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On Generalized Minimal-Open Set and Some Properties

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ABSTRACT

In this paper we introduced two topological concepts ,minimal regular open(briefly,$m_{i-RO(X)}$) set and minimal pre-regular open set(briefly,$m_{i-PRO(X)}$) and the concept minimal open(briefly,$m_{i-O(X)}$) set comes between them and we studied the relations betweenthese twoconcepts with the concept minimal α-open set(briefly,$m_{i-\alpha O(X)}$) .In addition to that we proved some of their proposition .At last we introduced other concepts like generalized* regular minimal closed set(briefly, g^*r-$m_{i-c(X)}$), regular generalized* minimal closed set (briefly, ,rg^*-$m_{i-c(X)}$), generalized** regular minimal closed set (briefly, $g^{**}r$-$m_{i-c(X)}$) andregular generalized** minimal closed set (briefly,rg^{**}-$m_{i-c(X)}$),we explained the relations between these concepts and generalized*minimal closed set and gave some important examples and some theorems.

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INTRODUCTION

The concept of minimal open set was introduced first byNakaoka. AndOda (2001,2003,2003).The concept of generalized closed set was first introduced by Levine(1970). In this paper we shall introduce , the concept of regular generalized* minimal-closed sets is to be introduced which lies between generalized*regular minimal-closed sets and regular generalized** minimal-closed sets, and introduce the generalized** regular minimal-closed set and some properties of this sets are studies and the corresponding topological space. Moreover It can be shown that the relation with regular generalized**minimal-closed set and generalized** regular minimal-closed set are independent and some properties and characteristics of these would be given.

2- Preliminaries:

Some important preliminaries required to go further through this paper are cited below.

Definition 2.1:

A subset A of a space X is called a generalized closed set (briefly,*g* - closed)(Bhattacharya, S., Halder,2011) if *ClA⊆U* whenever *A ⊆U* and *U* is an open set.

Definition 2.2:

A subset A of a space X is called a regular generalized closed (briefly, *rg*- closed)set(Vadival, A. and Variya Manickam, K., 2009) if *ClA⊆U* whenever *A ⊆U* and *U* is a regular open set

Definition 2.3:

A subset A of a space Xis called a generalizedregular closed[resp. a generalized regular open] set (briefly, *gr*- closed)[respbriefly,*gr*- open](Bhattacharya, S., Halder,2011) if *RCl(A) ⊆U* [resp*U ⊆R Int(A)*] whenever *A ⊆U* [resp*U ⊆A*] , and *U* is an open [resp. a closed] subset of *X*

Example 2.4:

Let X = {a, b, c}(Bhattacharya, S., Halder,2011)and the corresponding topological space be$\mathcal{T}$= {*φ*, X, {a}, {b,c}}. Let A = {b}. Here A is a generalized regular closed set of X. Though it is not a regular closed subset of X. Similarly it can be shown that, {b, c} is a generalized regular open subset of X.

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Definition .2.5:
A proper nonempty open subset U of a topological space X is called to be a minimal open set(Nakaoka, F. and Oda, N., 2003) if any open set which is contained in U is Ø or U.

Definition.2.6:
A proper nonempty open subset U of a topological space X is called to be maximal open set(Nakaoka, F. and Oda, N., 2003) if any open set which contains U is X or U.

Definition. 2.7:
A proper nonempty closed subset F of a topological space X is called to be a minimal closed set(Nakaoka, F. and Oda, N., 2003) if any closed set which is contained in F is Ø or F.

Definition.2.8:
A proper nonempty closed subset F of a topological space X is called to be maximal closed set(Nakaoka, F. and Oda, N., 2003) if any closed set which contains F is X or F.

Definition.2.9:
A subset A of X is called to be a generalized* minimal closed [resp. generalized* maximal open] set(Bhattacharya, S., Halder,2011) if A is contained [resp. A contains] in at least one minimal open [resp. at least one maximal closed] subset U of X such that $Cl(A) \supseteq U$ [resp .$Int(A) \subseteq U$]

Example.2.10:
Let $X=\{a,b,c,d\}$(Bhattacharya, S., Halder,2011) and the topology be $\mathcal{T}=\{\emptyset,\{b,c\},\{a\},\{a,b,c\},X\}$.Let $A=\{c\}\subseteq \{b,c\}$.Then $Cl(A)=\{b,c,d\} \supseteq \{b,c\} \Rightarrow A$ is a generalized* minimal closed

Definition 2.11:
Let A be a subset of a space X .the closure of A(Lipschutz S. ,1965), denoted by $Cl(A)$ is the intersection of all closed supersets of A .In other words, if $\{F_i : i\in I\}$is the class of all closed subsets of X containing A then$Cl(A)=\cap_i F_i$.

Definition 2.12:
A subset A of space X is called
1) an pre- open set(Mashhour, A.S., Abd El-Monsef , M.E. and El-Deeb, S.N., 1982) if $A\subseteq Int(cl(A))$ and a pre closed set if $Cl(Int(A))\subseteq A$;
2) an α-open set (Njastad,O.,1965) if $A\subseteq Int(Cl(Int(A)))$ and α-closed set if $Cl(Int(Cl(A)))\subseteq A$;
3) an regular open set(Palaniappan , N. and Rao, K.C., 1993) if $A= Int (Cl(A))$ and a regular closed set if $A = Cl(Int(A))$.

Definition 2.13:
Let A be a subset of a space X .the interior of A(Lipschutz S. ,1965), denoted by $Int(A)$ is the union of all open subsets of X, contained in A In other words ,if $\{F_i : i\in I\}$is the class of all open subsets of X contained in A then $Int(A)=\cup_i F_i$

Definition 2.14:
A subset A of a space X is called a generalized*regular closed set[resp. generalized*regular open set] (briefly g*r-closed set, .g*r-open set)((Bhattacharya, S., Halder,2011)if $RCl(A)\supseteq U$ [resp .$U\supseteq RInt(A)$] whenever $A\subseteq U$ [resp .$U\subseteq A$] and U is a regular open [resp. regular closed] subset of X.

