Australian Journal of Basic and Applied Sciences, 6(13): 228-232, 2012
ISSN 1991-8178

On the Investigation of Quantum Interactions of Rubidium 87 with Optical Cavity Modes using Monte Carlo Wavefunction Method

Md. Mijanur Rahman and R. Badlishah Ahmad

School of Computer and Communication Engineering Universiti Malaysia Perlis P.O. Box 77, d/a Pejabat Pos Besar 01007 Kangar, Perlis, Malaysia.

Abstract: We investigated quantum interaction of Rubidium (^{87}Rb) atom with modes of an optical cavity using Monte Carlo Wavefunction technique. In our Cavity Quantum Electrodynamics approach, a ^{87}Rb atom in the cavity could move in parallel to the cavity axis and both the cavity mode and the atom were pumped. We modeled the ^{87}Rb atom as a two-level system, where two hyperfine energy levels of ^{87}Rb, namely, $\left(5\ ^{2}S_{1/2}, F=1\right)$ and $\left(5\ ^{2}P_{1/2}, F=1\right)$, constituted the ground and excited states respectively. In our approach, the interaction of the system with the environment was taken into account by identifying an effective Hamiltonian and a jump operator. These Hamiltonian and jump operators were combined into a non-Hermitian Hamiltonian. This non-Hermitian Hamiltonian was used in the Monte Carlo Wavefunction method in order to simulate time evolution of the system. Simulation results show efficacy of the approach.

Key words: Monte Carlo Wavefunction, Cavity Quantum Electrodynamics, Jump Operator.

INTRODUCTION

With the discovery of the cavity effects on the vacuum modes, cavity quantum electrodynamics (CQED) has been an attractive research area for studying interaction of particles with radiation modes. A high quality cavity practically contains modes with a certain fundamental frequency and its integer multiples, thus facilitating investigation of interaction of a particle with a single mode. Quantum effects in these interactions become dominant particularly in the strong coupling regime that is when the coupling between the particle and the cavity mode is stronger than the associated dissipative processes.

Monte Carlo Wavefunction (MCWF) (Carmichael, H.J., 1987; Dalibard, J., 1992; Dum, R., 1992; Molmer, K., 1993) method is an attractive simulation technique for time evolution of open quantum system. This method is particularly suitable for problems involving large dimensionality. As the degrees of freedom in a CQED system is generally quite high, this method proves useful for CQED systems. As a result, this method has drawn considerable attention of researchers from relevant disciplines. Among them, Nakano and Yamaguchi (Nakano, M. and K. Yamaguchi, 2003) investigated quantum interactions of a dissipative molecular system with a single quantized mode using MCMW method. Plenio and Knight (Plenio, M.B. and P.L. Knight, 1998) provided a comprehensive account on different aspects of MCWF. Also, Chen and Meystre (Chen, W. and P. Meystre, 2009) performed CQED characterization of many-body atomic states in double-well potentials. They used Monte Carlo Wavefunction technique to estimate two-time correlation function and instantaneous physical spectrum of the retro-reflected field of a simple two-well lattice.

In this article, we present MCWF simulation of a Rubidium atom (^{87}Rb) moving in an optical cavity and interacting with the cavity modes. The next two sections provide an overview of the system under simulation. Then the system Hamiltonian and equations of motions for the system are discussed. A brief account of MCWF follows and finally, simulation of the system using MCWF is presented.

Overview:

The system under consideration consists of an optical cavity with a Rubidium atom (^{87}Rb) inside (Fig. 1). The cavity has a mode frequency ω_C and is pumped with a laser beam with detuning Δ_C and pumping strength η_C. The ^{87}Rb atom is modeled as a two-level system as described in the next section. The atom is also pumped with a laser beam standing wave with detuning Δ_A and pumping strength η_A. The atom moves parallel to the cavity axis and couples to the cavity mode with coupling constant g. The atom decays with a spontaneous emission rate γ, while the cavity mode decays out of the cavity with the decay rate k.

Corresponding Author: Md. Mijanur Rahman, School of Computer and Communication Engineering Universiti Malaysia Perlis P.O. Box 77, d/a Pejabat Pos Besar 01007 Kangar, Perlis, Malaysia.

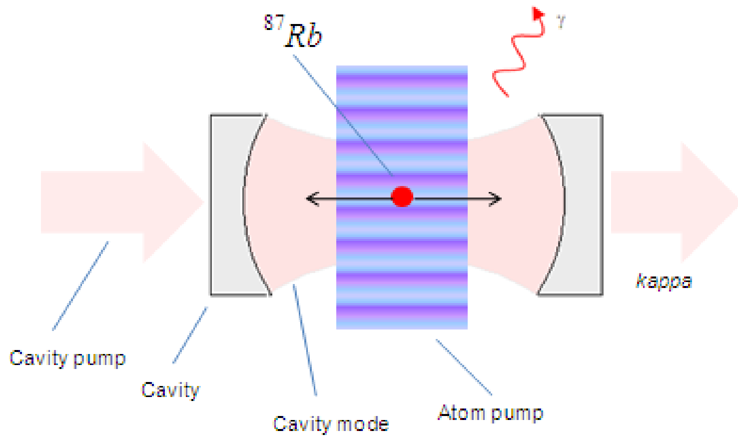

Fig. 1: Overview of the CQED system.

