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Vector Variational–like Inequalities with Respect to (η,F)

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Abstract: In this paper, we consider a new class of Vector variational inequalities with multivalued pseudomonotone with respect to (η,F). The results presented in this paper are extension and improvement of the authores.

Key words: Vector variational–like inequalities, C–pseudomonotone.

INTRODUCTION

In (F. Giannessi, 1980) initially introduced and considered a vector variational inequality (VVI) in a finite–dimensional Euclidean space which is the vector–valued version of the variational inequality of Harman and Stampacchia. Ever since then, vector variational inequalities have been extensively studied and generalized in infinite–dimensional spaces since they have played very important roles in many fields such as mechanics, physics, optimization, control, nonlinear programming, economics and transportation equilibrium and so forth. On account of their very valuable applicability 20 years, see (G.Y. Chen, 1992; Y. Chiang, 2005; F. Giannessi, 1980; J. Huang and Y.P. Fang, 2006).

Let X and Y be two normed spaces and let L(X,Y) denote the family of all continuous linear operators From X into Y equipped with the uniform convergence norm When Y is the set ℝ of real numbers L(X,Y) is the usual dual space X* of X. For any x∈K and u∈L (X,Y) we shall write the valued u(x) as ⟨u,x⟩. We suppose through out this chapter that K is a nonempty and convex subset of X, T:$K \to L(X,Y)$ is a set–valued mapping, η:K×K→X and f: K×K→Y are functions, and {C(x): x∈K} of closed, convex and pointed cone of Y.

Lemma 1.1:

Let (Y,C) be a topological vector space ordered closed convex cone and pointed to have for each x,y∈K
(1) y−x∈ intC and y ∉ intC⇒ x∉intC withintC≠ ∅.
(2) y−x ∈ C and y ∉ intC⇒ x ∉intC.
(3) y−x∈ −intC and y ∉−intC⇒ x ∉−intC.
(4) y−x∈ −C and y∉−intC⇒ x∉−intC.

Definition 2.1:

Let X, and Y are two normed spaces. A set–valued mapping T:K → $L(X,Y)$ with compact values is said to be $\mathcal{H}$ −hemicontinuous on K if for x,y∈ K, the mapping t↦$\mathcal{H}$(T(x+t(y−x)),T(x)) is continuous at 0^{+}, where $\mathcal{H}$ is the Hausdorff metric defined on CBL(X,Y) of all nonempty, closed, and bounded subsets of L(X,Y).

Let X be a nonempty set, we sall denote by $\boldsymbol{\mathcal{F}}$(K) the family of all nonempty finite subsets of X. Let Y be a nonempty set, X a topological space and F:Y→X a set–valued mapping. Then F is said to be transfer closed–valued iff ∀(y,x) ∈ $Y \times X$ with x ∉ F(y), there exists y'∈Y, such that x ∉ clF(y').

Definition 3.1:

Let K be a convex subset of vector space X. Then a mapping F:K→X is called a KKM mapping iff for each nonempty finite subset A of K, convA⊂F(A), where convA denotes the convex hall of A and F(A)=∪{F(x): x∈A}.

Let C be a closed, convex and pointed cone with intC≠∅. Then a partial order $\leq_C$ in Y is defined as fory_1 , $y_2 \in Y$.

$y_1 \geq_C y_2 \Leftrightarrow y_1 - y_2 \in C$; $y_1 \ngeq_C y_2 \Leftrightarrow y_1 - y_2 \notin C$.

Definition 4.1:

The set valued function F:K→Y is said to C–convex that a cone in Y if for x, $y \in K$,t ∈ [0,1] we have.
(1−t)F(x)+tF(y)⊆F((1−t)x+ty)+C

Definition 5.1:

The Clarke's generalized derivative of f:K→ℝ at x in direction v∈ X is defined by

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$$f^0(x;v) = \lim_{y\to x,\lambda\to 0^+} sup \frac{f(y+\lambda v)-f(y)}{\lambda},$$

and the Clarke's generalized subdifferential of f at x∈ X is defined by

$$\partial f(x) = \{\xi \in X^* : \langle \xi, v\rangle \leq f^0(x;v), \forall v \in X\}.$$

Lemma 6.1:

(S.jr. Nadler, 1961) (Nadler's theorem). Let (X,‖. ‖) be a noremed vector space and let H be the Hausdorff metric on the collection CB(X) of all nonempty closed and bounded subsets of X induced by a metric d in term of d(x,y)=$\|x-y\|$, which is defined by

H(U,V)=max($sup_{x\in U} inf_{y\in V}\|x-y\|$,$sup_{y\in V} inf_{x\in U}\|x-y\|$)

For U and V in CB(X).If U and V are compact sets in X, then for each x∈ U, there exists y ∈ V such that

$\|x-y\|$ ≤H(U,V).

