

Mixed Convection Flows Toward a Stagnation-point on a Stretching Sheet

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Abstract: The problem of mixed convection stagnation-point flow of a viscous and incompressible fluid towards a stretching vertical sheet with prescribed surface heat flux is considered. The governing partial differential equations are first transformed into a system of ordinary differential equations, before being solved numerically by a finite-difference scheme known as Keller-box method. The features of the flow and heat transfer characteristics for different values of the governing parameters are analyzed and discussed. Both assisting and opposing flows are considered. The results indicate that dual solutions exist for the opposing flow, whereas for the assisting flow, the solution is unique.

Key words: dual solutions, heat transfer, stagnation point, stretching sheet, surface heat flux, fluid mechanics

INTRODUCTION

The flow in the confines of a stagnation point on a stretching sheet has attracted many investigations during the past several decades because of its wide applications in industrial and practical applications as mentioned in papers by Ishak *et al.* (2006, 2007). Crane (1970) was the first who has studied the two-dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate. This problem has later been extensively studied in Newtonian and non-Newtonian fluids, steady and unsteady flows, hydrodynamic and hydromagnetic fluids and in many other situations. For example, Chiam (1995), Mahapatra and Gupta (2001) and Ishak *et al.* (2008a) studied the stretching sheet in the presence of a magnetic field, considering some other physical features such as power-law velocity and buoyancy effect. The problems of heat transfer from a stretching surface with uniform or variable heat flux have been studied by Elbashbeshy (1998), Liu (2005), Dutta and Roy (1985) and Lin and Chen (1998). The problem of stagnation flow towards a heated vertical surface was studied by Ramachandran *et al.* (1988), who considered both an arbitrary wall temperature and arbitrary surface heat flux variations. They found that a reversed flow developed in the buoyancy opposing flow region, and dual solutions are found to exist for a certain range of the buoyancy parameter. This problem was then extended by Ishak *et al.* (2008b) by considering permeable flat plate, and reported the existence of dual solutions for both assisting and opposing flows.

The aim of the present paper is to study the mixed convection stagnation-point flow towards a stretching vertical sheet with prescribed surface heat flux. The case of prescribed surface temperature was studied by Ishak *et al.* (2007). To the best of our knowledge, this problem has not been studied before.

Problem Formulation:

Consider a steady, two-dimensional flow of a viscous and incompressible fluid near the stagnation-point on a vertical, continuously stretching sheet placed in the plane $y = 0$ of a Cartesian system of coordinates xy with the x -axis along the sheet, while the y -axis is measured normal to the surface of the sheet and is positive in the direction from the sheet to the fluid. It is assumed that the surface heat flux, the stretching velocity and the external velocity impinging the stretching sheet vary in a power-law with the distance from the stagnation-point, i.e. $q_w(x) = ax^n$, $u_w(x) = bx^m$ and $u_e(x) = cx^m$, respectively, where a , b and c are constants and m and n are the exponents. Under these assumptions along with the Boussinesq and boundary-layer approximations, the equations which model the problem under consideration are

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + v \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

$$u = u_w(x), v = 0, \frac{\partial T}{\partial y} = -\frac{q_w}{k} \text{ at } y = 0, \tag{4}$$

$$u \rightarrow u_e(x), T \rightarrow T_\infty \text{ as } y \rightarrow \infty.$$

The continuity equation can be satisfied by introducing a stream function ψ , such that $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$. The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation (see Ishak *et al.* (2006, 2007)):

$$\eta = \left(\frac{u_w}{vx}\right)^{1/2} y, f(\eta) = \frac{\psi}{(vxu_w)^{1/2}}, \theta(\eta) = \frac{k(T - T_\infty)}{q_w} \left(\frac{u_w}{vx}\right)^{1/2} \tag{5}$$

where the functions $f(\eta)$ and $\theta(\eta)$ are given by the ordinary differential equations

$$f''' + \frac{m+1}{2} f f'' - m f'^2 + m\varepsilon^2 + \lambda\theta = 0 \tag{6}$$

$$\frac{1}{Pr} \theta'' + \frac{m+1}{2} f \theta' - n f' \theta = 0 \tag{7}$$

where primes denote differentiation with respect to η , $\varepsilon = c/b$ is velocity ratio parameter, $Pr = \nu/\alpha$ is the Prandtl number, $Gr_x = g\beta q_w x^4 / (kv^2)$ is the local Grashof number and $Re_x = u_w x / \nu$ is the local Reynolds number. It can be shown that $\lambda (= Gr_x / Re_x^{5/2})$ is independent of x if $n = (5m - 3) / 2$.

