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Solving Risk Conditions Optimization Problem in Portfolio Models

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Abstract: In this paper formation model of stock portfolios M from N is studied in order to minimize the total risk of all portfolios. In this study, it is assumed that risk of each stock portfolio is expressed by definite function. This function depends only on invested amount for portfolio formation, and total risk is a function of risks of separated portfolios. The discussed model is a non-linear minimization bi-parametric model which can be converted into one parametric minimization problem for certain varieties of objective.

Key words: stock portfolio, risk, goals matrix, aggregation, non-descending function, definite recurrent function.

INTRODUCTION

Financial management of economic agencies requires creation of numerous investment portfolios that aim at distributing risks which are generated under the effects of unexpected factors. Distribution (2)of risks usually includes solving optimization problems in which quantitative parametric alternations of portfolios are considered as objective (Markwitz, N., 1952).

For the first time, similar optimization problems were investigated by Gary Markwitz (1969). His works that are the basis of stock portfolio theory have been assigned to optimization problem and investment decisions in uncertainconditions and risk. It means that, in Markwitz's works the significantrole of covariance (4) has been clear in reducing unsystematic risk among prices (interests) of stocks. Also, his works have had influence on generation of two theories:

Funds valuation theory (1) and arbitrary computation theory (4) which have provided ground for advent of other financial models (Markwitz. N., 1959).

In general, in all of these models discussion is about the formation of a portfolio from risky and non-risky stock portfolios. Invested amount in stocks is assumed to be equal to unit. Portfolio structure is assigned by unitary investment and for this reason the invested amount is not shared in practical models. Thus, if we don't take the uniformity condition of total invested amount into account, the expected structure and interest, and portfolio risk will depend on this investment. Therefore, there will be a possibility for risk control and skillful distribution of funds (A.S.RS., 1976). This issue is of high importance in simultaneous formation of varies portfolios. So, it is recommended to analyze one of such problems. Distinguishing feature of this problem is that quantities computation of portfolio risk is conducted on the basis of allocated investment, and other factors influencing the risk are ruled out.

Statement of the Problem:

We assume that financial structure of an agency is based on the aim of N risky and current stockinvestment portfolio formation.

Moreover, stocks are classified into N groups and each portfolio can be laid in one of these groups. For creating portfolio i-th from group j-th of stocks, money amountx_{ij} ,$i = 1, m$, $i = 1, n$are assigned and the formation of portfolio i-th from group j-th of stocks relates to portfolio risk. This risk with the function r_{ij} (x_{ij}) depends only on allocated investment. In the case of function r_{ij} (x_{ij}), it is assumed that they will contain eventual non-negative quantities in a definite field. Portfolio i-th constructed from stock group j-th, (i ,j) is called Portfolio (Shiryaoo, A.N., 1998).

We insert matrix S for describing a set of M portfolio. Elements of the matrix are demonstrated in figure below:

$$S_{ij} = \begin{cases} 1, & \text{If portfolio : } i = \overline{1, m}, j = \overline{1, m} \\ 0, & \text{otherwise we have} \end{cases}$$

This matrix can be called goal matrix among portfolios and stock groups, because constructing portfolio i-th from stock group j-th can be analyzed as the aim of portfolio i-th in relation to group j-th (or vice versa).

Therefore, the condition in which each portfolio is composed of one set of stocks means that matrix S is a unitary matrix.

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The set of allgoals matrix which satisfy this condition are shown with N (the number of elements of set N equal n^m).

In general, quantitative computation of total risk of all portfolios which are selectedaltogether can be carried on like specified function in the sets:

$$\sum_{j=1}^{n} S_{1j}\ .r_{1j}(x_{1j}), \sum_{j=1}^{n} S_{2j}\ .r_{2j}(x_{2j}) \quad , \dots , \qquad \sum_{j=1}^{n} S_{mj}\ .r_{mj}(x_{mj})$$

So we have:

$$R = R(\sum_{j=1}^{n} S_{1j}\ .r_{1j}(x_{1j}) \ \ , \dots , \sum_{j=1}^{n} S_{mj}\ .r_{mj}(x_{mj})$$

Considering below conditions beside formation of investment portfolio m, financial structure should minimize each total risk:
a)Goal matrix formation
b) Investment distribution among portfolios.

Therefore the model requires goal matrix formation and limited investment distribution (K), in a way beside which function R reach its minimum amount R*:

$$R^* = \begin{matrix} min min \\ S\epsilon N\ xij \end{matrix} \quad R(\sum_{j=1}^{n} S_{1j}\ .r_{1j}(x_{1j}),\ \dots , \sum_{j=1}^{n} S_{mj}\ .r_{mj}(x_{mj}) \tag{1.1}$$

In which matrix S satisfies the condition below:

$$\sum_{j=1}^{n} S_{ij} = 1, \qquad i = \overline{1, m.} \tag{1.2}$$

In addition, since quantities x_{ij} in function R imply invested amount distributed among portfolios, provided that this investment is distributed completely among portfolio M, below conditions should be carried on for x_{ij}:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} S_{ij}\, .x_{ij} = K \tag{1.3}$$

$$x_{ij} \geq 0, \quad i = \overline{1, m}, \quad j = \overline{1, m} \tag{1.4}$$

For some problems, we can add the condition of being integer (or discreteness) of variables x_{ij}.

