Australian Journal of Basic and Applied Sciences, 4(9): 4379-4389, 2010
ISSN 1991-8178

Robust Controller Design for Single Axis Magnetic Levitation System

Basheer Noaman Hussein, Nasri Sulaiman, Hazem I. Ali

Department of Electrical & Electronic Engineering, Faculty of Engineering,
University Putra Malaysia, Malaysia

Abstract: This paper demonstrates theoretically the main idea of magnetic force control in magnetic levitation system using flux density measurements. A Hall-effect sensor is used to sense the flux density in the air gap. The magnetic force is obtained by its proportional relation to the flux density. A simple magnetic levitation system which consists of a U-shaped electromagnet and a manipulator is used. First, the system dynamics are described in state space form using air gap displacement, velocity of the magnetically levitated manipulator, and the flux density as state variables. Second, the magnetic force regulated using *H*f controller to achieve robust stability, disturbance/noise rejection and asymptotic tracking. Simulation results in terms of speed and accuracy are presented.

Key words: robust control, Hf *control, uncertain systems, nonlinear systems.*

INTRODUCTION

A magnetic levitation vehicle (MAGLEV) is a train-like vehicle that is suspended in the air above the track, and propelled forward using repulsive and attractive forces of magnetism. Because of the lack of physical contact between the track and the vehicle, the only friction is that between the carriages and the air. Maglev is a system in which the vehicle runs levitated from the guide-way (corresponding to the rail tracks of conventional railways) by using electromagnetic forces between superconducting magnets on-board the vehicle and coils on the ground (Boldea and Nasar, 1985). Usually, Maglev systems are open-loop unstable, and highly nonlinear. There have been many attempts on Maglevs for stabilization and linearization through the design of many types of controllers such as PID, H2, *H*f , etc. to give good results and performances. The main point of a micro-machine is that it has a frictionless movement; therefore it can be used for accuracy (Yi *et al.,* 1995; kuo *et al.,* 2005).

In some applications, micro-machines require a strategy of force control. For example, in semiconductor fabrication, force control is needed for mechanical quality control testing for bonds between circuits on devices (Busch-Vishniac *et al.,* 1990).

One possible conceptual design in the field of Micro-machines is the shear force tester shown in Figure 1. The tester consists of U-shaped electromagnets and a manipulator (a rectangular piece of metal), both are made of iron. It is assumed that the *xyz* coordinate system is fixed at the center of the manipulator, and Ψ, Φ, and θ are the Eulerian angles of the *xyz* coordinate system. Due to the magnetic suspension, spring forces serve as stabilizing forces on the manipulator and hence making the manipulator stable in both Ψ and Φ directions, and unstable in θ direction. A stabilizing controller (feedback or feed forward) is needed for planner motion (x, y, θ) and z motion. A known force can be applied to the bond in the z direction (Yi *et al.,* 1995).

In this application, only one degree-of-freedom motion in the z direction will be examined, with the assumption that the rest of the motions are stabilized. The force is determined from measurement of the flux density.

The goal of robust systems design is to retain assurance of system performance in spite of model uncertainty. A system is robust when his performance performs adequately under model uncertainty. A control system is robust when (i) it has low sensitivity, (ii) it is stable over the range of parameter variations and (iii) the performance continues to meet the specification in the presence of uncertainty (Ali *et al.,* 2009).

*H*f is one of the most known techniques available nowadays for robust control. It is an optimization method that takes into consideration a strong definition of the mathematical way to express the ability to include both classical and robust control concepts within a single design framework. With this method, model uncertainties and performance requirements can be incorporated into a single framework of *H*f controller achieve very robust stability and good performance in theory (http://en.wikipedia.org/wiki/H_infinity.).