Lemma7.1:

(M. Fakhar and J. Zafarani, 2005) Let K be a nonempty and convex subset of a Hausdorff topological vector space X. suppose that Γ ,$\hat{\Gamma}$:K→ K are two set–valued mappings such that the following conditions are satisfied:

(A_1) For for all x∈ K we have $\hat{\Gamma}$(x)⊆Γ(x).

(A_2) $\hat{\Gamma}$ is a KKM map,

(A_3)∀A∈$\mathcal{F}$(K), Γ is transfer closed−valued on convA,

(A_4) ∀A∈ $\mathcal{F}$(K), cl_K($\cap_{x\in convA} \Gamma(x)$)∩convA=($\cap_{x\in convA} \Gamma(x)$)∩convA,

(A_5) there is a nonempty compact convex set B ⊆K, such that cl_K($\cap_{x\in B} \Gamma(x)$) is compact

Then, $\cap_{x\in K} \Gamma(x) \neq \emptyset$.

Vector variational−like inequalities:

In this chapter, we introduce a new class vector variational inequalities and prove the solvability of vector variational inequality with respect to (η,F) (VVIF) for set–valued mapping T which is C–pseudomonotone with respect to (η,F). we consider the vector variational inequalities with respect to (η,F) (VVIF):

Find x ∈ K and x*∈ T(x) such that

⟨x*,η(y,x)⟩+F(y)−F(x) ∉−intC(x), ∀y∈K.

(1) If F=0, then (VVIF) reduce to (VVI): Find x ∈ K and x*∈ T(x) such that

⟨x*,η(y,x)⟩ ∉−intC(x), ∀y∈K.

(2) If f: K⟶$\mathbb{R}^n$, Y=$\mathbb{R}^n$, T=∂f and F=0, then (VVIF) reduce to : Find x∈ K and x*∈ ∂f such that

⟨x*,η(y,x)⟩ ∉–intC(x), ∀ y ∈K.

(3) If T=g: K→Y is a singel–valued mapping, then (VVIF) reduces to:

⟨g(x),η(y,x)⟩+F(y)−F(x)∉–intC(x), ∀y∈K.

(4) If η(y,x)=y−x, then (VVIF) is: Find x ∈ K and x*∈T(x) suchthat

⟨x*,y−x⟩+F(y) −F(x)∉–intC(x), ∀y∈K.

Definition 1.2:

T is said to be (1) Weakly C–pseudomonotone with respect to (η,F) on K whenever for each x,y∈ K⟨T(x),η(y,x)⟩+F(y)−F(x)⊈–intC(x)

⟹⟨T(y),η(x,y)⟩+F(x)−F(y)⊈intC(y);

(2) C–pseudomonotone with respect to (η,F) on K whenever for each x,y∈ K

⟨T(x),η(y,x)⟩+F(y)−F(x)⊈–intC(x)

⟹⟨T(y),η(x,y)⟩+F(x)−F(y)⊈–C(y);

(3) Strictly C–pseudomonotone with respect to (η,F) on K whenever for each x,y∈ K

⟨T(x),η(y,x)⟩+F(y)−F(x)⊈–intC(x)

⟹⟨T(y),η(x,y)⟩+F(x)−F(y)⊈–intC(y).

Remark 2.2:

It is easy to see that strict C–pseudomonotone implies that C–pseudomonoton nicity and the C–pseudomonotonicity implies that weak C–pseudomonotonicity. But the converse is not necessarily true.

