Australian Journal of Basic and Applied Sciences, 4(8): 3622-3634, 2010
ISSN 1991-8178

# Investigation of the Regularity Conditions in Data Envelopment Analysis

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**Abstract:** Duality plays a fundamental role in mathematics, especially in optimization. For every linear programming model, there is a corresponding one which called '*dual*'. A dual linear programming problem has some special specifications associated with the basic problem in which all characteristics, specially in order to meet the results for the basic problem, is used. Its variables provide extremely useful information about the optimal solution to the original linear program. This leads to rich economic interpretations related to the original linear programming problem. In fact, the roots of this problem lie in the characterization of the optimality conditions for the original linear program. For the sake of expository reference, we shall call the original linear programming problem, the *primal* (linear programming) problem, and we shall call this related linear program, the *dual* (linear programming) problem. Actually, the terms primal and dual for this related pair of linear programming problems are only relative, because the dual of the '*dual*' is the '*primal*' itself. One of the most interesting specifications of primal-dual, is to investigate the regularity conditions. In this paper, the regularity conditions is going to be inspected in two models, BCC and CCR, and then, the conclusions will be discussed.

**Key words:** Data Envelopment Analysis(DEA), Regularity, Efficiency, Primal, Dual.

## INTRODUCTION

Data Envelopment Analysis (DEA), as developed by Charnes et al.(1978), is a technique that has been used widely in the supply chain management literature. This non-parametric, multi-factor approach enhances our capability to capture the multi-dimensionality of performance discussed earlier. More formally, DEA is a mathematical programming technique for measuring the relative efficiency of decision making units (DMUs), where each DMU has a set of inputs used to produce a set of outputs.

On the one hand, duality is very important in optimization.It is an old idea to study, additionally to a given optimization problem, a corresponding dual problem. Duality has resulted in many applications within optimization, and it has provided many unifying conceptual insights into economics and management science. One of the important subjects about DEA, is the investigation in regularity conditions in which the relations and conclusions would be considerable. In fact, the regularity condition, expresses those conditions that, by meeting them, a collection of optimum solutions in non-empty and bounded, both for primal and dual forms, will be.

The current paper proceeds as follows. Section 2 discusses the basic DEA models and relations between primal and dual problems and investigating regularity conditions. Section 3 develops the regularity conditions in the basic DEA model, such as BCC and CCR. And finally, conclusions are given in section 4.

### *2. Background:*

Consider $DMU_j, (j = 1,\dots,n)$ , where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of $DMU_j$ are $X_j = (x_{1j},\dots,x_{mj})$ and $Y_j = (y_{1j},\dots,y_{sj})$

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respectively, and let $X_j \geq 0$ , $X_j \neq 0$ , $Y_j \geq 0$ , and $Y_j \neq 0$ , (Cooper, W., 2002).

The production possibility set $T_c$ is defined as:

$$T_c = \{(X,Y)| \ X \geq \sum_{j=1}^{n} \lambda_j X_j, \ Y \leq \sum_{j=1}^{n} \lambda_j Y_j \ , \lambda_j \geq 0, \ \ j = 1, \dots, n\} .$$

By the above definition the CCR model (Charnes, A., 1978) is as follows:

$$\begin{aligned} & Min \quad \theta \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \qquad i = 1, \dots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \dots, s \\ & \lambda_j \geq 0, \qquad\qquad j = 1, \dots, n \end{aligned} \tag{1}$$

Moreover the production possibility set $T_v$ is defined as:

$$T_v = \{(X,Y)| \ X \geq \sum_{j=1}^{n} \lambda_j X_j, \ Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \ \ j = 1, \dots, n\} .$$

By the above definition the BCC model (Banker, R.D., 1984) is as follows:

$$\begin{aligned} & Min \quad \theta \\ s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \qquad i = 1, \dots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \dots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0, \qquad\qquad j = 1, \dots, n \end{aligned} \tag{2}$$

In continue, regularity conditions in primal and dual problems will be considered. In order to do so, a couple problems of primal and dual with the followed scheme is mentioned.
First suppose that the primal linear program is given in the (canonical) form (Murty, Katta G., 1983):

$$\begin{aligned} (P) \quad & Min \quad CX \\ s.t \quad & AX \geq b \\ & X \geq 0, \end{aligned} \tag{3}$$

Then the dual linear program is defined by:

$$\begin{aligned} (D) \quad & Max \quad Wb \\ s.t \quad & WA \leq C \\ & W \geq 0, \end{aligned} \tag{4}$$

Suppose that feasible region of primal (P) and the feasible region of dual (D), are defined as follows:

$$S_p = \{X| \ AX \geq b, X \geq 0\} .$$

$S_D = \{W| \ WA \leq C, W \geq 0\}$.

