partial derivative $\frac{\partial e}{\partial \theta}$ is called as the sensitivity derivative of the system. If the parameter changes are slower than the other parameters of the system the derivative $\frac{\partial e}{\partial \theta}$ can be evaluated under the assumption that θ is a constant.

$J(\theta) = |e|$ then

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \frac{\partial e}{\partial \theta} \text{signe} \tag{7}$$

The above procedure applies when there are many parameters to adjust, then the symbol θ is a vector and $\frac{\partial e}{\partial \theta}$ is the gradient vector of the error with respect to the parameters.

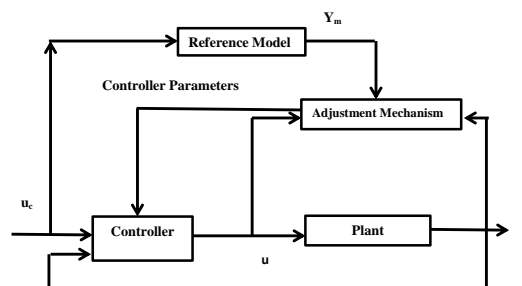


Fig. 3: Adaptation with Reference Model.

$$\frac{dy}{dt} = -ay + bu \tag{8}$$

where u is the control variable and y is the measured variable output

To obtain the closed loop system, described by the model

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c \tag{9}$$

Where u is the control signal and u_c is the command signal

Let the structure of the controller be

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \tag{10}$$

The controller has two parameters and chosen to be

$$\theta_1 = \theta_1^0 = \frac{b_m}{b} \tag{11}$$

$$\theta_2 = \theta_2^0 = \frac{a_m - a}{b} \tag{12}$$

The input-output relations of the system and the model are same then it is called as perfect model following.

To Apply MIT rule we introduce the error

$$e = y - y_m \quad (13)$$

$$y = \frac{b}{s+a} u = \frac{b}{s+a} (\theta_1 u_c(t) - \theta_2 y(t)) \quad (14)$$

$$\frac{b}{s+a} \theta_1 u_c(t) - \frac{b}{s+a} \theta_2 y(t) = \frac{b\theta_1}{s+a+b\theta_2} u_c(t) \quad (15)$$

The sensitivity derivatives are obtained by taking the partial derivatives of the error with respect to the controller parameters as

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{s+a+b\theta_2} u_c(t) \quad (16)$$

$$\frac{d\theta_2}{dt} = -\gamma \left(\frac{a_m}{s+a_m} y(t) \right) e \quad (17)$$

These cannot be used directly because the parameters a and b are unknown. Approximations are therefore required.

$$s+a+b\theta_2 \approx s+a_m$$

Since under perfect model following $a=a_m$ and $\theta_2=0$

Finally the sensitivity derivatives becomes as

$$\frac{d\theta_1}{dt} = -\gamma \left(\frac{a_m}{s+a_m} u_c(t) \right) e \quad (18)$$

$$\frac{d\theta_2}{dt} = -\gamma \left(\frac{a_m}{s+a_m} y(t) \right) e \quad (19)$$

B Lyapunov Theory:

Lyapunov Stability theory used to construct adaptive control algorithms to adjust the parameters of control scheme. differential equation for the error is derived which has the adjustable parameters of the system which attempts to find lyapunov function and an adaptation mechanism in such a error tends to zero, asymptotically.

The first order system is considered here

$$\dot{y} = -ay + bu \quad (20)$$

Where y is the measured output and u is the control variable.

The model of the closed loop system is given by

$$\dot{y}_m = -a_m y_m + b_m u_c \quad (21)$$

Where u_c is a reference signal which is bounded and $y_m > 0$.

Let the control structure be

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \quad (22)$$

Introducing the error

$$e = y - y_m$$

$$\dot{e} = \dot{y} - \dot{y}_m$$

The aim is to reduce the error and the differential equation is given as

$$\dot{e} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c \quad (23)$$

The error approaches zero if the parameters of θ_1 and θ_2 given by

$$\theta_1 = \theta_1^0 = \frac{b_m}{b} \quad (24)$$

$$\theta_2 = \theta_2^0 = \frac{a_m - a}{b} \quad (25)$$

Considering the lyapunov candidate as

$$v(e, \theta_1, \theta_2) = \frac{1}{2} \left(e^2 + \frac{1}{b\gamma} (b\theta_1 - b_m)^2 + \frac{1}{b\gamma} (b\theta_2 + a - a_m)^2 \right) \quad (26)$$

The time derivative of the lyapunov function is

$$\begin{aligned} \dot{v} &= e\dot{e} + \frac{1}{\gamma}(b\theta_1 - b_m)\dot{\theta}_1 + \frac{1}{\gamma}(b\theta_2 + a - a_m)\dot{\theta}_2 \\ &= -a_m e^2 + \frac{1}{\gamma}(b\theta_1 - b_m)\dot{\theta}_1 + \gamma_c e + \frac{1}{\gamma}(b\theta_2 + a - a_m) \\ &\quad (\dot{\theta}_2 - \gamma e) \end{aligned} \tag{27}$$

Assuming that $b\gamma > 0$ $v(e, \theta_1, \theta_2) > 0$; $v(0) = 0$

And the parameters are updated as $\dot{\theta}_1 = -\gamma_c e$ and $\dot{\theta}_2 = \gamma e$

Simulation Results:

Assumptions (Shankar and Bodson, 1980)

(1) Plant Assumption:

The plant is single-input single output (SISO), Linear Time Invariant (LTI) system, described by a transfer function. The plant is strictly proper and minimum phase. The sign of high frequency gain is known and it is strictly positive.