Example2.15:
Let $X=\{a,b,c\}$((Bhattacharya, S., Halder,2011)and the corresponding topological space be
$\mathcal{T}=\{\emptyset,\{a\},\{a,b\},X\}$
Here the only regular open sets are X and $\emptyset$
Let $A=\{a,c\}$.Clearly $A\subseteq X$,the regular open set $RCl(A)=X$,which is also subset of X. So A is a generalized*regular closed set.

Definition 2.16**:**
Let X be a space and $A \subseteq X$ an α - open set. Then A is called a minimal α-open set(Mohammed, M. , Nokhas, 2013) if ϕ and A are the only α-open subsets of A.

Theorem.2.17:

A subset *A* of *X* is a generalized * minimal closed(Bhattacharya, S., Halder,2011)*iff* there exist a minimal open set *U* containing *A* such that *Cl(A)=Cl(U)*

Remark.2.18:

Ø and *X* are not generalized * minimal closed((Bhattacharya, S., Halder,2011)**.**

Theorem.2.19:

Arbitrary union of generalized * minimal closed set((Bhattacharya, S., Halder,2011) is a generalized * minimal closed set if it is contained in a minimal open set

Remark.2.20:

Finite intersection of generalized * minimal closed set((Bhattacharya, S., Halder,2011)need not be a generalized* minimal closed set which follows example

Let *X={a ,b ,c}* and the corresponding topological space be $\mathcal{T}$*={Ø,{a,b},{c},X}*.Let *A={c}* be a subset of *X* .Obviously *A* is a generalized * minimal closed set. Let *B={a}* be another subset of *X*. *B* is also a generalized * minimal closed set of *X* .But *A∩B=Ø* is not a generalized * minimal closed set of *X*.

Theorem 2.21:

Non–null intersection of a generalized * minimal closed set(Bhattacharya, S., Halder,2011) and a closed is a generalized minimal closed set.

Theorem 2.22:

Let *A* be a generalized * minimal closed set(Bhattacharya, S., Halder,2011) and *B* be a subset of *X* contained in the same minimal open set if *A⊆B* ⊆ *Cl(A)*,then *B* is also a generalized minimal closed set.

Theorem 2.23:

A proper nonempty subset *F* of *X*(Bhattacharya, S., Halder,2011) is maximal gpr-closed set *iff* (*X-F*) is a minimal gpr-open set.

3- On generalized minimal-open set and some properties:

Now we introduced a new concepts in a topological space and some of their properties with many examples

Definition 3.1:

Let *X* be a space and *A* ⊆ *X* an regular open set then *A* is called a minimal regular open set if *Ø* and *A* are only regular open subsets of *A*. The family of all minimal regular open set is denoted by $m_{i-RO(X)}$

Example 3.2:

Let *X ={a ,b ,c} and* the topology be $\mathcal{T}$*={ Ø,{a},{a,b},{b},X}*, *Ro(X)={Ø,{a},{b},X}*, *A={a} is* $m_{i-R-Open}$ set⇒ *A* is minimal regular open set.

Definition 3.3:

Let *X* be a space and *A* ⊆ *X* an regular closed set then *A* is called a maximal regular closed set if *Ø* and *A* are only regular closed subsets of *A*. The family of all maximal regular closed set is denoted by $m_{a-Rc(X)}$

Recall Example 3.2.

A={b, c} is$m_{a-R-closed}$ set

Definition 3.4:

Let *X* be a space and *A* ⊆ *X* an pre regular open set then *A* is called a minimal pre regular open set if *Ø* and *A* are only pre regular open subsets of *A*. The family of all minimal pre regular open set is denoted by$m_{i-pRO(X)}$

Recall Example 3.2.

Example 3.5:

PRo(X)={Ø,{a},{b},{a ,b},X}

PRc(X)={X,{b ,c},{a ,c},{c}, Ø}

A={b} is $m_{i-pR-Open}$ set

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Definition 3.6:

Let X be a space and $A \subseteq X$ an pre regular closed set then A is called a maximal pre regular closed set if Ø and A are only pre regular closed subsets of A. The family of all maximal pre regular closed set is denoted by$m_{i-pRc(X)}$

Recall example 3.5

A={c} is $m_{a-pR-closed}$ set

Now we introduce some relations among these definitions as

Proposition 3.7*:*

Every minimal regular open set is minimal pre regular open set.
but the converse is not true.

Proof:

Let A is a minimal regular open set.To prove A is a minimal pre regular open set.
Since A is regular open set and every regular open set is pre regular open set.
Then minimal regular open set is minimal pre regular open set.
Therefore A is a minimal pre regular open set ■

Example 3.8:

Let *X={a ,b, c}and* the topology be *𝒯={Ø,{a},{c},{a,c},{b,c},X}*
PRo(X)={ Ø,{a},{b},{c},{a ,b},{a ,c},{b ,c},X},
A={c} is $m_{i-pR-Open}$ set but not $m_{i-R-Open}$ set

Proposition 3.9:

Every minimal α –open set is minimal pre regular open set. but the converse is not true.

Proof:

Let A is a minimal α –open set.To prove A is minimal pre regular open set.
Since A is $m_{i-\alpha-Open}$ *set*
Since every *α-open set* is *Pre-open* set And every *α*-open setis *PR*-open set
Then $m_{i-\alpha-Open}$ set is $m_{i-pR-Open}$ set
Therefore A is a minimal pre regular open set ■

Example 3.10:

Let *X={a, b ,c ,d} and* the topology be $\mathcal{T}=\{ Ø, \{a\}, \{c, d\}, \{a, c, d\}, x\}$
PRo(X)={ Ø,{a},{c},{d},{c ,d},{a, c},{a ,d},{a, b, c},{a ,b ,d},X}
αo(X)={ Ø,{a},{c ,d},{a ,c ,d},X}
A={a, b} is $m_{i-pR-Open}$ *set* but not $m_{i-\alpha-Open}$ *set.*

Proposition 3.11:

Every minimal regular open set minimal α –open set. but the converse is not true.