Corresponding Author: Basheer Noaman Hussein, Department of Electrical & Electronic Engineering, Faculty of Engineering, University Putra Malaysia, Malaysia
E-mail:Nasaheer_s@yahoo.com

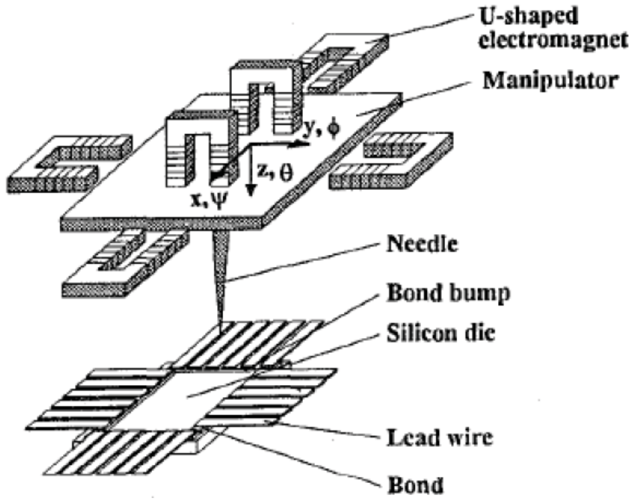


Fig. 1: Conceptual design for the shear force tester.

2. System Dynamics:

A single axis force controlled MLS shown in Figure 2 will be considered to derive a mathematical formulation of system motion. Besides the electromagnet and the manipulator, a spring and a linear bearing are used for providing a reaction force and constraining the manipulator motion to the vertical direction respectively. The U-shaped iron core magnet can produce a strong magnetic force which is a good advantage. The parameters for the system are listed in table (1) below.

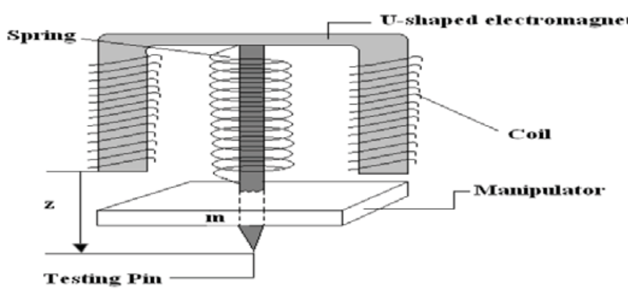


Fig. 2: A single Axis Magnetic Levitation System.

Table 1: System Parameters

Parameter	Description	Value
A	Pole face area	0.004 m2
B0	Nominal flux density	0.0687 Tesla
E0	Nominal voltage	7.37 V
h	Electromagnet width	0.02 m
F0	Nominal force	15.04 N
m	Manipulator mass	0.153 Kg
N	Coil Turns	410 Turns
R	Resistance	4.6 Ω
kf	Force constant	3187.4 N/Tesla^2
Z0	Nominal air gap	6 mm
z n	Air gap with no spring force	6 mm
μ 0	Permeability of free space	1.25 MN/A^2
ks	Spring constant	2257.3 N/m

The equation of motion is derived from Newton's second law of motion

$$mz''(t) = \sum forces \tag{1}$$

Four physically different forces are acting on the manipulator mass. They are the gravitational force, the electromagnetic force (attraction or repulsion), the spring force and any possible internal or external disturbance. It is important to note here that the disturbance force is also introduced to compensate for any neglecting of specific physical modes which may be considered in the mathematical formulation as will be shown below. Therefore, the equation of motion of the system is (Yi *et al*, 1995)

$$mz''(t) = mg - f(t) - k_s(z(t) - z_n) + f_{dis}(t) \tag{2}$$

where m is the manipulator mass, g is the gravity of earth (acceleration), $z(t)$ is the air gap length, z_n is the air gap length when no spring force exists (assumed neglected), $f(t)$ is the time function electromagnetic force, k_s is the spring constant, and $f_{dis(t)}$ is a time function of disturbance force or model uncertainty.

The magnetic force can be expressed in terms of the flux density B by assuming the magnetic flux is uniform in the air gap and the permeability of iron is high enough to be considered. The equation of magnetic force f in terms of flux density is as follows

$$f(y) = k_f B^2(t) \tag{3}$$

where B is the flux density, and k_f is

$$k_f = \frac{A}{\mu_0} \tag{4}$$

where A is the magnetic pole face area, and μ_0 is the permeability of free space.