Moreover, if $S_P^*$ and $S_D^*$ be the collections of optimum solutions of primal (P) and dual (D), the followed theorem that is necessary and sufficient condition for bounding the $S_P^*$ and $S_D^*$ will be defined and proved.

***Theorem 1:***

(a). $S_P^*$ is non-empty and bounded, if only if, $S_P \neq \emptyset$ and no $d \neq 0$ to be that:

$Ad \geq 0$ , $Cd \leq 0$ , $d \geq 0$ .

b). $S_D^*$ is non-empty and bounded, if only if, $S_D \neq \emptyset$ and no $\mu \neq 0$ to be that:

$\mu A \leq 0$ , $\mu b \geq 0$ , $\mu \geq 0$ .

***Proof:***

Suppose that $S_P^* \neq \emptyset$ and bounded. According to strong duality theorem, both, primal (P) and dual (D) should be feasible. Since dual (D) is feasible, by introducing the slack variables to the dual problem, the followed system would have a solution.

$$WA + IV = C, W \geq 0, V \geq 0$$

So, according to the **Farkas** lemma, the below system has no solution.

$$Ad \geq 0 \ , Cd < 0 \ , d \geq 0 \ . \qquad (1)$$

And so, it is needed to prove that followed system has no solution as well.

$$Ad \geq 0 \ , Cd = 0 \ , d \geq 0 \ , \ d \neq 0 \ . \qquad (2)$$

By contradiction, suppose that system (2) has solution. On the other hand, it is ok to consider $S_P^*$ as below.

$$\begin{cases} AX \geq b \\ X \geq 0 \\ CX = CX^* \end{cases} \qquad (3)$$

Because of being the system (2) of homogenized system (3), so if (2) has a solution, then (3) has a extreme direction.And in this way, the collection (3) will be unbounded (Mokhtar, S. Bazaraa, 1990). It means, $S_P^*$ would be unbounded and the subject is contradiction. So, the proof is completed.

*Theorem vice versa*. Suppose that $S_P \neq \emptyset$ and there is no $d \neq 0$ for considering the bellow conditions:

$$Ad \geq 0 \ , Cd \leq 0 \ , \ d \geq 0 \ . \qquad (4)$$

We obtain from (4), the below system has no solution:

$Ad \geq 0$ , $Cd < 0$ , $d \geq 0$ .

It means, in accordance with **Farkas** lemma, the below system has solution.

$WA + IV = C, \quad W \geq 0, \quad V \geq 0$

It can be concluded that, because of $S_D \neq \emptyset$ and assuming $S_P \neq \emptyset$ , both problems, primal and dual are feasible and also optimum solutions, means, $S_P^* \neq \emptyset$ . Now it may be claimed that $S_P^*$ is bounded.

By contradiction, let $S_P^*$ is unbounded, we reach another conclusion from (4), the followed system has no solution as well.

$Ad \geq 0$ , $Cd = 0$ , $d \geq 0$ , $d \neq 0$ . (5)

$S_P^*$ is considered as follows:

$$\begin{cases} AX \geq b \\ X \geq 0 \\ CX = CX^* \end{cases} \qquad (6)$$

Because of $S_P^* \neq \emptyset$ , so it may be an assumption that $x^* \in S_P^*$ . If $S_P^*$ is unbounded, the homogenous system has non-zero solution and it leads us, the system:

$$\begin{cases} Ad \geq 0 \\ Cd = 0 \\ d \geq 0 \\ d \neq 0 \end{cases} \qquad (7)$$

has a solution. It is a contradiction. So, the proof is completed.
Part (b) has a similar proof.
Above proved conditions, means:

$(a) Ad \geq 0$ , $Cd \leq 0$ , $d \geq 0$ , $d \neq 0$ have no solution.