Proof:

Let A is a minimal regular open set.To prove A is a minimal α –open set
Since A is regular open set and every regular open set is α –open set]
Therefore A is $m_{i-\alpha-Open}$ *set* ■

Example 3.12:

Let *X={a ,b ,c ,d} and* the topology be *𝒯={ Ø,{a,d},{b,c},{d}, {b,c,d},X},Ro(X)={ Ø,{a, d},{b ,c},X},αo(X)={ Ø,{a ,d},{b ,c},{d}, {b ,c ,d},X},A={b, c}* is $m_{i-\alpha-Open}$ set but not $m_{i-R-Open}$ set

Proposition 3.13:

Every minimal open set is minimal pre regular open set. but the converse is not true.

***Proof*:**

Let A is a minimal open set. To prove A is a minimal pre regular open set.
Since A is open set and every open set is pre regular open set
Then minimal open set is minimal pre regular open set
Therefore A is a minimal pre regular open set ■

Example 3.14:

Let $X=\{a, b, c, d\}$ *and* the topology be $\mathcal{T}=\{\emptyset,\{b,c\},\{a\},\{a,b,c\},X\}$
$PRo(X)=\{\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},\{a,b,d\},X\}$
$A=\{b\}$is$m_{i-pR-Open}$ setbut notm_{i-Open} set

Proposition 3.15:

Every minimal regular open set is minimal open set. . but the converse is not true.

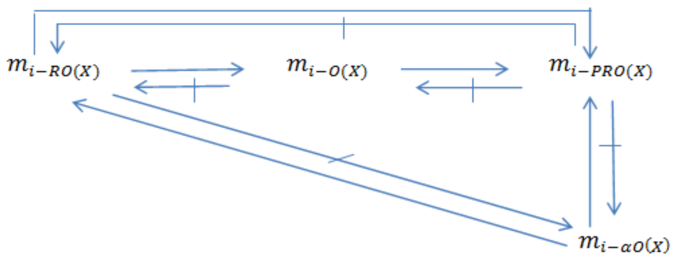
Proof:

Let *A* is a minimal regular open set. To prove *A* is a minimal open set .
Since *A* is regular open set and every regular open set is open set
Then minimal regular open set is minimal open set
Therefore *A* is minimal open set ■

Example 3.16:

Let $X=\{a,b,c,d\}$ *and* the topology be $\mathcal{T}=\{\emptyset,\{a\},\{a,c,d\},\{b\},\{a,b\},X\}$
$Ro(X)=\{\emptyset,\{a,c,d\},\{b\},X\}$
$A=\{a\}$ is m_{i-Open} *set* but not $m_{i-R-Open}$ *set*
Now we introduce the diagram which is explain the relation among these concepts and prove it as a propositions

Diagram 3.1.A



Definition 3.17:

A subset *A* of a space *X* is called to be generalized*regular minimal closed[resp. generalized*regular maximal open] set (briefly, $g^{*}r\text{-}m_{i-c(X)}$, $g^{*}r\text{-}m_{a-o(X)}$) if $U\subseteq RCl(A)$ [resp.$RInt(A)\subseteq U$] whenever $A\subseteq U$ [resp.$U\subseteq A$] and *U* is minimal regular open [resp. maximal regular closed] subset of *X*.

Example 3.18:

Let $X=\{a,b,c,d\}$ *and* the topology be $\mathcal{T}=\{\emptyset,\{b,c\},\{a\},,\{a,b,c\},X\}$
$Ro(X)=\{\emptyset,\{b,c\},\{a\},X\}$,$m_{i-RO(X)}=\{\{b,c\},\{a\}\}$,$A=\{b\}$,$U=\{b,c\}$
$RCl\{b\}=\{b,c,d\}$,$U\subseteq RCl(A)\Rightarrow A$ *is* $g^{*}r\text{-}m_{i-closed}$ *set*
A^{c} is , $g^{*}r\text{-}m_{a-open}$ *set* because $A^{c}=\{a,c,d\}$,$U^{c}=\{a,d\}$
$RInt\{a,c,d\}=\{a\}\Rightarrow RInt(A^{c})\subseteq U^{c}$

Definition 3.19:

A subset *A* of a space *X* is called to be regular generalized*minimal closed [resp. regular generalized*maximal open]set (briefly, $rg^{*}\text{-}m_{i-c(X)}$, $rg^{*}\text{-}m_{a-o(X)}$) if $U\subseteq Cl(A)$ [resp.$Int(A)\subseteq U$] whenever $A\subseteq U$ [resp. $U\subseteq A$] and *U* is minimal regular open [resp. maximal regular closed] subset of *X*.
Recall Example 3..18

Example 3.20:

Let $X=\{a,b,c,d\}$ *and* the topology be $\mathcal{T}=\{\emptyset,\{b,c\},\{a\},,\{a,b,c\},X\}$
$Ro(X)=\{\emptyset,\{b,c\},\{a\},X\}$,$A=\{b\}$,$U=\{b,c\}$, $Cl\{b\}=\{b,c,d\}$
$U\subseteq Cl(A)\Rightarrow A$ *is* $rg^{*}\text{-}m_{i-closed}$ *set.*
A^{c} *is* ,$rg^{*}\text{-}m_{a-open}$ *set* because $A^{c}=\{a,c,d\}$,$U^{c}=\{a,d\}$
$Int\{a,c,d\}=\{a\}\Rightarrow Int(A^{c})\subseteq U^{c}$

Definition 3.21:

A subset *A* of a space *X* is called to be a regular generalized** minimal closed [resp. regular generalized** maximal open] set (briefly, $rg^{**}\text{-}m_{i-c(X)}$, $rg^{**}\text{-}m_{a-o(X)}$) if $U\subseteq Cl(A)$ [resp.$Int(A)\subseteq U$] whenever $A\subseteq U$ [resp.$U\subseteq A$] and *U* is minimal pre regular open [resp. maximal pre regular closed] subset of *X*.

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Recall Example 3.2

Example 3.22:

PRo(X)={ Ø,{a},{b},{a,b},X},A={a} , U={a}
Cl{a}={a,c}⇒ *U⊆Cl(A)*⇒*A is* rg^{**}-$m_{i-closed}$ *set*
A^c *is* rg^{**}- m_{a-open} *set* because A^c*={b,c}* ,U^c *={b,c},Int{b,c}={b}*⇒*Int*(A^c) ⊆ U^c

Definition 3.23:

A subset *A* of a space *X* is called to be a generalized** regular minimal closed [resp. generalized** regular maximal open] set (briefly, $g^{**}r$-$m_{i-c(X)}$,$g^{**}r$- $m_{a-o(X)}$) if *U⊆RCl (A)* [resp. *RInt(A)* ⊆ *U*] whenever *A⊆U* [resp. *U* ⊆*A*] and *U* is minimal pre regular open [resp. maximal pre regular closed] subset of *X*.