The instantaneous flux density $B(t)$ and hence the instantaneous electromagnetic force $f(t)$ are produced by the electrical coil circuit. The equation relating the coil voltage e, flux density B, and the air gap length z is expressed by (Theraja, 1984).

$$e(t) + e_{dis}(t) = NA\frac{dB(t)}{dt} + \frac{2R}{\mu_0 N}B(t)z(t) \tag{5}$$

where N is the number of coil turns, R is the coil resistance, and e_{dis} is the voltage noise. Again this noise voltage is added not only to compensate for changes in the voltage source but also for any mathematical simplification which may be carried in Eq. (5).

Equation (2) and Eq. (5) now represent the basic two differential equations of the considered MLS. These equations depict nonlinear elements, the square of flux density B^2 and the product Bz. Moreover, if one assumes that f_{dis} and/or e_{dis} are random quantities then the solution would not be found analytically. One way to solve the problem is to apply the theory of small perturbation around nominal steady state parameter values.

Let F_0, B_0, Z_0, and E_0 be the nominal values of the magnet force, flux density, air gap length and coil voltage respectively, and let δf, δB, δz, δe be the deviation of these quantities from the nominal values. To apply the theory of deviation to the time function variables, it yields:

$$f(t) = F_0 + \sigma f(t) \tag{6}$$

$$B(t) = B_0 + \sigma B(t) \tag{7}$$

$$z(t) = Z_0 + \sigma z(t) \tag{8}$$

$$e(t) = E_0 + \sigma e(t) \tag{9}$$

The nominal flux density can be measured by the following equation (Vestgard *et al.,* 1994).

$$B_0 = \frac{0.5\mu_0 N E_0 \sqrt{1 + \frac{2Z_0}{\pi h}}}{R Z_0} \tag{10}$$

Next, the independent variable (t) will be dropped for simplicity of writing. The magnetic force can be related to the flux density by substituting Eq.s (6) and (7) in Eq. (3):

$$F_0 + \sigma f = k_f (B_0 + \sigma B)^2 \tag{11}$$

$$F_0 + \sigma f = k_f (B_0^2 + 2B_0 \sigma B + \sigma B^2) \tag{12}$$

$$F_0 + \sigma f = k_f B_0^2 + 2K_f B_0 \sigma B + K_f \sigma B^2 \tag{13}$$

By neglecting the second order term, which is valid if $|\delta B| << |B_0|$ (Yi *et al.,* 1995), then

$$F_0 = k_f B_0^2 \tag{14}$$

$$\sigma f = 2k_f B_0 \sigma B \tag{15}$$

The last two equations show that the force can be determined from flux density measurement. By substituting these two equations and Eq. (8) in Eq. (2), the system dynamics can be written as follows:

$$m(Z_0 + \sigma z)'' = mg - k_f B_0^2 - 2k_f B_0 \sigma B - k_s (Z_0 + \sigma z) + f_{dis} \tag{16}$$

$$mZ_0'' + m\sigma z'' mg - k_f B_0^2 - 2k_f B_0 \sigma B - k_s Z_0 - k_s \sigma z) + f_{dis} \tag{17}$$

The terms mZ_0'', mg, $-k_s Z_0$, and $-k_f B_0^2$ are put with model uncertainty (will be renamed as f_d), as shown below, and the final form of equation of motion will be:

$$m\sigma z'' = -k_s \sigma z - 2k_f B_0 \sigma B + f_d \tag{18}$$

On the other hand, after substituting Eq.s (7), (8), and (9) in Eq. (5), it yields

$$NA \frac{d(B_0 + \sigma B)}{dt} + \frac{2R}{\mu_0 N}(B_0 + \sigma B)(Z_0 + \sigma z) = (E_0 + \sigma e) + e_{dis} \tag{19}$$

$$NA \frac{d\sigma B)}{dt} + \frac{2R}{\mu_0 N}(B_0 Z_0 + B_0 \sigma z + \sigma B Z_0 + \sigma B \sigma z) = E_0 + \sigma e + e_{dis} \tag{20}$$

Again the terms $B_0 Z_0$, $\delta B \delta z$, and E_0 are put with voltage noise (will be renamed e_d), and the final form of electrical circuit equation will be:

$$NA\sigma B' = -\frac{2R}{\mu_0 N} B_0 \sigma z - \frac{2R}{\mu_0 N} \sigma B Z_0 + \sigma e + e_d \tag{21}$$

Therefore, Eq. (18) and Eq. (21) represents a linear version of the original system equations and all nonlinear modes are put in the model uncertainties f_d and e_d.