$(b) \mu A \leq 0$ , $\mu b \geq 0$ , $\mu \geq 0$ , $\mu \neq 0$ have no solution.

as regularity conditions for primal and dual problems.

***Interpretation of Theorem from the Geometric Point of View:***

Suppose that the primal linear program is minimization.

Based on figure (1), the $S_P$ feasible region is unbounded and two extreme directions are exist, called $d_1$ , $d_2$ .

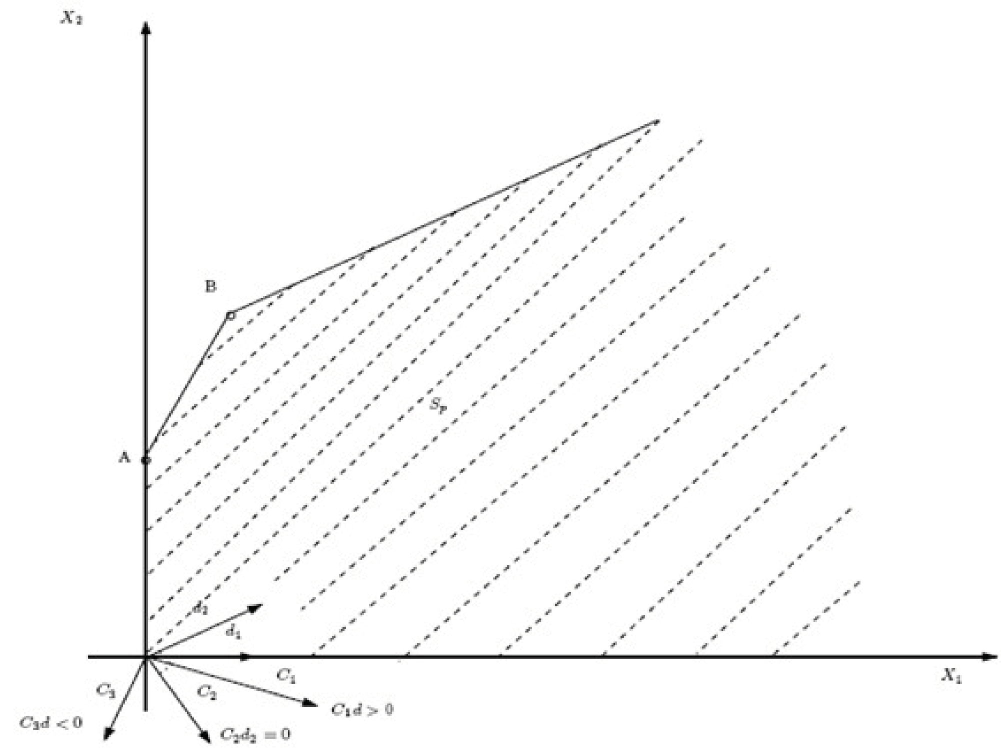


**Fig. 1:** Feasible region of primal (p).

In figure (1), by considering $C_1 d > 0$ , $S_p^*$ is bounded and the A is the optimum point of the problem. Moreover, we have $C_2 d_2 = 0$ , and a unbounded ray as well, so $S_p^*$ is also unbounded. For vector $C_3$, we have $C_3 d < 0$ and so, as figured out, $S_p^*$ is empty.

***3. Researching and Expressing the Regularity Conditions in DEA Basic Models as BCC and CCR:***
***3.1. Regularity Condition in CCR Model:***

In this part, we are going to analyze the regularity condition in DEA basic models. As shown in previous part, investigating the regularity condition in linear programming problems allow us to reach a pair of primal and dual problems which meet necessary and sufficient conditions for these non-empty and bounded problems. Scrutinizing these conditions in CCR model is important, because of decision making units under study in different kind of conditions from input and output vectors point of view, have regularity condition.

Meeting or not meeting these conditions, because of explaining the non-empty and bounded the optimum solution envelopment and multiplier forms of CCR model, have a great value.