Example 3.24:

Let *X ={a,b,c} and* the topology be $\mathcal{T}$ *={ Ø,{a},{b,c},X}*
PRo(X)={ Ø,{a},{b},{c},{a, b},{a ,c},{b, c},X},A={b} , U={b}
RCl{b}={b,c}⇒ *U⊆ RCl(A)*⇒*A is* $g^{**}r$-$m_{i-closed}$ set
A^c *is* $g^{**}r$- m_{a-open} *set* because A^c*={a}* ,U^c *={a}*
RInt{a}={a}⇒ *RInt*(A^c) ⊆ U^c
Now we introduce the explain the relation among them

Proposition 3.25:

Every generalized* regular minimal closed set is regular generalized* minimal closed set but the converse is not true.

Proof:

Let *A* is $g^{*}r$-$m_{i-c(X)}$ **.**To prove *A is* rg^{*}-$m_{i-c(X)}$.
Since *A is* $g^{*}r$-$m_{i-c(X)}$ ⇒ *U⊆ RCl(A)*
Since *RCl(A)* ⊆*Cl(A)* ⇒ *U⊆ RCl(A)* ⊆ *Cl(A)* ⇒ *U⊆Cl(A)*
Then A is rg^{*}-$m_{i-c(X)}$ ■

Example 3.26:

Let *X ={a,b,c,d} and* the topology be $\mathcal{T}$ *={ Ø,{a},{a,b},{b,c,d},{b},X}*
Ro(X)={Ø,{a},{b,c,d},X}, A={b} ,U={b},Cl{b}={b},U⊆Cl(A)⇒*A is* rg^{*}-$m_{i-closed}$ *set*but A is not $g^{*}r$-$m_{i-closed}$ *set* because
RCl{b}={b,c,d}⇒ *RCl{b}≠{b}*⇒*A* is not $g^{*}r$-$m_{i-closed}$ *set*.

Proposition 3.27:

Every regular generalized* minimal closed set is generalized* minimal closed setbut the converse is not true.

Proof:

Let *A* is rg^{*}-$m_{i-c(X)}$ .To prove *A is* g^{*}-$m_{i-c(X)}$.
Since *A is* rg^{*}-$m_{i-c(X)}$ ⇒ *U⊆Cl(A)*
Since every minimal regular open set is minimal open set
Then *U* is minimal open set
Therefore *A is* g^{*}-$m_{i-c(X)}$ ■

Example 3.28:

Let *X ={a ,b ,c, d} and* the topology be $\mathcal{T}$ *={ Ø,{d},{a,b},{a,b,c},{a,b,d},X},Ro(X)={Ø,{a,b,c},{d},X},A={a} , U={a,b},Cl{a}={a,b,c},U⊆Cl(A)*⇒*A is* g^{*}-$m_{i-closed}$ *set*but *A* is not rg^{*}-$m_{i-closed}$ *set* because, *Cl{a}={a,b,c},U⊆Cl(A)* but *U* is not $m_{i-R-0pen}$ set .

Proposition 3.29:

Every generalized* minimal closed set is regular generalized** minimal closed setbut the converse is not true.

Proof:

Let *A* is g^{*}-m_{i-cX} .To prove *A is* rg^{**}-m_{i-cX} .

Since *A is* g^*-$m_{i-c(X)}$ ⇒ *U⊆Cl(A)*
Since every minimal open set is minimal pre regular open set
Therefore *A* is *rg***-$m_{i-c(X)}$ ■

Example 3.30:

Let *X ={a ,b ,c} and* the topology be *𝒯={Ø,{a},{a,b},X},PRo(X)={Ø,{a},{b},{c},{a,b},{a,c},{b,c},X},A={c} ,U={c},Cl{c}={c}⇒ U⊆Cl(A)⇒A* is*rg***-$m_{i-closed}$ *set*but *A* is not g^*-$m_{i-closed}$ set because *,Cl{c}={c}⇒ U⊆Cl(A)* but *U* is not m_{i-Open} set

Proposition 3.31:

Every generalized** regular minimal closed set is regular generalized** minimal closed setbut the converse is not true.

Proof:

Let *A* is $g^{**}r$-$m_{i-c(X)}$ .To prove *A* is *rg***-$m_{i-c(X)}$ .Since *A* is $g^{**}r$-$m_{i-c(X)}$ ⇒ *U⊆ RCl(A)*
Since *RCl(A) ⊆ Cl(A)* ⇒ *U⊆ RCl(A) ⊆ Cl(A)* ⇒ *U⊆ Cl(A)*Therefore *A* is *rg***-$m_{i-c(X)}$ ■
Recall Example 3..30

Example 3.32:

Let *X ={a,b,c} and* the topology be *𝒯={Ø,{a},{a,b},X},A={a} ,U={a}*
Cl{a}=X ⇒ *U⊆Cl(A)⇒A* is *rg***-$m_{i-closed}$ *set*but *A* is not $g^{**}r$-$m_{i-closed}$ *set* because *,RCl{a}=X* ⇒ *RCl(A)≠* (*A*) ⇒*A* is not $g^{**}r$-$m_{i-closed}$ *set*

Proposition 3.33:

The relations between regular generalized* minimal closed set with generalized** regular minimal closed set are independent
Recall Example 3..32

Example 3.34:

$m_{i-RO(X)}$={{a},{b,c}},$m_{i-pRO(X)}$={{a},{b},{c}},*A={b} , U={b,c}*
Cl{b}={b,c} ⇒ *U ⊆ Cl(A)*⇒and *U* is $m_{i-R-Open}$ *set* ⇒ *A* is *rg*-*$m_{i-closed}$ *set*,but *A* is not $g^{**}r$-$m_{i-closed}$ set because *,RCl{b}={b,c}*⇒ *U⊆ RCl(A)*,but *U* is not $m_{i-pR-Open}$ *set* ⇒*A* is not $g^{**}r$-$m_{i-closed}$ set
B={c} ,U={c},RCl{c}={b,c}⇒ *U⊆ RCl(B),* and *U* is $m_{i-pR-Open}$ *setA* is $g^{**}r$-$m_{i-closed}$ *set*,but *B* is not *rg**-$m_{i-closed}$,because *,Cl{c}={b,c}*⇒ *U⊆Cl(B)*,but *U* is not $m_{i-R-Open}$ *set* ⇒*B* is not *rg**-$m_{i-closed}$ *set*.