There are three physical quantities in the system to be assigned as system states. They are the air gap length *(z),* the flux density *(B),* and the electric coil voltage *(e).*

Let the system states be defined as follows:

$$\begin{aligned} x_1(t) &= \sigma z(t) \\ x_2(t) &= \sigma z'(t) \\ x_3(t) &= \sigma B(t) \\ u(t) &= \sigma e(t) \end{aligned} \tag{22}$$

Then the system dynamics can be expressed in the following state space form:

$$x_1' = x_2 \tag{23}$$

$$x_2' = -\frac{k_s x_1}{m} - \frac{2k_f B_0 x_3}{m} + \frac{f_d}{m} \tag{24}$$

$$x_3' = -\frac{2RB_0 x_1}{\mu_0 N^2 A} - \frac{2RB_0 x_3}{\mu_0 N^2 A} + \frac{u}{NA} + \frac{e_d}{NA} \tag{25}$$

To prepare the model for H_∞ -synthesis the followings are defined

$$x_p(t) = [x_1(t)\, x_2(t)\, x_3(t)]^T \tag{26}$$

$$x_p(0) = [Z_0 \;\; 0 \;\; B_0]^T \tag{27}$$

$$d(t) = [f_d \;\; e_d]^T \tag{28}$$

$$y_f(t) = \sigma f(t) \tag{29}$$

Then the dynamic model is

$$x_p'(t) = A_p x_p(t) + B_p u(t) + W_p d(t) \tag{30}$$

$$y_f(t) = C_p x_p(t) + n(t) \tag{31}$$

where n(t) represents the sensor noise (random disturbance), and

$$A_p = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k_s}{m} & 0 & -\frac{2k_f B_0}{m} \\ -\frac{2RB_0}{\mu_0 N^2 A} & 0 & -\frac{2RZ_0}{\mu_0 N^2 A} \end{bmatrix} \quad B_p = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{NA} \end{bmatrix}$$

$$C_p = [0 \;\; 0 \;\; 2k_f B_0] \quad W_p = \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & \frac{1}{NA} \end{bmatrix} \tag{32}$$

3. Controller Design:

Converting of the system state space defined by Eq. (32) gives the following transfer function:

$$G_p(s) = \frac{b_2 s^2 + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{33}$$

And we have disturbance and voltage noise (model uncertainty)

$$G_d(s) = \frac{b_1 s}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{34}$$

$$G_n(s) = \frac{b_2 s^2 + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{35}$$

Where

$$b_2 = (2k_f B_0)(\frac{1}{NA})$$
$$b_1 = (\frac{4k_f^2 B_0^2}{m^2})$$
$$b_0 = \left(\frac{k_s}{m}\right)(2k_f B_0)(\frac{1}{NA})$$
$$a_2 = (\frac{2RZ_0}{\mu_0 N^2 A}) \tag{36}$$
$$a_1 = (\frac{k_s}{m})$$
$$a_0 = [\left(\frac{k_s}{m}\right)\left(\frac{2RZ_0}{\mu_0 N^2 A}\right) - (\frac{2k_f B_0}{m})(\frac{2RB_0}{\mu_0 N^2 A})]$$

The system dynamics can be described by linear control system as shown in Figure 3.

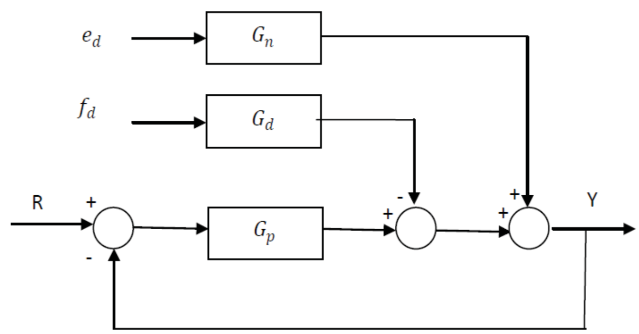


Fig. 3: Block diagram of the system.