Monte Carlo Wavefunction (MCWF) approach is adopted to simulate the dynamics of the system. First the atom is modeled as a two level system and based on this model, overall Hamiltonian of the system is determined. Then MCWF method is used to track the time evolution and quantum jumps of the system. During time evolution, dynamic variables such as position and momentum of the atom, expectation value of the number operator are estimated.

Rubidium ^{87}Rb as a Two-Level System:

In order to model ^{87}Rb as a two-level system, we consider its hyperfine state, $|\psi_g\rangle = (5\ ^2S_{1/2}, F=1)$ as the ground state and $|\psi_e\rangle = (5\ ^2P_{1/2}, F=1)$ as the excited state. Relevant hyperfine states of ^{87}Rb are shown in Fig. 2. As seen in the figure, frequency difference between these two states, $\omega_A = 377.107463380THz - 509.05MHz + 4.271676631GHz$ =377.111226006631THz.

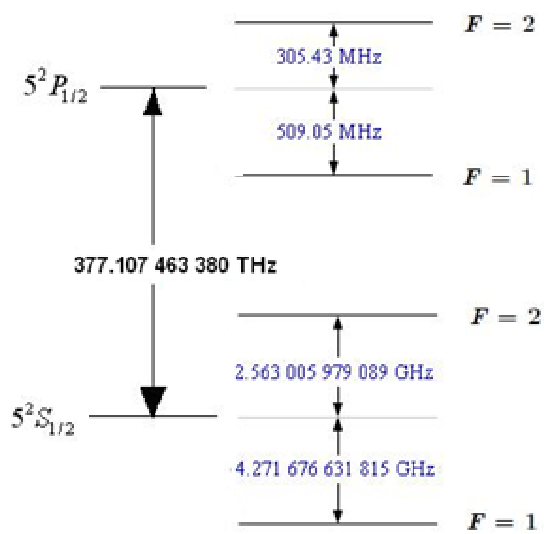

Fig. 2: Some of the hyperfine states of ^{87}Rb (Steck, D.A., 2009).

System Hamiltonian:

We adopt the Hamiltonian of the overall system from ref (Vukics, A. and H. Ritsch, 2007). From the basic Jaynes-Cummings model (Jaynes, E.T. and F.W. Cummings, 1963), the Hamiltonian for the system can be written as,

$$\hat{H} = -\Delta_C a^+ a + i(\eta_C a^+ - \eta_C{}^* a) + \frac{\hat{p}^2}{2\mu} - \Delta_A \sigma_Z + i(\eta_A{}^*(\hat{r})\sigma - \eta_A(\hat{r})\sigma^+) - i(g(\hat{r})\sigma^+ a - g^*(\hat{r}) a^+ \sigma) \quad (1)$$

Where, a^+ and a are cavity field creation and destruction operators respectively. σ^+ and σ are, respectively, atomic raising and lowering operator, and $\hat{r}$ and $\hat{p}$ position and momentum operators.

The interaction of the system with environment, which is normally given by Liouvillean of the system (Lindblad, G., 1976), can be absorbed into an effective Hamiltonian and a jump operator as follows (Vukics, A. and H. Ritsch, 2007),

$$\hat{H}_{eff} = -(iZ_C - U_0 |f(\hat{r})|^2) a^+ a + i(\eta a^+ - \eta^* a) + \frac{\hat{p}^2}{2\mu} + \eta_{eff} |\zeta(\hat{r})|^2 + sign(U_0)\sqrt{U_0 \eta_{eff}}\,(f^*(\hat{r})\zeta(\hat{r}) a^+ + h.c.) \quad (2a)$$

$$J_C = \sqrt{2k}\; a \quad (2b)$$

In Eq. (2), $Z_C = k - i\Delta_C$, $U_0 = |g|^2 / \Delta_A$ and $\eta_{eff} = |\eta_A|^2 / \Delta_A$. The two functions, $f(\hat{r})$ and $\zeta(\hat{r})$ are introduced as $g(\hat{r}) = g\ f(\hat{r})$ and $\eta_A(\hat{r}) = \eta_A \zeta(\hat{r})$.