For the above title, we consider the envelopment form of the CCR model in input oriented as follows:

$$\begin{aligned} & Min \quad \theta \\ & s.t \quad X\lambda \le \theta x_o \\ & \qquad Y\lambda \ge y_o \\ & \qquad \lambda \ge 0 \end{aligned} \tag{5}$$

The dual multiplier form of this linear program is expressed as:

$$\begin{aligned} & Max \quad UY_o \\ & s.t \quad UY - VX \leq 0 \\ & \quad VX_o = 1 \\ & \quad U \geq 0, V \geq 0 \end{aligned} \tag{6}$$

As the first step, as above said, the model (5) should be standardized. So:

$$\begin{aligned} & Min \quad \theta_1 - \theta_2 \\ & s.t \quad -X\lambda + \theta_1 X_o - \theta_2 X_o \geq 0 \\ & \quad Y\lambda \geq y_o \\ & \quad \lambda \geq 0, \theta_1 \geq 0, \theta_2 \geq 0 \end{aligned} \tag{7}$$

The next step is to scrutinize the regularity condition in (a) in order to know that the below system has any solution or not.

$(a) Ad \geq 0$ , $Cd \leq 0$ , $d \geq 0$ , $d \neq 0$

Suppose that, $d = (d_1, d_2, d_3)$ in which : $d_1 \in R^n, d_2 \in R, d_3 \in R$ . So the above system will be:

$$\begin{cases} -Xd_1 + X_o d_2 - X_o d_3 \geq 0 \\ Yd_1 \geq 0 \\ d_1 \geq 0, d_2 \geq 0, d_3 \geq 0 \\ (d_1, d_2, d_3) \neq (0,0,0) \end{cases} \tag{1}$$

Moreover $Cd \leq 0$ is constructed:

$Cd = (0,1,-1)(d_1, d_2, d_3) = d_2 - d_3 \leq 0 \Rightarrow d_2 \leq d_3$ .

It is known that:

$$\begin{cases} X_o \leq X_o \\ d_2 \leq d_3 \\ d_2, d_3 \geq 0 \end{cases}$$

So $X_o d_2 \leq X_o d_3$ , that is $X_o d_2 - X_o d_3 \leq 0$ (2)

By considering (1) and (2) as a whole, we have:

$$\begin{cases} -Xd_1 + X_o d_2 - X_o d_3 \geq 0 \\ X_o d_2 - X_o d_3 \leq 0 \end{cases}$$

What concluded is: $-Xd_1 \geq 0$ so $Xd_1 \leq 0$ , means that, we should have:

$$\begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} * \begin{pmatrix} d_{11} \\ \vdots \\ d_{n1} \end{pmatrix} = \begin{pmatrix} x_{11}d_{11} & + \cdots + & x_{1n}d_{n1} \\ \vdots & & \vdots \\ x_{m1}d_{11} & + \cdots + & x_{mn}d_{n1} \end{pmatrix} \le 0$$

The claim is: $d_1 \in R^n$ has no non-zero element. By contradiction, we assume: $d_{11} \neq 0$. So what from above equation gained will be:

$$\begin{pmatrix} x_{11} \\ \vdots \\ x_{m1} \end{pmatrix} d_{11} + \cdots + \begin{pmatrix} x_{1n} \\ \vdots \\ x_{mn} \end{pmatrix} d_{n1} \le 0$$

By supposing:

$$\begin{cases} X_{ij} \ge 0, & i = 1, \dots, m \\ d_{j1} \ge 0, & j = 1, \dots, n \\ d_{11} \neq 0 \end{cases}$$

It must be $\begin{pmatrix} x_{11} \\ \vdots \\ x_{m1} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ and it means that the first DMU has all zero input component and it is on the opposite side of the assumption ($X_j \neq 0, j = 1, \dots, n$ ).

It is concluded that the vector ($d_1 \in R^n$ ) has no non-zero component, so $d_1 = 0$ .
Considering

$$\begin{cases} d_1 = 0, \\ d_2 \le d_3, \end{cases}$$

Two conditions are easily distinguishable.

*Condition one*: $d_2 = d_3 \neq 0$ . As an example: $d = (d_1, d_2, d_3) = (0 \in R^n, 1 \in R, 1 \in R)$.

What above mentioned, is one of result of the system (a), and in this case, the envelopment form of CCR model is irregular in the input oriented.