Proposition 3.35:

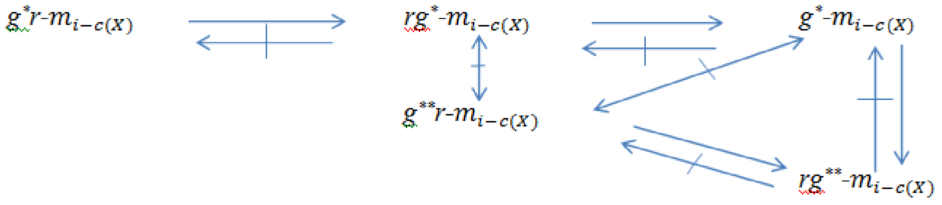
The relations between generalized* minimal closed set with generalized** regular minimal closed set are independent
Recall Example 3.18

Example 3.36:

A= {b} ,U={b,c},Cl{b}={b,c,d}⇒ *U⊆Cl(A)*⇒ and *U* is m_{i-Open} *set* ⇒*A* is g^*-$m_{i-closed}$ *set*,but *A* is not $g^{**}r$-$m_{i-closed}$ *set* because *,RCl{b}={b,c,d}*⇒ *U⊆ RCl(A)*,but *U* is not $m_{i-pR-open}$ *set* ⇒*A* is not $g^{**}r$-$m_{i-closed}$ *set,B={c} ,U={c},RCl{c}={b,c,d}*⇒ *U⊆ RCl(B),* and *U* is $m_{i-pR-Open}$ *set* ⇒*B* is $g^{**}r$-$m_{i-closed}$ *set*,but *B* is not is g^*-$m_{i-closed}$ *set*,because *,Cl{c}={b,c,d}*⇒ *U⊆Cl(B)*,but *U* is not m_{i-Open} *set* ⇒*B* is not g^*-$m_{i-closed}$ *set*.

Now we introduce the diagram which is explain the relation among these concepts and prove it as a propositions

Diagram 3..B



Theorem 3.37:

A subset *A* of *X* is a regular generalized* minimal closed set *iff*∃ aminimal regular open set *U* containing *A* such that *Cl(A)= Cl(U)*

Proof:

Let *A* be a rg^*-$m_{i-c(X)}$ .To prove *Cl(A)= Cl(U).*
From definition ,*A*⊆*U* ,*U* is minimal regular open set ⇒ *U*⊆ *Cl(A)*
Then *Cl(U)* ⊆ *Cl Cl (A)* ⇒ *Cl(U)* ⊆ *Cl (A)* ⋯⋯⋯ (1)
But , *A*⊆*U* ⇒ *Cl(A)* ⊆ *Cl(U)* ⋯⋯⋯ (2)
From (1) and (2) we have
Cl(A)= Cl(U)

Conversely:

Since *A*⊆*U* , *Cl(A)= Cl(U)* ⊇ *U*
Then *A* is rg^*-$m_{i-c(X)}$ ■

Remark 3.38:

∅ and *X* are not regular generalized* minimal closed[resp. regular generalized** minimal closed ,generalized** regular minimal closed, generalized*regular minimal closed] set follows from definition.

Proof:

∅ ⊆ any $m_{i-RO(X)}$[resp.$m_{i-pRO(X)}$,$m_{i-pRO(X)}$,$m_{i-RO(X)}$]
Similarly *X* ⊈ any $m_{i-RO(X)}$[resp.$m_{i-pRO(X)}$,$m_{i-pRO(X)}$,$m_{i-RO(X)}$] ■

Theorem 3.39:

Arbitrary union of regular generalized* minimal closed[resp. regular generalized** minimal closed ,generalized** regular minimal closed, generalized*regular minimal closed] set is regular generalized* minimal closed[resp. regular generalized** minimal closed ,generalized** regular minimal closed, generalized*regular minimal closed] set if it is contained in a minimal regular open[resp. minimal pre regular open, minimal pre regular open, minimal regular open]set.

Proof:

Let$\{A\}_{i\in I}$ be a collection of rg^*-m_{i-cX} [resp.rg^{**}-$m_{i-c(X)}$, $g^{**}r$-$m_{i-c(X)}$, g^*r-$m_{i-c(X)}$] set
Let ∪ $\{A\}_{i\in I}$ ⊆*U* ,*U is* $m_{i-RO(X)}$[resp.$m_{i-pRO(X)}$,$m_{i-pRO(X)}$,$m_{i-RO(X)}$]
Then A_i ⊆*U* ,for all *i*∈*I*
Since $\{A\}_{i\in I}$ are a collection of rg^*-$m_{i-c(X)}$ [resp. rg^{**}-$m_{i-c(X)}$, $g^{**}r$-$m_{i-c(X)}$, g^*r-$m_{i-c(X)}$] set
Then *U*⊆ *Cl*$\{A\}_{i\in I}$ [resp. *U*⊆ *Cl*$\{A\}_{i\in I}$, *U*⊆ *RCl*$\{A\}_{i\in I}$, *U*⊆ *RCl*$\{A\}_{i\in I}$] ,now∪ $\{A\}_{i\in I}$ ⊇ A_i for all *i*∈*I* .
So *Cl* [∪ $\{A\}_{i\in I}$] ⊇ *Cl*$\{A\}_{i\in I}$ ⊇ *U* [resp. *Cl*[∪ $\{A\}_{i\in I}$] ⊇ *Cl*$\{A\}_{i\in I}$ ⊇ *U*, *RCl*[∪ $\{A\}_{i\in I}$] ⊇ *RCl*$\{A\}_{i\in I}$ ⊇ *U*, *RCl*[∪ $\{A\}_{i\in I}$] ⊇ *RCl*$\{A\}_{i\in I}$ ⊇ *U*]
So arbitrary union of rg^*-$m_{i-c(X)}$ [resp. rg^{**}-$m_{i-c(X)}$, $g^{**}r$-$m_{i-c(X)}$, g^*r-$m_{i-c(X)}$] set is rg^*-$m_{i-c(X)}$ [resp. rg^{**}-$m_{i-c(X)}$, $g^{**}r$-$m_{i-c(X)}$, g^*r-$m_{i-c(X)}$] set ■