Monte Carlo Wavefunciton Simulation:

As mentioned, time evolution of the system is simulated using MCWF method. Time propagation was simulated in few steps. First, system state vector $|\psi\rangle$ is evolved non-unitarily with the equation of motion,

$$i\hbar \frac{d|\psi\rangle}{dt} = H_{nh}|\psi\rangle \quad (3)$$

Where, the non-hermitian Hamiltonian, H_{nh}, is constructed as follows.

$$\hat{H}_{nh} = \hat{H}_{eff} - \frac{i\hbar}{2} J_C{}^+ J_C \quad (4)$$

As expected, the evolved statevector would not be normalized. As such, the norm of the state vector is computed and state vector is normalized.

Finally, based on a randomly generated number, it is decided whether a quantum jump should occur. If a quantum jump should occur, the state vector is further transformed as follows,

$$|\psi\rangle_{jump} = J_C |\psi\rangle \quad (5)$$

Simulation Results:

In our simulation of the aforesaid CQED system, we used a unit system in which $\hbar = 1$ and one unit of time was equal to 10^{-6} second. In this unit system, the cavity pump detuning, Δ_C, was set 3 and the cavity mode decay rate, k, to 10. Also, U_0, the parameter related atom-cavity coupling as defined above, was set to -4. The simulation was performed using software package C++QED (Vukics, A. and H. Ritsch, 2007).

The simulation results are shown in Figs. 3-8. Out of these results, Fig. 3 and Fig. 4 show, respectively, expectation values of Fock space number operator and ladder operator. The expectation value of the position operator $\hat{x}$ and associated standard deviation are shown in Figs. 5 and 6 respectively. Finally, Figs. 7 and 8, respectively , show the expectation values of the momentum operator $\hat{p}$ and the associated.

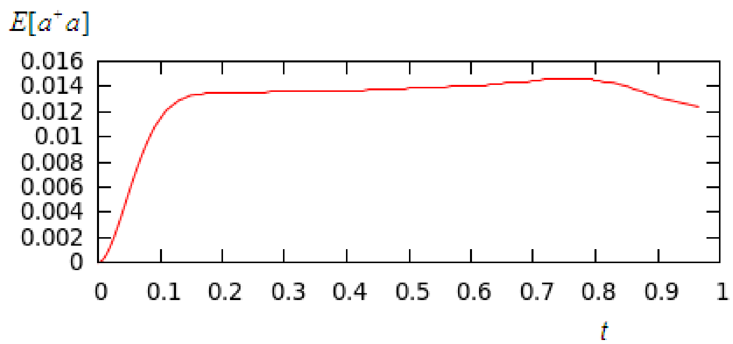

Fig. 3: Expectation value of the Number operator a^+a

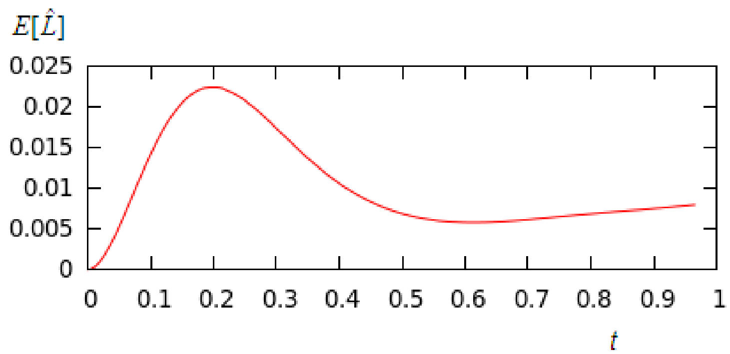

Fig. 4: Expectation value of the Ladder operator $\hat{L}$ of cavity Fock space.

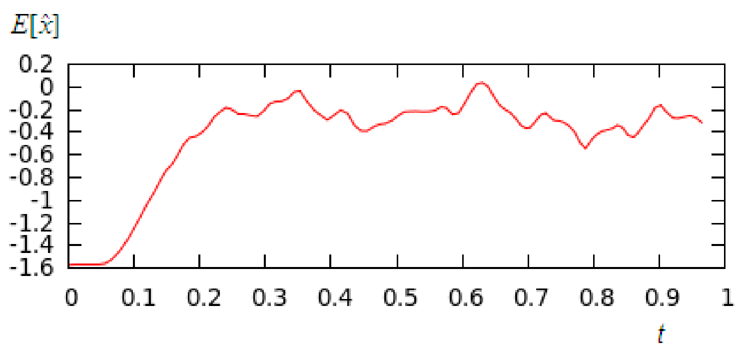

Fig. 5: Expectation value of the atomic position operator $\hat{x}$.