Remark 3.2:

If T=g: K→Y is a single–valued mapping then Definition 2.1, reduces to the following: g is said to (1) Weakly C–pseudomonotone with respect to (η,F) on K whenever for Each x,y ∈ K⟨g x),η(y,x)⟩+F(y)−F(x)∉–intC(x)
⟹⟨g(y),η(x,y)⟩+F(x)−F(y) ∉intC(y);

(2) C–pseudomonotone with respect to (η,F) on K whenever for each x,y ∈ K
⟨g(x),η(y,x)⟩+F(y)−F(x) ∉–intC(x)
⟹⟨g(y),η(x,y)⟩+F(x)−F(y)∊ –C(y);

(3) Strictly C–pseudomonotone with respect to (η,F) on K whenever for each x,y ∈ K
⟨g(x),η(y,x)⟩+F(y)−F(x) ∉–intC(x)
⟹⟨g(y),η(x,y)⟩+F(x)−F(y)∊–intC(y).

Proposition 4.2:

Let g:$K \to L(X,Y)$be a single–valued map strictly C–pseudomonotonewith respect to (η,F) on k. If (VVIF) has a solution, then the solution is unique.
Proof: Let x_1 ,x_2 ∈ K be solutions of (VVIF), then
⟨g(x_1),η(x_2 ,x_1)⟩+F(x_2)–F(x_1) ∉–intC(x_1), (2.1).
(x_2),η(x_1 ,x_2)⟩+F(x_1)–F(x_2) ∉–intC(x_2), (2.2).

Since g is strictly C–pseudomonotone with respect to (η,F) on k, it follows from (2.1) that
⟨g(x_2),η(x_1 ,x_2)⟩+F(x_1)–F(x_2) ∊–intC(x_2)

Which contradict (2.2). ■

We consider the generalized vector variational–like inequalities with respect To (η,F) (VVLIF): Find x∈ K and x*∈ T(x) Such that
⟨x*,η(x,x)⟩+F(x) ∉ intC(x) and ⟨x*,η(y,x)⟩+F(y) ∉–intC(x), ∀y∈K.

If F=0, then (VVLIF) reduces to (VVI): Find x∈ K and x*∈ T(x) such that
⟨x*,η(x,x)⟩ ∉ intC(x) and ⟨x*,η(y,x)⟩ ∉–intC(x), ∀y∈K.

(2) If T=$\partial f, f: K \to \mathbb{R}^n$, Y=$\mathbb{R}^n$ and F=0, then (VVLIF) reduce to : Findx∈ K and x*∈ ∂f(x) such that
⟨x*,η(x,x)⟩ ∉ intC(x) and ⟨x*,η(y,x)⟩ ∉–intC(x), ∀y∈K.

(3) If T=g:K→Y is a singel–valued mapping, then (VVLIF) reduces to:
⟨g(x),η(x,x)⟩+F(x) ∉intC(x) and ⟨g(x),η(y,x)⟩+F(y) ∉intC(x), ∀y∈K.

(4) If η(y,x)=y, for each y ∈K, (VVLIF): Find x∈ K and x*∈ T(x) such that
⟨x*,η(x,x)⟩+F(x) ∉ intC(x) and ⟨x*,η(y,x)⟩+F(y) ∉–intC(x), ∀y∈K.

Theorem 5.2:

(a) If (x,x*) is a solution of (VVLIF) and there exists z_0 ∊ K such that
(1) ⟨x*,η(z_0 ,x)⟩+F(z_0 –x) ∈C;
(2) Foreach y∈K, F(y–z_0)=F(y)–F(z_0) and F(y–x)=F(y)–F(x);
(3)If η (y–z_0 ,x)=η(y,x)–η(z_0 , x); then (x,x*) is a solution of (VVIF),

(b) Let (x,x*) be a solution of (VVIF) and y+x∈ K, for each y∈K. If
(1) ⟨x*,η(x,x)⟩+F(x+y)–F(y)–F(x) ⊆–C(x), for each y ∈K,
(2)η (y+x,x)=η(y,x)+ η(x,x), for each y∈K,
(3) ⟨x*,η(x,x)⟩+F(x) ∉ intC(x),
Then (x,x*) is a solution of (VVLIF).

Proof: (a) Let (x,x*) be a solution of (VVIF). Then x ∈ K and x*∈ T(x) such that ⟨x*,η(y,x)⟩+F(y) ∉–intC(x), ∀y∈K.