As is clear in the structure of the model, the problem is a bi-parametric optimization problem; Moreover, this problem is a variance of classic non-linear problem in relation to parameter S, and non-linear optimization resource distribution problem in relation to parameter x_{ij}.

So, these two problems can't be analyzed separately, because for solving this problem the amount of optimized investment distribution and optimized resource distribution, and optimized goal matrix is needed. Thus, this problem should be solved in a way that provides possibility for finding aim and optimized distribution simultaneously.

Method of Solving Theproblem:

In general, function R can have a complicated structure which causes some calculation problems; But analyzing the structure of problem shows that the aim of this problem can be converted into one parametric optimum resources distribution problem for some types of functions. One of these types is descending functions in accordance with each Argoman in [0, K] coordinate. We briefly explain the process of converting for this type of functions.

We show the matrix from the set N which includes S_{ki} (k, 1)- 1 with S(K, 1) sign. The rest of the elements are constant and the subset N_1={S(1,1), S(1,2),…,S(1,n)} is separated from set N. N1 elements are only different from each other in first lines. Now we analyze the minimization problem which is extended in accordance with subset N1:

RESULTS AND DISCUSSION

$$\min_{S\epsilon N_1'}\{\quad \min_{0 \leqq x_{11} \leqq k} \mathrm{R}(r_{11}(x_{11}), \sum_{j=1}^{n} S_{2j}\,(1{,}1) r_{2j}(x_{2j}), \dots, \sum_{j=1}^{n} S_{mj}\,(1{,}1) r_{mj}(x_{mj}))$$

$$\sum_{1=2}^{m}\sum_{j=1}^{n} S_{ij}\,(1{,}1) x_{ij} = k - x_{11}$$

$$\min_{x_{2j,\dots,}x_{mj}j} \min_{\leqq x_{12} \leqq k} \mathrm{R}(r_{12}(x_{12}), \sum_{j=1}^{n} S_{2j}\,(1{,}2) r_{2j}(x_{2j}), \dots, \sum_{j=1}^{n} S_{mj}\,(1{,}2) r_{mj}(x_{mj}))$$

$$\sum_{1=2}^{m}\sum_{j=1}^{n} S_{ij}\,(1{,}2) x_{ij} = k - x_{12}$$

$$\min_{x_{2j,\dots,}x_{mj}j} \min_{\leqq x_{1n} \leqq k} \mathrm{R}(r_{1n}(x_{1n}), \sum_{j=1}^{n} S_{2j}\,(1,n) r_{2j}(x_{2j}), \dots, \sum_{j=1}^{n} S_{mj}\,(1,m) r_{mj}(x_{mj}))$$

Here we have:

$$N' = N/N_1$$

The features of problem (2, 1)are explained below:
a) Minimization variables x_{11}, x_{12},…,$x_{1n}$are occurred at the same[0, K] coordinate.
b) Regarding features of N_1elements, it is concluded that

$$S_{2j}(1{,}1) = S_{2j}(1{,}2) = \cdots = S_{2j}(1,n)$$

....

$$S_{mj}(1,m) = S_{mj}(1,n) = \cdots = S_{mj}(1,n\}$$

For total:$i = \overline{2,m}\,,\ j = \overline{1,n},$

According to the stated items and regarding function R as a descending function, it is concluded that problem (1, 2) is equivalent to following problem:(2. 2).

$$\min_{0 \leqq y_1 \leqq k} \min_{S\epsilon N'_1} \min_{x_{2j},\dots,x_{mj}} \mathrm{R}(F_{1(}y_1), \sum_{j=1}^{n} S_{2j}\,(1{,}1) r_{2j}(x_{2j}), \dots, \sum_{j=1}^{n} S_{mj}\,(1,m) r_{mj}(x_{mj}))$$

$$\sum_{1=2}^{m}\sum_{j=1}^{n} S_{ij}\,(1{,}2) x_{ij} = k - y_1\ \ , i = \overline{1,m}$$

$$F_1(y_1) = \min_{j = 1,n}\{r_{1j}(y_i)\} y_1\ \epsilon\ [0,\mathrm{k}]$$

In which we have for total y_1=arg$\min_{j = 1,m}\{r_{1j}(x_{1j})\}$

We will call the explained process as an aggregation of problem(2, 1) in relation to problem (2, 2).
Then with converting a single element in each line except the first line, we separate the subset (1, 1):

$$N_2 = \{\acute{S}, \acute{S}(1{,}1), \acute{S}(1{,}2), \dots, \acute{S}(1,n)\}$$

Again and compute the aggregation of related problems in N_2. We continue this process to the point that there is no first Argoman of R in all aggregated problems of F_1 (y_1) function. With doing this item, the process of aggregation in the first Argoman of function R is finished.