(2) Reference model Assumption:

The reference model is stable, minimum phase and is the same degrees as the corresponding plant polynomial

(3) The reference input is piecewise continuous and bounded. The reference model is considered as $1/s+1$ and

the system is $G(s) = \frac{2.452}{0.5125s + 1}$

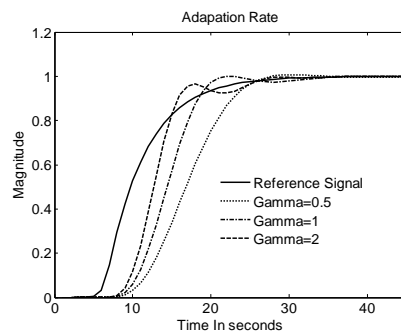


Fig. 3: Various Adaptation Gains.

Table 1:

Adaptation Gain	Integral Absolute Error
0.5	1.158
1	0.6806
2	0.4082

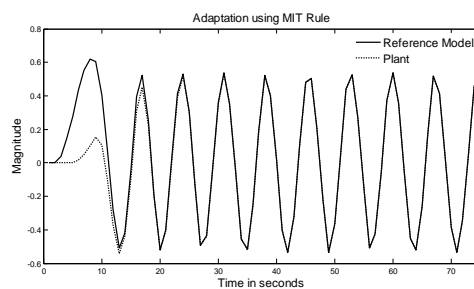


Fig. 4: Adaptation using MIT Rule.

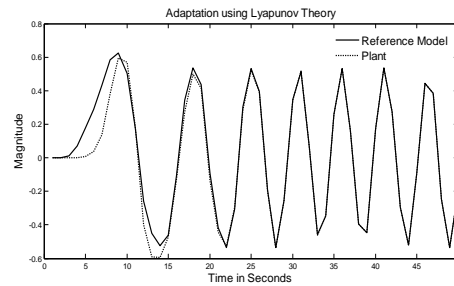


Fig 5: Adaptation using Lyapunov Theory.

Table 2:

Methods	Integral Absolute Error
MIT Rule	0.352
Lyapunov Theory	0.2528

Conclusion:

The combined parameters b and a_m with the adaptation gain γ they appear as $\gamma^2 b/a_m$. Comparing MIT rule and lyapunov theory the parameter adjustment is where θ is vector of parameters. For Lyapunov rule $\phi = (-u_c \ y) T$ and for MIT Rule $\phi = a_m/s + a_m (-u_c \ y) T$. The Lyapunov rule is simpler due to the absence of the filters although the error becomes zero the parameters will not assume its correct values, the parameters are not bounded and the sensitivity derivatives are not used here as in the case of MIT rule, so the order of the controller is third order instead of five in the case of five in MIT rule. The sign of the parameter should be known to fix the correct sign of γ and the filter has been normalized to steady state gain as unity.

REFERENCES

- Deepa, P., D. Manamalli and C. Rames, R.C. Panda, 2013. Control of Liquid Level in Spherical Tank, Wolfania Journal Klagenfurt ,Austria, 20(6): 343-354.
- Vijayan, V. and C. Rames, Panda, 2012a. Design of PID controllers in double feedback loops for SISO systems with set-point filters, ISA Transactions., 51(4): 514-521.
- Padmasree, R. and M. Chidambaram, 2006. Control of Unstable Systems, Narosa Publishing House, India.
- Wayne Bequette , B ., 2003. Process Control – Modelling, Design and Simulation, Prentice – Hall of India Pvt Ltd.
- Suresh Manic, K., A. Sarath and V. Rajinikanth, 2014. Refined Double Search Optimization Methodology to Design PID Controller for Unstable Systems, Australian Journal of Basic and Applied Sciences, 8(10): 44-51.
- Landau, ID, 1974. Survey of Model-Reference adaptive Techniques-Theory and applications, Journal Automatica, 10(4): 353-379.
- Broussard, J.R. and M.J. O'Brien, 1980. Feedforward Control to track the output of a forced model, IEEE Transactions on Automatic Control Ac, 25: 851-853.
- Landau, I.D., 1979. Adaptive Control-The model Reference Approach, Marcel dekker, New York.
- Shankar Sastry, Marc bodson, 1980. Adaptive Control Stability, Convergence and Robustness, Prentice Hall, Inc, Englewood Cliffs NJ., USA.
- Latha, K., V. Rajinikanth and P.M. Surekha, 2013. PSO-Based PID Controller Design for a Class of Stable and Unstable Systems, ISRN Artificial Intelligence, Article ID, 543607: 11.
- Rajinikanth, V. and Latha, K. 2012. Controller Parameter Optimization for Nonlinear Systems Using Enhanced Bacteria Foraging Algorithm, Applied Computational Intelligence and Soft Computing, Article ID 214264, 12 pages,.
- Senthil Kumar, B. and K. Suresh Manic, 2014. Sliding Mode Control for a Minimum Phase Second Order System, Applied Mechanics and Materials, 573: 285-290.
- Vijayan, V. and Rames, C. Panda, 2012. Design of setpoint filter for minimizing overshoot for low order process”, ISA Transactions., 51(2): 271-276.