The H_∞ synthesis (Zhou and Doyle, 1998; Anselmo and Moura, 1998) has been used to design a force controller for the system such that the following requirements are achieved: robust stability against various model uncertainties, disturbance/noise attenuation, asymptotic tracking to command signals, avoidance of saturation of the actuators, and limiting closed loop bandwidth to achieve good robustness and noise rejection.

Consider the standard H_∞ control problem shown in Figure 4, where w represents all exogenous input signals such as command signals *(R),* voltage noise (e_d), and disturbances (f_d); z represents all variables that need to be regulated, including weighted error signals (e_f), and control inputs (u_f); *u* represents the actuator input; *e* is the error signal; *P* is the standard plant which includes the plant model plus weighting functions that reflect the design specifications and goals. *P* is defined as:

The input-output transfer functions of interest here are: the transfer function between tracking error and disturbance or command signal, which is called the sensitivity function *S(s);* the transfer function between the plant output and the command or noise signals, which is called the complementary sensitivity function,

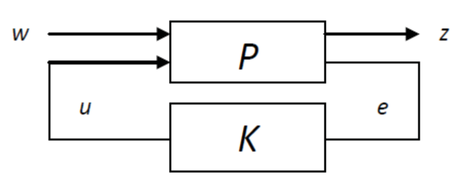


Fig. 4: Standard H_∞ control problem.

The input-output transfer functions of interest here are: the transfer function between tracking error and disturbance or command signal, which is called the sensitivity function S(s); the transfer function between the plant output and the command or noise signals, which is called the complementary sensitivity function, *T(s)*. Here, the *S(s)* and *T(s)* are defined as

$$S(s) = \frac{1}{1 + G_p(s)K(s)} \tag{37}$$

$$T(s) = \frac{G_p(s)K(s)}{1 + G_p(s)K(s)} \tag{38}$$

The design goal is to keep *S(s)* as small as possible to minimize the effect of the disturbance, and to keep *T(s)* large at low frequencies for good tracking and small at high frequencies for good noise rejection. The relation between *S(s)* and *T(s)* can be expressed as:

$$S(s) + T(s) = 1 \tag{39}$$

In order to achieve robust stability and good performance, *S(s)* must be weighted with proper weighting functions, $W_p(s)$ so that the following requirements are met:

$$\left\| \begin{matrix} S(s)W_p(s) \\ KS(s)W_u(s) \end{matrix} \right\| \le 1 \tag{40}$$

To achieve good tracking, less control energy and output disturbance/noise rejection. The weighting functions are given as

$$W_p(s) = \frac{0.66S^2 + 111S + 4624}{S^2 + 4.3S + 4.62} \tag{41}$$

$$W_u(s) = 0.1 \tag{42}$$

The following figure shows the augmented system where the transfer functions in Eq.s (33), (34) and (35) are incorporated with the weighting functions in Eq.s (41) and (42):