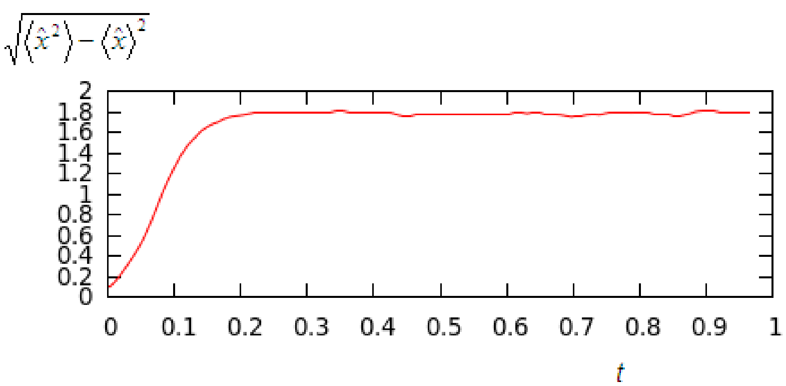

Fig. 6: Deviation associated with $\hat{x}$

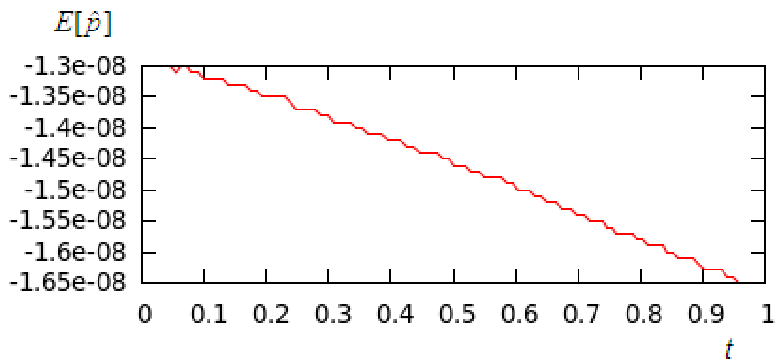

Fig. 7: Expectation value of $\hat{p}$.

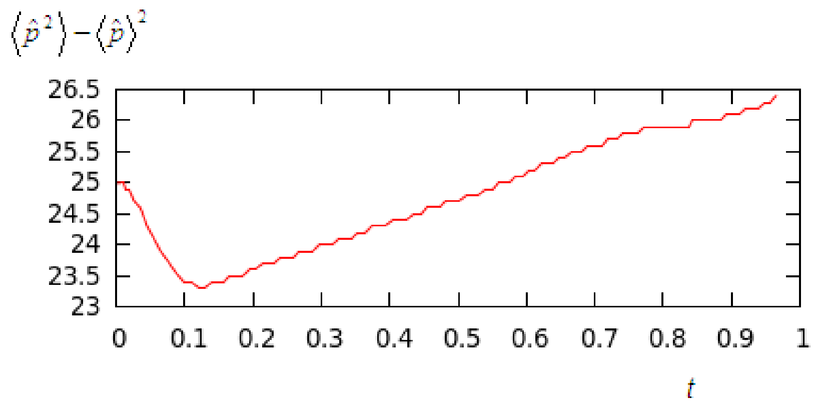

Fig. 8: Variance associated with $\hat{p}$.

Conclusion:

Cavity quantum electrodynamics of Rubidium (^{87}Rb) atom inside an optical cavity was investigated using Monte Carlo Wavefunction technique. The ^{87}Rb atom was modeled as a two-level system, the ground and excited energy levels were represented by two hyperfine energy levels. In order to take into account the interaction of the system with the environment, an effective Hamiltonian and a jump operator were identified. These Hamiltonian and jump operators were combined into a non-Hermitian Hamiltonian which was used in the Monte Carlo Wavefunction method in order to simulate time evolution of the system. Simulation results showed efficacy of the approach.

REFERENCES

Carmichael, H.J., 1987. *J. Opt. Soc. Am., B*4: 1588.

Chen, W. and P. Meystre, 2009. *Phys. Rev., A* 79: 043801.

Dalibard, J., Y. Castin and K. Molmer, 1992. *Phys. Rev. Lett*., 68: 580.

Dum, R., P. Zoller and H. Ritsch, 1992. Phys. Rev., A 45: 4879.

Jaynes, E.T. and F.W. Cummings, 1963. *Proc. IEEE*, 51: 89.

Lindblad, G., 1976. Commun. in Math. Phys., 48: 119.

Molmer, K., Y. Castin and J. Dalibard,1993. *J. Opt. Soc. Am., B* 10: 524.

Nakano, M. and K. Yamaguchi, 2003. *Int. J. of Quant. Chem.*, 95(4-5): 461.

Plenio, M.B. and P.L. Knight, 1998. *Rev. Mod. Phys*., 70(1): 101.

Steck, D.A., 2009. Rubidium-87 D Line Data, available online http://steck.us/alkalidata (revision 2.1.1, 30 April).

Vukics, A. and H. Ritsch, 2007. *Eur. Phys. J., D* 44: 585.