Remark 3.40:

Finite intersection of regular generalized* minimal closed[resp.generalized*regular minimal closed] set need not be regular generalized* minimal closed[resp. generalized*regular minimal closed] set which follows from the example
Recall Example 3.1.23

***Example 3.41**:*

Let *X ={a ,b, c}*
𝒯={ Ø,{a},{b,c},X}
$\mathcal{T}^C$*={X,{b ,c},{a}, Ø}*
Ro(X)={ Ø,{a},{b c},X}
Rc(x)={X,{b c},{a}, Ø}
A={b ,c}⇒*A* is rg^*-$m_{i-closed}$ *set* [resp. g^*r-$m_{i-closed}$ *set*] set of *X*, but
B={a}⇒ *B* is rg^*-$m_{i-closed}$ *set* [resp. g^*r-$m_{i-closed}$ *set*] set of *X* , but
A∩*B*=∅ is not rg^*-$m_{i-closed}$ *set* [resp. g^*r-$m_{i-closed}$] of *X*.

Theorem 3.42:

Non –null intersection of regular generalized* minimal closed [resp. generalized*regular minimal closed] set and closed set is regular generalized* minimal closed[resp. generalized*regular minimal closed] set.

Proof:

Let A be rg^*-$m_{i-c(X)}$ [resp. g^*r-$m_{i-c(X)}$] set and let F be a closed set U is$m_{i-RO(X)}$ Since A is rg^*-$m_{i-c(X)}$ [resp. g^*r-$m_{i-c(X)}$] set

Therefore $U \subseteq Cl(A)$ [resp. $U \subseteq RCl(A)$]. $\Rightarrow U \subseteq Cl(A) \cap F$ [resp. $U \subseteq RCl(A) \cap F$].

$\Rightarrow U \subseteq Cl(A \cap F)$ [resp. $U \subseteq RCl(A \cap F)$].

Then $(A \cap F)$ is rg^*-$m_{i-c(X)}$ [resp. g^*r-$m_{i-c(X)}$] set■

Theorem 3.43:

Let A be a regular generalized* minimal closed set and B be a subset of X contained in the same minimal regular open set

If $A \subseteq B \subseteq Cl(A)$, then B is also regular generalized* minimal closed set

Proof:

Let A be a rg^*-$m_{i-c(X)}$ then $A \subseteq U$,$U \subseteq Cl(A)$

Then $Cl(U) \subseteq Cl\ Cl(A) \Rightarrow Cl(U) \subseteq Cl(A)$ ⋯⋯⋯ (1)

But , $A \subseteq U \Rightarrow Cl(A) \subseteq Cl(U)$⋯⋯⋯ (2)

From (1) and (2) we have

$Cl(A) = Cl(U)$

B is subset of $X \Rightarrow B \subseteq U$, here $A \subseteq B \subseteq Cl(A)$

i.e$Cl(A) = Cl(B) = Cl(U)$

Therefore B also a rg^*-$m_{i-c(X)}$ ■

Theorem 3.44:

A proper non empty subset F of X is regular generalized* minimal closed set *iff*$(X-F)$ is regular generalized* maximal open set.

Proof:

Let F be a rg^*-$m_{i-c(X)}$ .To prove $(X-F)$ is a rg^*-$m_{a-o(X)}$.

Suppose $(X-F)$ is not is arg^*-$m_{a-o(X)}$

Then ∃ a minimal regular open set $U \neq (X-F) \Rightarrow (X-F) \subseteq U$

Therefore $(X-U) \subseteq F$, and $(X-U)$ is a maximal regular closed set which is a contradiction for F is a rg^*-$m_{i-c(X)}$ ■

Conversely:

Let $(X-F)$ be a rg^*-$m_{a-o(X)}$ .To prove F is a rg^*-$m_{i-c(X)}$.

Suppose F is not rg^*-$m_{i-c(X)}$ .Then ∃a maximal regular closed set $E \neq F$ Such that $X \neq E \subseteq F$. That is $(X-F) \subseteq (X-E)$, and $(X-E)$ is a minimal regular open set which is a contradiction for $(X-F)$ a rg^*-$m_{a-o(X)}$. There fore F is rg^*-$m_{i-c(X)}$ ■ .

Theorem 3.45:

A subset A of X is a generalized*regular minimal closed set *iff*. ∃ a minimal regular open set U containing A such that$RCl(A) = Cl(U)$

Proof:

Let A be a g^*r-$m_{i-c(X)}$ .To prove $RCl(A) = Cl(U)$.

From definition ,$A \subseteq U$,U is minimal regular open set$\Rightarrow U \subseteq RCl(A)$

Then $Cl(U) \subseteq Cl[RCl(A)] \Rightarrow Cl(U) \subseteq RCl(A)$ ⋯⋯⋯ (1)

But, $A \subseteq U \Rightarrow Cl(A) \subseteq Cl(U)$

Since $RCl(A) \subseteq Cl(A)$.

Then $RCl(A) \subseteq Cl(A) \subseteq Cl(U) \Rightarrow RCl(A) \subseteq Cl(U)$⋯⋯⋯ (2)

From (1) and (2) we have

$RCl(A) = Cl(U)$

Conversely:

Since $A\subseteq U$,$RCl(A)= Cl(U) \supseteq U$
Therefore A is $g^{*}r\text{-}m_{i-c(X)}$ ■

Theorem 3.46:

Let A be a generalized*regular minimal closed set and B be a subset of X contained in the same minimal regular open set ,If $A\subseteq B \subseteq RCl(A)$, then B is also generalized* regular minimal closed set .