Thus, in the presence of buoyancy force, similarity is achieved under this limitation. For $n = (5m - 3) / 2$,

Eq. (7) becomes

$$\frac{1}{Pr} \theta'' + \frac{m+1}{2} f \theta' - \frac{5m-3}{2} f' \theta = 0 \tag{8}$$

The transformed boundary conditions are:

$$\begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -1, \\ f'(\infty) \rightarrow \varepsilon, \quad \theta(\infty) \rightarrow 0. \end{aligned} \tag{9}$$

Further, $\lambda > 0$ and $\lambda < 0$ correspond to assisting (aiding) and opposing flows, respectively. It is worth mentioning that for $\lambda = 0$, Eqs. (6) and (8) are decoupled and this case corresponds to the forced convection flow past a stretching sheet.

The physical quantities of interest are the skin friction coefficient and the local Nusselt number, which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2 / 2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{10}$$

where the wall shear stress τ_w and wall heat flux q_w are given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \tag{11}$$

with μ being the dynamic viscosity. Using the non-dimensional variables (5), we obtain

$$\frac{1}{2} C_f Re_x^{1/2} = f''(0), \quad Nu_x / Re_x^{1/2} = 1 / \theta(0) \tag{12}$$

We notice that when both external flow and buoyancy force are absent, the analytical solution of Eq. (6) for $m=1$ was reported by Crane (1970) as

$$f(\eta) = 1 - e^{-\eta} \tag{13}$$

and the solution for the thermal field is

$$\theta(\eta) = \frac{1}{Pr} e^{-\eta Pr} \frac{M(Pr-n, Pr+1, -Pr e^{-\eta})}{M(Pr-n, Pr, -Pr)} \tag{14}$$

where $M(a, b, z)$ denotes the confluent hypergeometric function (see Abramowitz and Stegun (1965)) with

$$M(a, b, z) = 1 + \sum_{k=1}^{\infty} \frac{a_k}{b_k} \frac{z^k}{k!}$$

$$a_k = a(a+1)(a+2) \cdots (a+k-1)$$

$$b_k = b(b+1)(b+2) \cdots (b+k-1)$$

Further, from Eqs. (13) and (14), the skin friction coefficient $f''(0)$ and the surface temperature $\theta(0)$ can be shown to be given by

$$f''(0) = -1$$

$$\theta(0) = \frac{1}{Pr} \frac{M(Pr-n, Pr+1, -Pr)}{M(Pr-n, Pr, -Pr)} \tag{15}$$

On the other hand, when $\lambda = 0$ and $\varepsilon = 0$, the solution of equation (6) subject to the boundary condition (9) is given by

$$f(\eta) = \eta \tag{16}$$

and integrating Eq. (8) subject to the boundary conditions (9) gives

$$\int_0^\infty f'\theta d\eta = \frac{1}{(3m-1)Pr} \tag{17}$$

RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (6) and (8) subjected to (9) were solved numerically for some values of velocity ratio parameter ε , velocity exponent parameter m and Prandtl number Pr using a finite-difference scheme known as the Keller-box method described in the book by Cebeci and Bradshaw (1988). Comparison of the values of $\theta(0)$ with those obtained by Elbashbeshy (1998) and Liu (2005) for several values of parameters is listed in Table 1. It is observed that the results show a very good agreement.

The numerical results for the skin friction coefficient $f''(0)$ and the local Nusselt number $1/\theta(0)$ for various values of velocity ratio parameter ε when Pr and m are unity are presented in Figures 1 and 2, respectively. It can be seen that at the upper branch of the curves, as the buoyancy parameter increases, both the wall shear stress and the local heat transfer rate increase, due to the increased velocity caused by the external flow and buoyancy forces. Apart from that (see Fig. 2), at the lower branch of the curves, the buoyancy parameter decreases the velocity near the wall and causes the local heat transfer rate to decrease with increasing value of the buoyancy parameter. We also notice that $f''(0) = -1$ when both buoyancy force and

external flow are absent which is in agreement with the exact solution (15). Furthermore, dual solutions are found to exist for the opposing flow ($l < 0$), see Figs. 1 and 2. The solution for a particular value of ε exists up to a critical value of λ , say λ_c . Based on our computations, we found that $MBOL108\}f''Symbol"s12_c = -4.730, -9.108$ and -5.200 for $\varepsilon = 1, 1.5$ and 2 , respectively, all for $Pr = 1$ and $m = 1$. It is worth mentioning that the existence of dual solutions in the mixed convection problems in the case of wall heat flux was also reported by Ishak *et al.* (2008c, 2009a,b).