Thus, with continuing this process for second and thirdArgomans and for m-Argoman, We consequently get the following completely aggregated problem:

$$\min_{0 \leqq y_m \leqq k_m} \min_{0 \leqq \mathrm{m}-1 \leqq \mathrm{k}_{\mathrm{m}-1}} \min_{0 \leqq \mathrm{y}_1 \leqq \mathrm{k}} R, F_1(y_1), F_2(y_2), \dots, F_m(y_m) \quad (2.3)$$

Or in equivalent form we have:

$$R\left(F_1(y_1), F_2(y_2), \dots, F_m(y_m)\right) \rightarrow min \quad (2.4)$$

Or we will have following conditions:

$$\sum_{j=1}^{m} y_i = k \quad (2.5)$$

$$y_1 \geq 0 \quad (2.6)$$

In which we have:

$$F_i(y_i) = \min_{j=1,n} \{r_{ij}(y_i): y_i \epsilon [0, k]\} \quad (2.7)$$

$$\mathrm{i} = \overline{1, n}, k_i = k_{i-1} - y_i, k_0 = k \; y_i = arg \min_{j=1,n} \{r_{ij}(x_{ij})\} \quad (2.8)$$

Solving problem (1. 1)-(1-4) can be started with specifying F_i (y_i) functions along with helping formula (2. 7). In this case using formula (2. 8), we assume the problems of x_{ij} variables in y_i variables to be constant; Then like y_1 variables and F_i (y_i) functions we can solve (2. 4)-(2. 6) problems.

The complexity of (2. 4)-(2. 6) problem mostly depends on the degree ofnon-linearity of function R. Solving (2. 4)-(2. 6) problem can be simpler for some partial functions of function R.

Definite recurrent functions can be regarded one of its partial functions.

Function R (Z_1, Z_2,…..Z_m) is called definite recurrent function if it provides the following problem:

$$R\,(z_1, z_2, \dots, z_m) = R\,\left(z_1, R\,(z_2, \dots, z_m)\right)$$

It is clear that the optimization problems of the type (2. 4)-(2. 6) are simply solvable based on the BLMAN`s optimality principle.

As a result using R.BLMAN optimized Axiom, We compute following recurrent equation:

$$B_l(Z) = \min_{0 \leqq y_i \leqq z} R\left(F_l(y_l),\; B_{l-1}(z - y_l)\right),\; \mathrm{l} = 2, \overline{m} \quad (2.9)$$

In the initial conditions we have:

$$B_1(Z) = F_1(\mathrm{z}), \mathrm{z}\epsilon[0, \mathrm{k}] \quad (2.10)$$

For example, we indicate partial functions R which are mostly noticeable in following part.

1) If the risk of all portfolios of M means the risk of total separated portfolios, then after conversion, R function and related recurrent equation of (2. 9) will be like this

$$R = \sum_{i=1}^{m} f_i\,(y_i)$$

$$B_l(Z) = \min_{0 \leqq y_i \leqq z} \left\{\{F_l(y_l), B_{l-1}(z - y_l)\}\right\}$$

$$B_l(Z) = \min_{0 \leqq y_i \leqq z} R(F_l(y_l), B_{l-1}(z - y_l))$$

2) If as a total risk, the maximum risk of all M-portfolios is chosen, then we have:

$$\mathrm{R} = \max\{F_1(y_1), F_2(y_2), \dots, F_m(y_m)\}$$

$$B_l(Z) = \min_{0 \leqq y_i \leqq z} \left\{\max\{F_l(y_l), B_{l-1}(z - y_l)\}\right\}$$

When the total risk is considered as a minimum risk of all M-portfolios, then recurrent equations have the same forms as well.

For solving (2. 9)-(2. 10) equations we have this explanation:

If in problem (1. 1)-(1. 4) the condition of being integer for X_{ij} variables is hold, then y_i variables are also integer and the computations of B_1 (z) functions at integers' points of [0, Z] will be occurred, but if in problem (1. 1)-(1. 4) the conditions of being integer isn't found, then [0, Z] point in (2. 9)-(2. 10) equations will be discontinuousin a certain way (Man R.BL 1960).

Conclusion:

1. Total risk in multi-portfolio model can be started by a linear function which depends on the risk of all portfolios.
2. Total risk of Minimization problem is regarded as a bi-parametric non-linear optimization problem. One of these parameters is goals matrix and the other is invested amounts which are specified for providing portfolios.
3. With using the method of successive aggregation of objective, this problem aggregates like one parametric non-linear optimization problem.
4. The aggregated problem can be solved by using dynamic programming.
5. In this paper M-numbers of investment stocks which include the minimum of general risk among N-numbers of stocks are analyzed. In this case, it is assumed that the risk of each stocks of i-th which is belonged to the set of j-th is related to these stocks with a dependent function to the degree of specified investment. The total risk of all stocks is a function of the risk of all stocks separately. This problem includes a bi-parametric non-linear optimization problem and it is available to generalize as a factor of a one parametric optimum for special type of objective.

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