Since we have
$\langle x^*, \eta(y - z_0, x)\rangle + F(y - z_0) - \{\langle x^*, \eta(y, x)\rangle + F(y - x)$
$= \langle x^*, -\eta(z_0, x)\rangle + F(x - z_0) \in -C \subseteq -C(x), \forall y \in K.$

It follows from Lemma 1.1 (4)
$\langle x^*, \eta(y, x)\rangle + F(y) - F(x) \notin -\operatorname{int}C(x), \forall y \in K.$

There fore (x,x*) is a solution of (VVIF).
(b) Let (x,x*) be a solution of (VVIF). Then $x \in K$ and $x^* \in T(x)$ such that
$\langle x^*, \eta(y, x)\rangle + F(y) - F(x) \notin -\operatorname{int}C(x), \forall y \in K.$
$\langle x^*, \eta(y + x, x)\rangle + F(y) - F(x) \notin -\operatorname{int}C(x), \forall y \in K.$
$\langle x^*, \eta(y + x, x)\rangle + F(y + x) - F(x) \notin -\operatorname{int}C(x), \forall y \in K.$ (2.3)
$\langle x^*, \eta(y + x, x)\rangle + F(y + x) - F(x) - \{\langle x^*, \eta(y, x)\rangle + F(y)\} \forall y \in K.$
$= \langle x^*, \eta(x, x)\rangle + F(y + x) - F(y) - F(x) \in -C(x), \forall y \in K.$

By Lemma 1.1 (4), it follows (2.1) and (1) that
$\langle x^*, \eta(y, x)\rangle + F(y) \notin -\operatorname{int}C(x), \forall y \in K.$

Which it shows that (x,x*) is a solution of (VVLIF). This completes the proof.

Lemma 6.2:

Let X and Y be two normed spaces. Assume that $T: K \to L(X, Y)$Weakly pseudomonotone on K with respect to (η,F), $\mathcal{H}$ −hemicontinuous with compact values. Suppose that the following conditions are satisfied:
(1) The set–valued mapping $W: K \to Y$ defined by $W(x) = Y \setminus \{-\operatorname{int}C(x)\}$ is $w \times \boldsymbol{\tau}$ −closed.
(2) F and η are continuous in the second argument.
(3) For each $x \in K$, $\langle T(x), \eta(x, x)\rangle = \{0\}$.
(4) For each $x, y, z \in K$, the set–valued mapping $y \mapsto \langle T(z), \eta(y, x)\rangle + F(y) - F(x)$ is C(x)–convex.
Then (VVIF) is equivalent to the problem: Find $x_0 \in K$ such that for each $y \in K$, $\langle T(y), \eta(x_0, y)\rangle + F(x_0) - F(y) \not\subseteq \operatorname{int}C(y)$.

Proof: Since T is weakly pseudomonotone on K with respect to (η,F) therefore any solution of problem (VVIF)
$\langle T(y), \eta(x_0, y)\rangle + F(x_0) - F(y) \not\subseteq \operatorname{int}C(y), \forall y \in K.$

So is a solution.

Conversely, suppose that can find $x_0 \in K$, such that
$\langle T(y), \eta(x_0, y)\rangle + F(x_0) - F(y) \not\subseteq \operatorname{int}C(y), \forall y \in K.$
Now, we have, $x_0 \in K$, such that for each $y \in K$
$\langle T(y), \eta(x_0, y)\rangle + F(x_0) - F(y) \not\subseteq \operatorname{int}C(y), \forall y \in K.$

We consider y_t for $t \in (0,1)$. Replacing y by $y_t = x_0 + t(y - x_0) \in K$ in the condition (3) we deduce
$\{0\} = \langle T(y_t), \eta(y_t, y_t)\rangle + F(y_t) - F(y_t) \not\subseteq \operatorname{int}C(y_t),$ (2.4).

By condition (4), we have
$t[\langle T(y_t), \eta(y, y_t)\rangle + F(y) - F(y_t)] + (1-t)[\langle T(y_t), \eta(x_0, y_t)\rangle + F(x_0) - F(y_t)]$
$\subseteq \langle T(y_t), \eta(y_t, y_t)\rangle + F(y_t) - F(y_t) + C(y_t),$ (2.5).