The samples of velocity and temperature profiles for selected values of parameters are depicted in Figures 3 and 4, respectively. These figures show that the far field boundary conditions (9) are satisfied asymptotically, hence support the validity of the numerical results, besides supporting the existence of the dual solutions shown in Figs. 1 and 2.

Figure 5 presents the same trend of the skin friction coefficient $f''(0)$ as in Fig. 1 for some values of velocity exponent parameter m when $\varepsilon = 1.5$, which indicates that the surface shear stress increases as the buoyancy force increases.

Conclusions:

The problem of mixed convection flow towards a vertical plate with a prescribed surface heat flux immersed in an incompressible micropolar fluid has been studied theoretically. The governing partial differential equations were transformed into ordinary differential equations using similarity transformation, before being solved numerically by a finite-difference scheme known as the Keller-box method. We discussed the effects of the material parameter, buoyancy parameter and the Prandtl number on the fluid flow and heat transfer characteristics. We found that dual solutions exist for both assisting and opposing flows. The solutions for

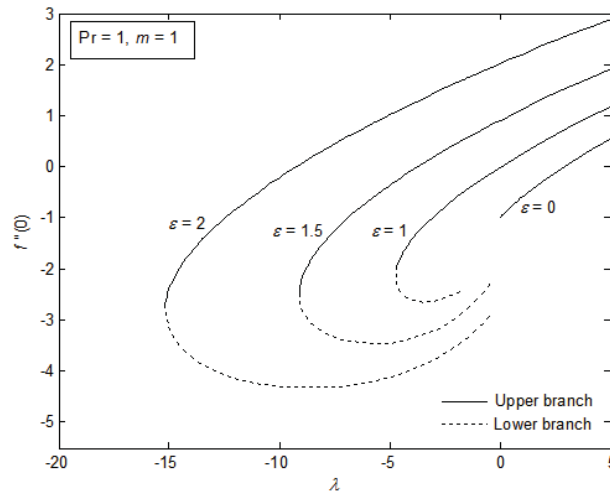


Fig. 1: Variation of the skin friction coefficient $f''(0)$ with λ for some values of ε when $Pr = 1$ and $m = 1$.

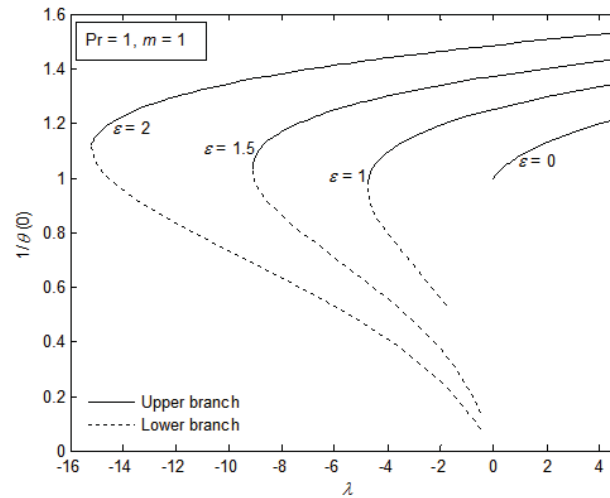


Fig. 2: Variation of the local Nusselt number $1/\theta(0)$ with λ for some values of ε when $Pr = 1$ and $m = 1$