Proof:

Let A be a $g^{*}r\text{-}m_{i-c(X)}$ then $A\subseteq U$,$U\subseteq RCl(A)$
Then $Cl(U) \subseteq Cl\,[RCl\,(A)] \Rightarrow Cl(U) \subseteq RCl\,(A)$ ……… (1)
But , $A \subseteq U \Rightarrow Cl(A) \subseteq Cl(U)$
Since $RCl(A) \subseteq Cl(A)$.
Then $RCl(A) \subseteq Cl(A) \subseteq Cl(U) \Rightarrow RCl(A) \subseteq Cl(U)$……… (2)
From (1) and (2) we have
$RCl(A)= Cl(U)$.
Since B is subset of $X \Rightarrow B \subseteq U \Rightarrow Cl(B) \subseteq Cl(U)$
Since $RCl(B) \subseteq Cl(B) \subseteq Cl(U) \Rightarrow RCl(B) \subseteq Cl(U)$,
But $A\subseteq B \subseteq RCl(A) \Rightarrow Cl(A) \subseteq Cl(B) \subseteq RCl(A)$
Since $RCl(A) \subseteq Cl(A) \subseteq RCl(B) \subseteq Cl(B) \subseteq RCl(A) = Cl(U)$
$Cl(U) \subseteq RCl(B) \subseteq Cl(U)$
i.e$Cl(U)= RCl(B)=RCl(A)$
Then $U\subseteq Cl(U)= RCl(B)=RCl(A)$
Therefore B a $g^{*}r\text{-}m_{i-c(X)}$ ■

Theorem 3.47:

A proper non empty subset F of X is generalized* regular minimal closed set *iff*$(X\text{-}F)$ is generalized* regular maximal open set.

Proof:

Let F be a $g^{*}r\text{-}m_{i-c(X)}$ .To prove $(X\text{-}F)$ is a $g^{*}r\text{-}m_{a-o(X)}$.
Suppose $(X\text{-}F)$ is not is a $g^{*}r\text{-}m_{a-o(X)}$
Then $\exists$ a minimal regular open set $U\neq (X\text{-}F) \Rightarrow (X\text{-}F) \subseteq U$
Then $(X\text{-}U) \subseteq F$, and $(X\text{-}U)$ is a maximal regular closed set which is a contradaction for F is a $g^{*}r\text{-}m_{i-c(X)}$

Conversely:

Let $(X\text{-}F)$ be a $g^{*}r\text{-}m_{a-o(X)}$ .To prove F is a $g^{*}r\text{-}m_{i-c(X)}$.
Suppose F is not $g^{*}r\text{-}m_{i-c(X)}$ .Then $\exists$ a maximal regular closed set $E \neq F$ Such that $X\neq E\subseteq F$. That is $(X\text{-}F) \subseteq (X\text{-}E)$, and $(X\text{-}E)$ is a minimal regular open set which is a contradaction for $(X\text{-}F)$ a $g^{*}r\text{-}m_{a-o(X)}$
Therefore F is $g^{*}r\text{-}m_{i-c(X)}$ ■

Theorem 3.48:

A subset A of X is a regular generalized* minimal closed set *iff* $\exists$ a minimal pre regular open set U containing A such that $Cl(A)= Cl(U)$

Proof:

Let A be a $rg^{**}\text{-}m_{i-c(X)}$ .To prove $Cl(A)= Cl(U)$.
From definition ,$A\subseteq U$,U is minimal pre regular open set$\Rightarrow U\subseteq Cl(A)$
Then $Cl(U) \subseteq Cl\,Cl\,(A) \Rightarrow Cl(U) \subseteq Cl\,(A)$ ……… (1)
But , $A\subseteq U \Rightarrow Cl(A) \subseteq Cl(U)$……… (2)
From (1) and (2) we have
$Cl(A)= Cl(U)$

Conversely:

Since $A\subseteq U$, $Cl(A)= Cl(U) \supseteq U$
Therefore A is $rg^{**}\text{-}m_{i-c(X)}$ ■

Remark 3.49:

Finite intersection of regular generalized** minimal closed[resp. generalized**regular minimal closed] set need not be regular generalized** minimal closed[resp. generalized**regular minimal closed] set which follows from the following example
Recall Example 3.1.23

Example 3.50:

$m_{i-pRo\,(X)}$={{a},{b},{c}}
$m_{a-pRc\,(X)}$={{b,c},{a,c},{a,b}}
A={a}⇒A is rg^{**}-$m_{i-closed}$ [resp. $g^{**}r$-$m_{i-closed}$] set of X
B={c}⇒ B is rg^{**}-$m_{i-closed}$ [resp. $g^{**}r$-$m_{i-closed}$] set of X , but
A∩B=∅ is not rg^{*}-$m_{i-closed}$ [resp. $g^{*}r$-$m_{i-closed}$] set of X.

Theorem 3.51:

Non–null intersection of regular generalized** minimal closed[resp. generalized**regular minimal closed] set and closed set is regular generalized** minimal closed[resp. generalized**regular minimal closed] set.

Proof:

Let A be rg^{**}-$m_{i-c(X)}$ [resp. $g^{**}r$-$m_{i-c(X)}$] set and let F be a closed set ,U is $m_{i-PRO\,(X)}$.
Since A is rg^{**}-$m_{i-c(X)}$ [resp. $g^{**}r$-$m_{i-c(X)}$] set
Then U⊆Cl(A)[resp. U⊆ RCl(A)]. ⇒ U ⊆ Cl(A) ∩F[resp. U⊆RCl(A) ∩F].
⇒ U⊆Cl(A ∩F) [resp. U ⊆RCl(A ∩F)].
Therefore (A ∩F) is rg^{**}-$m_{i-c(X)}$ [resp. $g^{**}r$-$m_{i-c(X)}$] set ■

Theorem 3.52:

Let A be a regular generalized** minimal closed set and B be a subset of X contained in the same minimal pre regular open set
If A⊆ B ⊆Cl(A) , then B is also regular generalized** minimal closed set .