Then we have
$\langle T(y_t), \eta(y, y_t)\rangle + F(y) - F(y_t) + C(y_t) \not\subseteq -\operatorname{int}C(y_t),$ (2.6).

Therefore (2.5), (2.6) and condition (3) since $T(y_t)$ and $T(x_0)$ are compact from Lemma 5.1 it follows that for each $v_t \in T(y_t)$ there exists $u_t \in T(x_0)$ such that
$\|v_t - u_t\| \le H(T(y_t), T(x_0))$

Since $T(x_0)$ is compact, without loss of generalizes we suppose that when $u_t \to u_0 \in T(x_0)$ as $t \to 0^+$.
Since T is $\mathcal{H}$ −hemicontinuous thererfore $H(T(y_t), T(x_0)) \to 0$ as $t \to 0^+$ so
wehave
$\|v_t - u_0\| \le \|v_t - u_t\| + \|u_t - u_0\| \le H(T(y_t), T(x_0)) + \|u_t - u_0\|$as $t \to 0^+$.
So when
$\|\langle (v_t - u_0), \eta(y, x_0)\rangle\| \le \|v_t - u_0\| \|\eta(y, x_0)\| \to 0.$

since Y\–intC(x_0) is closed there for from (2.6) we deduce that
$\langle u_0, \eta(y, x_0)\rangle$+F(y)–F($x_0$)∉–intC($x_0$).

Consequently
$\langle T(x_0), \eta(y, x_0)\rangle$+F(y)–F($x_0$)⊈–intC($x_0$). ■

Theorem 7.2:

Let X and Y be two normed spaces. Assume that T: $K \to L(X,Y)$

Weakly pseudomonotone on K with respect to (η,F), $\mathcal{H}$ −hemicontinuous with compact values. Suppose that the following conditions are satisfied:

(1) The set–valued mapping W:$K \to Y$ defined by W(x)=Y\{−intC(x)} and W': $K \to Y$ defined by W'(x)=Y\{–intC(x)} are closed, where w is weak topology of X.

(2) F and η continuous in the second aregument.

(3) For each x, y ∈ K, ⟨T(x),η(x,x)⟩={0}.

(4) For each x,y,z∈ K, the set–valued mapping y ↦⟨T(z),η(y,x)⟩+F(y)–F(x) is C(x)–convex.

(5) There exist a nonempty compact set M⊂K, and a nonempty compact convex set B⊂K such that for each for all x∈K\M, there is y∈K such that
⟨T(y),η(x,y)⟩+F(x)–F(y)⊆ intC(y).

then (VVIF) has a solution.

Proof: we have that for y ∈K, the set
Γ(y)={x∈K :⟨T(y),η(x,y)⟩+F(x)–F(y)⊈intC(y)}

Is closed. Let $\{x_n\}$ be a sequence in Γ(y) convergent to x_0 ∈K. since x_n∈ Γ(y) there exists v_n∈Γ(y) satisfying
z_n=⟨v_n,η(x_n ,y)⟩+F(x_n)–F(y)∉intC(y),

Then therefore, z_n∈W(x_n) and hence (x_n ,z_n)∈G_r(W). sinceΓ(y) is compact, Let $\{v_m\}$ be a subsequence of $\{v_n\}$ that convergent to v_0∈T(y). by continuity ofη,{η(x_m, y)} is norm bounded and therefore by proposition 3.2 (G.Y. Chen, 1992) and continuity of F, we have
z_0=$lim_m z_m$=⟨v_0,η(x_0 ,y)⟩+F(x_0)–F(y).

SinceG_r(W) is closed,then (x_0 , z_0)∈G_r(W) and hence
⟨v_0,η(x_0 ,y)⟩+F(x_0)–F(y)∉intC(y).

Then, x_0 ∈ Γ(y), this means Γ(y) is subset closed. Now, for each y∈K, we define the set–valued mapping $\hat{\Gamma}$:K→K by
$\hat{\Gamma} := \{x \in K : \langle T(x), \eta(y,x)\rangle + F(y) - F(x) \nsubseteq -\mathrm{intC}(x)\}$.