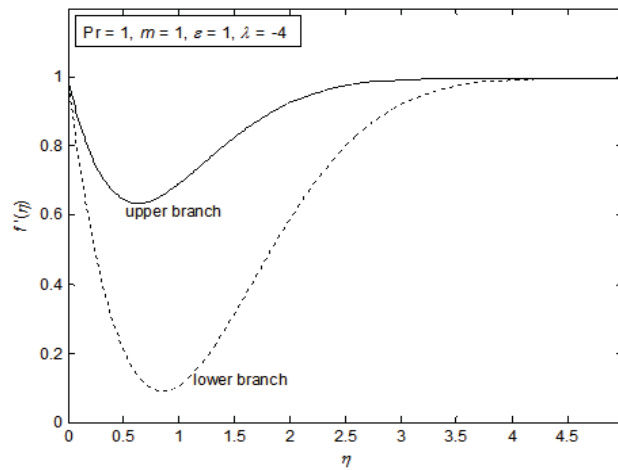


Fig. 3: Velocity profile $f'(h)$ for $Pr=1$, $m=1$, $\varepsilon=1$ and $l = -4$

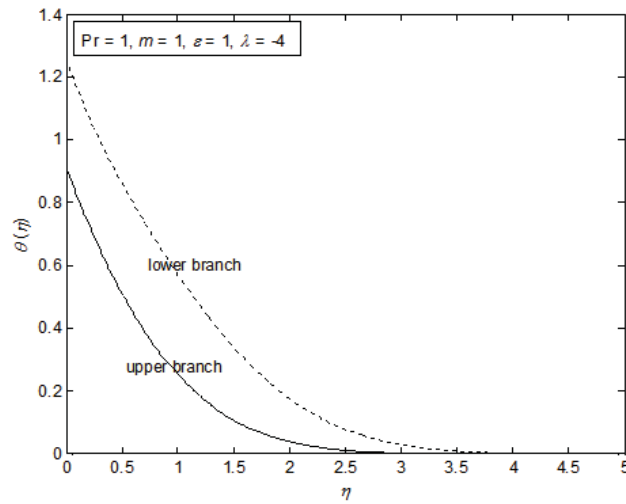


Fig. 4: Temperature profile $q(h)$ for $Pr=1$, $m=1$, $\epsilon=1$ and $l = -$

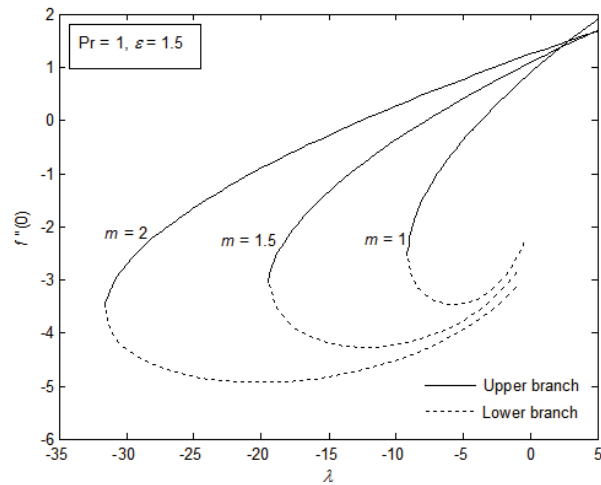


Fig. 5: Variation of the skin friction coefficient $f'(0)$ with λ for some values of m when $Pr = 1$ and $\epsilon=1.5$.

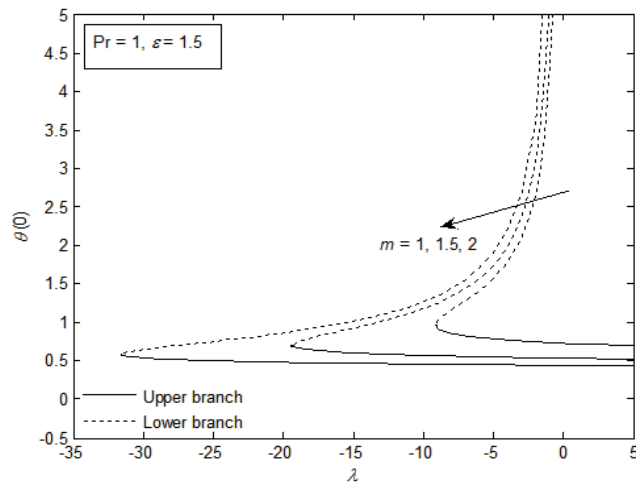


Fig. 6: Variation of the wall temperature $q(0)$ with λ for some values of m when $Pr = 1$ and $\epsilon=1.5$.

Table 1: Values of the surface temperature $q(0)$ for different values of ε , λ , m and Pr.

ε	λ	m	Pr	Numerical solution	Series solution, Eq. (15)	Elbashbeshy (1998)	Liu (2005)	
0	0	0.6	1	1.89				
			0.72	2.3703				
		1	1	1	1.2367	1.236657472	1.2253	
				1	1	1	1	
				10	0.2688	0.268768515	0.2688	
				6.7	0.3333	0.333303061		0.333303
		1	1	1	0.924			
				2	0.8842			
		1	1	1	1	0.7833		
		2				0.6679		

assisting flow ($\lambda > 0$) could be obtained for all values of the buoyancy parameter λ , while for the opposing flow ($\lambda < 0$), the solutions were obtained only in the range of $\lambda > \lambda_c (< 0)$ where λ_c is the minimum value of λ for which the solution exists. It is also found that micropolar fluids as well as fluids with larger Prandtl number increase the range of λ for which the solution exists.

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