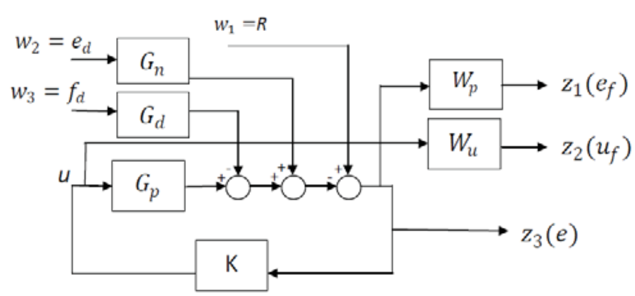


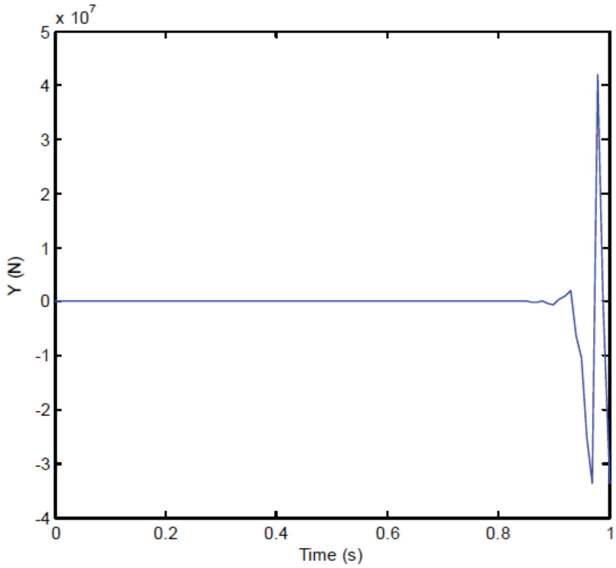
Fig. 5: Block diagram of the closed loop system with feedback structure, disturbance, and voltage noise and performance requirements.

The controller design has been carried out using the MATLAB Robust Control Toolbox [11]. After some iteration the *H*f controller which achieves the robustness and performance goals was found to be

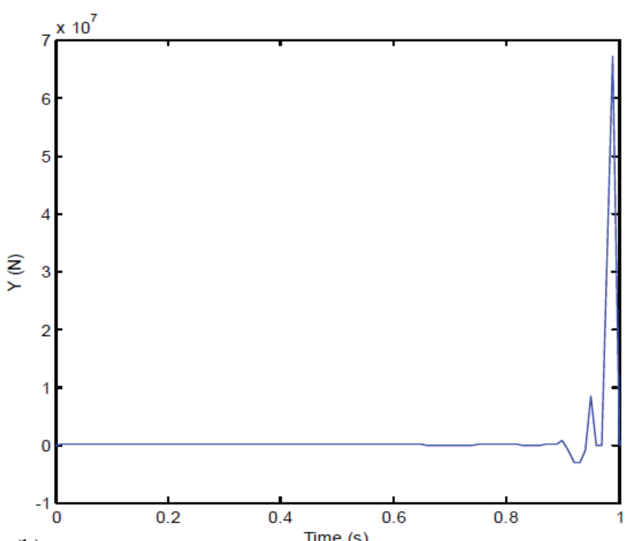
$$K = \frac{1.29*10^9 S^4 + 2.15*10^{11} S^3 + 3.59*10^{13} S^2 + 1.4*10^{15} S + 2.36*10^{16}}{S^5 + 1.78*10^8 S^4 - 1.74*10^{11} S^3 + 1.6*10^{13} S^2 + 7.12*10^{13} S + 7.75*10^{13}} \tag{43}$$

4. Simulation Results:

Figure 6 shows the system output response characteristics. It is clear that the system becomes unstable when a small disturbance or noise signals are applied. One of the important achievements of applying H_∞ controller is to ensure the robust stability of the system. Figure 7 shows the frequency characteristics of the sensitivity function compared with the inverse of the performance weighting function. It is clear that the performance criterion in Eq. (40) has been satisfied. The second performance criteria in Eq. (40) also have been achieved as shown in Figure 8. Figure 9 and 10 shows the time response characteristics of the system. The following time response specifications have been achieved: the rise time is 0.02 sec, the settling time is 0.13 sec and the percent overshoot is 30%.



(a)



(b)

Fig. 6: The system output response for a 0.1 N step input (a) with 0.1 N step disturbance force f_d applied without H_∞ controller and (b) with 0.1 N step voltage noise e_d applied without H_∞ controller.