We show that $\hat{\Gamma}$ is a KKM mapping. Since if $\hat{\Gamma}$ is not a KKM mapping, then there exists $\{x_1,x_2,\dots,x_n\}$⊂K, t_i ≥0 ,i=1,2,…,n, with $\sum_{i=1}^{n} t_i$=1 such that x=$\sum_{i=1}^{n} t_i\, x_i \notin \cup_{i=1}^{n} \hat{\Gamma}(x_i)$ then for each i=1,2,…,n we have $\langle T(x), \eta(x_i, x)\rangle + F(x_i) - F(x) \subseteq -\mathrm{intC}(x)$,
So

$$\sum_{i=1}^{n} t_i \langle T(x), \eta(x_i, x)\rangle + \sum_{i=1}^{n} t_i\,(F(x_i) - F(x)) \subseteq -\mathrm{intC}(x). \quad (2.7)$$

On the otherhand by (4)

$$\langle T(x), \eta(x,x)\rangle - \sum_{i=1}^{n} t_i [\langle T(x), \eta(x_i, x)\rangle + F(x_i) - F(x)] \subseteq -\mathrm{intC}(x). \quad (2.8)$$

By (2.7) , (2.8) and second part two condition (3)

$$\langle T(x), \eta(x,x)\rangle \in -\mathrm{intC}(x), \quad (2.9)$$

Thus by which contradict C(x)≠Y. $\hat{\Gamma}$is a KKM mapping.Since T is weakly pseudomonotone we have $\hat{\Gamma}$(y)⊆Γ(y) for each y∈K. by Lemma 6.2 have solution. ■

Theorem 8.2:

Let set–valued function T: $K \to L(X,Y)$ weakly pseudomonotone on K with respect to(η,F), $\mathcal{H}$ −hemicontinuous with compact values if

(1) The set– valued mapping W:$K \to Y$ defined by W(x)=Y\{–intC(x)} and W': $K \to Y$ defined by W'(x)=Y\{–intC(x)} are w× $\boldsymbol{\tau}$ −closed, where w is weak topology of X.

(2)F and η are weak–norm continuous in the first and second component.
(3) for each x,y ∈K, ⟨T(x),η(x,x)⟩={0}.
(4) For each x,y,z ∈ K, the set–valued mapping y ↦⟨T(z),η(y,x)⟩+F(y)–F(x) is C(x)–convex.
(5) There exist a nonempty weak compact set M⊂K, and a nonempty weak compact convex set B⊂K such that for each for all x ∈K\M, there is y ∈K such that ⟨T(y),η(x,y)⟩+F(x)–F(y)⊆intC(y).
then (VVIF) has a solution.

Proof: by a similar proof of theorem 7.2 can deduced the result. we have that for y∈K, the set
Γ(y)={x∈K :⟨T(y),η(x,y)⟩+F(x)–F(y)⊈intC(x)}

Is weak closed. If $\{x_\beta\}$ be a net in Γ(y) convergent to x_0 ∈K.since $x_\beta \in \Gamma$ (y) there exists v_β∈T(y) satisfying
z_β=⟨v_β,η(x_β ,y)⟩+F(x_β)–F(y)⊈intC(x_β),

Then, z_β∈W(x_β) and hence (x_β ,z_β)∈G_r(W). Let $\{v_\lambda\}$ be a subnet of $\{v_\beta\}$ that convergent to v_0∈T(y). by continuity of η, {η(x_λ , y)} is norm bounded. So there exists λ_0 such that {η(x_λ , y) : λ≥λ_0} is norm bounded. by proposition 3.2 (G.Y. Chen, 1992) and continuityof F, we have
z_0=$\lim_{\lambda \geq \lambda_0} z_\lambda$=⟨$v_0$,η($x_0$,y)⟩ + F($x_0$) –F(y)

Since G_r(W), w× $\boldsymbol{\tau}$ −closed then (x_0 , z_0)∈G_r(W) and so
⟨v_0,η(x_0 ,y)⟩+F(x_0)–F(y)⊈intC(x_0).
Then, x_0 ∈ Γ(y), this means Γ(y) is subset weak closed. by similar theorem 7.2 (VVIF) has a solution.

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