Proof:

Let A be a rg^{**}-$m_{i-c(X)}$ then A⊆U , U ⊆ Cl(A)
Then Cl (U) ⊆ Cl Cl (A) ⇒ Cl(U) ⊆ Cl (A) ⋯⋯⋯ (1)
But , A ⊆ U ⇒ Cl(A) ⊆Cl(U)⋯⋯⋯ (2)
From (1) and (2) we have
Cl(A)= Cl(U)
B is subset of X ⇒B⊆U , here A⊆ B ⊆Cl(A)
i.eCl(A)= Cl(B)= Cl(U)
Therefore B also a rg^{**}-$m_{i-c(X)}$ ■

Theorem 3.53:

A proper non empty subset F of X is regular generalized** minimal closed set *iff(X-F)* is regular generalized** maximal open set.

Proof:

Let F be a rg^{**}-$m_{i-c(X)}$ .To prove (X-F) is a rg^{**}-$m_{a-o(X)}$.
Suppose (X-F) is not is a rg^{**}-$m_{a-o(X)}$
Then ∃ a minimal regular open set U≠ (X-F) ⇒(X-F) ⊆U
Then (X-U) ⊆F , and (X-U) is a maximal pre regular closed set which is a contradiction for F is a rg^{**}-$m_{i-c(X)}$

Conversely:

Let (X-F) be a rg^{**}-$m_{a-o(X)}$ .To prove F is a rg^{**}-$m_{i-c(X)}$.
Suppose F is not rg^{**}-$m_{i-c(X)}$ .Then ∃ a maximal pre regular closed set E ≠ F Such that X≠ E⊆F. That is (X-F) ⊆ (X-E) , and (X-E) is a minimal pre regular open set which is a contradiction for (X-F) a rg^{**}-$m_{a-o(X)}$. There fore F is rg^{**}-$m_{i-c(X)}$ ■

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***Theorem 3.54**:*

A subset *A* of *X* is a generalized**regular minimal closed set *iff* ∃ a minimal pre regular open set *U* containing *A* such that *RCl(A)= Cl(U)*

Proof:

Let *A* be a $g^{**}r\text{-}m_{i-c(X)}$ .To prove *RCl(A)= Cl(U).*
From definition ,*A*⊆*U* ,*U* is minimal pre regular open set⇒*U*⊆ *RCl(A)*
Then *Cl(U)* ⊆ *Cl [RCl (A)]* ⇒ *Cl(U)* ⊆ *RCl(A)* ⋯⋯⋯ (1)
But, *A* ⊆ *U* ⇒ *Cl(A)* ⊆ *Cl(U)*
Since *RCl(A)* ⊆ *Cl(A)*
Then *RCl(A)* ⊆ *Cl(A)* ⊆ *Cl(U)* ⇒*RCl(A)* ⊆ *Cl(U)*⋯⋯⋯ (2)
From (1) and (2) we have
RCl(A)= Cl(U)

Conversely:

Since *A*⊆*U* ,*RCl(A)= Cl(U)* ⊇ *U*
Therefore *A* is $g^{**}r\text{-}m_{i-c(X)}$ ■

Theorem 3.55:

Let *A* be a generalized**regular minimal closed set and *B* be a subset of *X* contained in the same minimal pre regular open set,
If *A*⊆ *B* ⊆ *RCl(A)* , then *B* is also generalized** regular minimal closed set .

Proof:

Let *A* be a $g^{**}r\text{-}m_{i-c(X)}$ then *A*⊆*U* ,*U*⊆ *RCl(A)*
Then *Cl(U)* ⊆ *Cl[RCl (A)]* ⇒ *Cl(U)* ⊆ *RCl (A)* ⋯⋯⋯ (1)
But , *A*⊆*U* ⇒ *Cl(A)* ⊆ *Cl(U)*
Since *RCl(A)* ⊆ *Cl(A)* .Then *RCl(A)* ⊆ *Cl(A)* ⊆ *Cl(U)* ⇒ *RCl(A)* ⊆*Cl(U)*⋯⋯⋯ (2)
From (1) and (2) we have
RCl(A)= Cl(U).
Since *B* is subset of *X* ⇒*B*⊆*U* ⇒ *Cl(B)* ⊆*Cl(U)*
Since *RCl(B)* ⊆ *Cl(B)* ⊆*Cl(U)* ⇒ *RCl(B)* ⊆ *Cl(U),*
But *A*⊆ *B* ⊆ *RCl(A)* ⇒*Cl(A)* ⊆*Cl(B)* ⊆ *RCl(A)*
Since *RCl(A)* ⊆ *Cl(A)* ⊆ *RCl(B)* ⊆ *Cl(B)* ⊆ *RCl(A) = Cl(U)*
Cl(U) ⊆ *RCl(B)* ⊆ *Cl(U)*
i.e*Cl(U)= RCl(B)=RCl(A)*
Then *U*⊆ *Cl(U)= RCl(B)=RCl(A)*

***Theorem 3.56**:*

A proper non empty subset *F* of *X* is generalized** regular minimal closed set *iff(X-F)* is generalized** regular maximal open set.

Proof:

Let *F* be a $g^{**}r\text{-}m_{i-c(X)}$ .To prove *(X-F)* is a $g^{**}r\text{-}m_{a-o(X)}$.
Suppose *(X-F)* is not is a $g^{**}r\text{-}m_{a-o(X)}$
Then ∃ a minimal pre regular open set *U*≠ *(X-F)* ⇒*(X-F)* ⊆*U*
Then *(X-U)* ⊆*F* , and *(X-U)* is a maximal pre regular closed set which is a contradiction for *F* is a $g^{**}r\text{-}m_{i-c(X)}$.

Conversely:

Let *(X-F)* be a $g^{**}r\text{-}m_{a-o(X)}$ .To prove *F* is a $g^{**}r\text{-}m_{i-c(X)}$.

Suppose *F* is not $g^{**}r\text{-}m_{i-c(X)}$ .Then ∃ a maximal pre regular closed set *E* ≠ *F* Such that *X*≠ *E*⊆*F*. That is *(X-F)* ⊆*(X-E)* , and (X-E) is a minimal pre regular open set which is a contradiction for *(X-F)* a $g^{**}r\text{-}m_{a-o(X)}$ .Therefore *F* is $g^{**}r\text{-}m_{i-c(X)}$

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