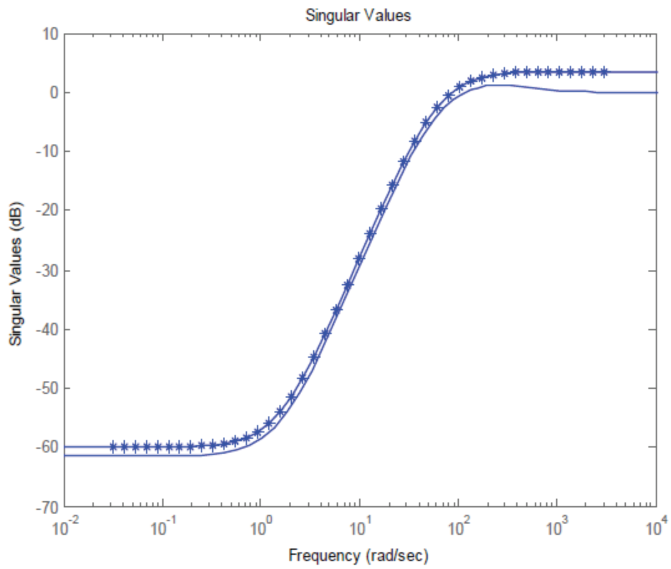


Fig. 7: The frequency response characteristics of the sensitivity function S(s)(-) compared with the inverse of the performance weighting function W_p(*).

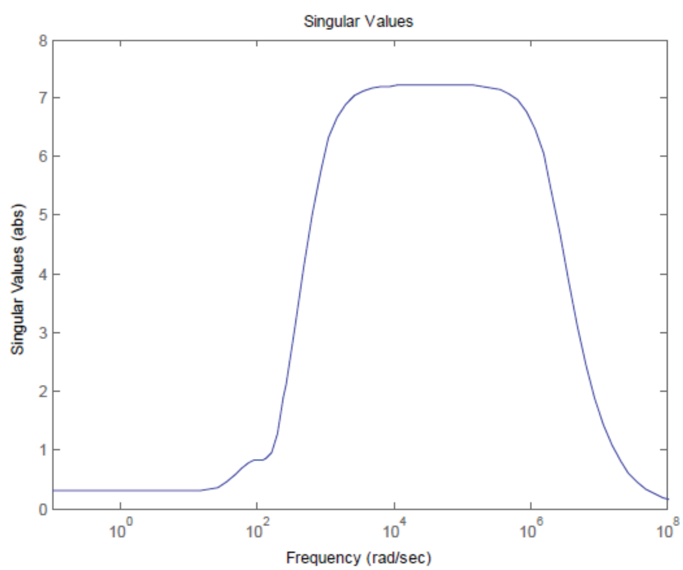


Fig. 8: The frequency response characteristics of *K(s)S(s)*.

Conclusion:

A robust *H*f controller for a single axis magnetic levitation system with model uncertainty is presented. First the linear mathematical model of the system is derived. The model uncertainty was taken into account as a disturbance to design the controller. Adjusting the performance of the simulated closed loop response carried out the selection of the weighting functions. The appropriate selection of the weighting functions led to obtain a robust controller that achieves the force control in magnetic levitation system. Finally, the objectives of the controller were verified by simulation.

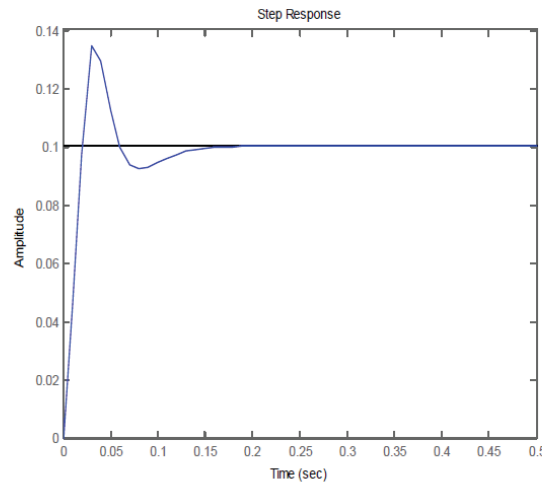


Fig. 9: The system output response for a 0.1 N step input with H_∞ controller.

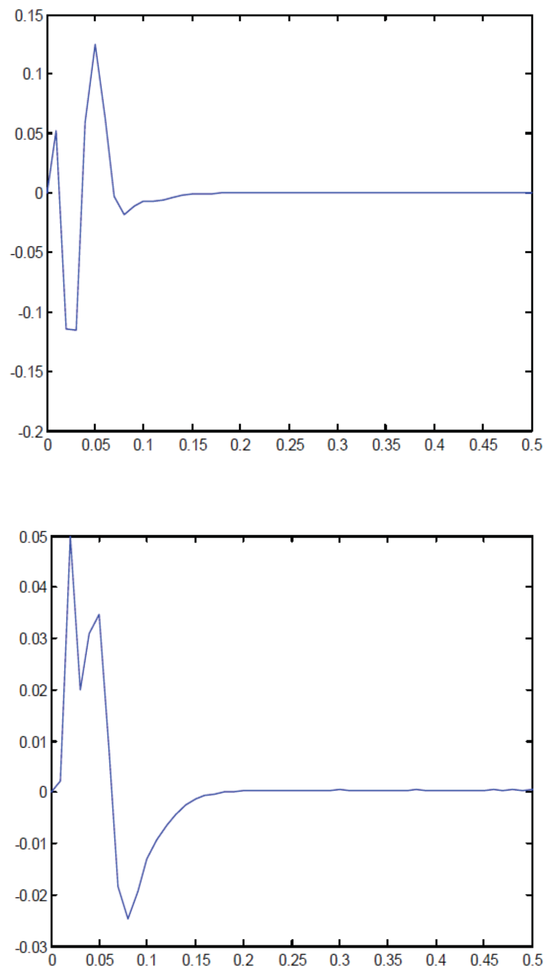


Fig. 11: The system output response for (a) 0.1 N step disturbance force f_d with H_∞ controller and (b) for 0.1 N step voltage noise e_d with H_∞ controller.

REFERENCES

Ali, H.I., S.B.M. Noor, S.M. Bashi and M.H. Marhaban, 2009. "Robust Controller Conference on Control, Instrumentation and Mechatronic Engineering. Malacca, Malaysia, Wikipedia, the free Encyclopedia.S.v., H infinity, http://en.wikipedia.org/wiki/H_infinity.

Anselmo, B. and S.R. Moura, 1998. "H_2 and $H\infty$ control for Maglev Vehicles". IEEE control systems, pp: 18-25.

Boldea, I., S.A. Nasar, 1985. *Linear Motion Electromagnetic Systems.* New York: John Wiley & Sons, Inc.

Busch-Vishniac, Ilene J., Chen, Shiaw-Jong; Jeong, Myung-Chan; Li, Shih-Hung and Wang, Ing-Yann, 1990. "Magnetic Levitation-Based Micro-Automation of the Mechanical Process in Semiconductor Fabrication". Proceedings of IEEE MEMS 90. Napa Valley, CA, pp: 142-146.

Grace, A., A.J. Laub, J.N. Little and C. Thompson, 1990. "Control System Toolbox". The Math Works Inc.

Kuo, Chao-Lin; Li, S. Tzuu-Hseng and Guo, Nai Ren. 2005. "Design of a Novel Fuzzy Sliding-Mode Control for Magnetic Ball Levitation System". Journal of Intelligent and Robotic Systems, 42: 295-316.

Theraja, B.L., 1984. A Text Book of Electrical Technology. Publication Division of Nirja Construction and Development Co. LTD. Ram Nagar, New Delhi.

Vestgard, Bard, M. Mohamed, Abdelfatah and Busch-Vishniac, 1994. Ilene. "Real Time Implementation of a Robust Hь Controller for a 2-DOF Magnetic Micro-Levitation Positioner". *Proceedings of the American Control Conference.* Baltimore, Maryland, pp: 3219-3223.

Yi, J.H., K.H. Park, S.H. Kim, Y.K. Kwak, M. Abdelfatah,and I. Busch-Vishniac, 1995. "Force Control for Magnetic Levitation System Using Flux Density Measurement". *Proceedings of IEEE of the 34th Conference on Decision and Control.* New Orleans, LA, pp: 153-2158.

Zhou, K. and J.C. Doyle, 1998. "Essentials of robust control". Prentice Hall.