*Condition two*: $d_2 = d_3 = 0$ . In this case, because of $d_1 = 0$ , so $d = (d_1, d_2, d_3) = (0 \in R^n, 0 \in R, 0 \in R)$. So, the system (a) has no resolution $d \neq 0$ and regularity condition in part (a) is satisfied.

***Attention:***

$d_2 < d_3$ would not be carried out because of the below system:

$$\begin{cases} -Xd_1 + X_o d_2 - X_o d_3 \geq 0 \\ d_1 = 0 \end{cases}$$

We obtain: $X_o d_2 - X_o d_3 \geq 0$. If $d_2 < d_3$, what expressed above, would be incorrect. In continue, the co-efficient of the regularity condition of the CCR model in input oriented will be paid attention. Fulfilling so, the follow system should be investigated to know as if, it has any solution.

$\mu A \leq 0$ , $\mu b \geq 0$ , $\mu \geq 0$ , $\mu \neq 0$

As it had been told before, in dual form of the problem (envelopment model), we had:

$$A = \begin{pmatrix} -x_1 & \cdots & -x_n & x_o & -x_o \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_1 & \cdots & y_n & 0 & 0 \end{pmatrix}, b = \begin{pmatrix} 0 \\ \vdots \\ y_0 \end{pmatrix}$$

For the expanded form of matrix A, vector μ will be as follows:

$$A = \begin{pmatrix} -x_{11} & \cdots & -x_{1o} & \cdots & -x_{1n} & x_{1o} & -x_{1o} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -x_{m1} & \cdots & -x_{mo} & \cdots & -x_{mn} & x_{mo} & -x_{mo} \\ y_{11} & \cdots & y_{1o} & \cdots & y_{1n} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{s1} & \cdots & y_{so} & \cdots & y_{sn} & 0 & 0 \end{pmatrix}, \mu = \begin{pmatrix} \mu_1, & \cdots & , \mu_m, & p_1, & \cdots & , p_s \end{pmatrix}$$

In this case, the condition $\mu A \leq 0$ exists. So (n+2) inequality would be resulted as follows:

$$\begin{cases} -\mu_1 x_{11} - \cdots - \mu_m x_{m1} + p_1 y_{11} + \cdots + p_s y_{s1} \leq 0 & non-equation\ the\ first \\ \vdots & \vdots \\ -\mu_1 x_{1o} - \cdots - \mu_m x_{mo} + p_1 y_{1o} + \cdots + p_s y_{so} \leq 0 & non-equation\ (o)\ th \\ \vdots & \vdots \\ -\mu_1 x_{1n} - \cdots - \mu_m x_{mn} + p_1 y_{1n} + \cdots + p_s y_{sn} \leq 0 & non-equation\ (n)\ th \\ \mu_1 x_{1o} + \cdots + \mu_m x_{mo} \leq 0 & non-equation\ (n+1)\ th \\ -\mu_1 x_{1o} - \cdots - \mu_m x_{mo} \leq 0 & non-equation\ (n+2)\ th \end{cases}$$

From the condition $\mu \mathrm{b} \geq 0$ :

$$p_1 y_{1o} + \cdots + p_s y_{so} \geq 0$$

We also know that $0 \leq (\mu_1, \dots, \mu_m, p_1, \dots, p_s) \neq 0$

Three cases should be considered:

1) If $X_j \geq 0$ , $X_j \neq 0$ , $Y_j \geq 0$ , and $Y_j \neq 0$ .

From (n+1)th and (n+2)th, we obtain:

$$\mu_1 x_{1o} + \dots + \mu_m x_{mo} = 0.$$

Because of $\mu_1, \dots, \mu_m \geq 0$ and $X_o \geq 0$, for every positive component of vector $X_o$ , there is a correspondent for μ which its amount is zero. But if $X_o$ has a zero value component, for the correspondence μ, there would be no conclusion. What above said is easily extended for every component of $X_j (j = 1, \dots, n)$ . So in this situation, no conclusion may be reached for vector $\mu = \begin{pmatrix} \mu_1, & \cdots & ,\mu_m, & p_1, & \cdots & ,p_s \end{pmatrix}$.

2) If we assume $X_o > 0$ , $Y_o > 0$ , we conclude what followed from (n+1)th and (n+2)th:

$$\mu_1 x_{1o} + \dots + \mu_m x_{mo} = 0.$$

Because of $X_o > 0$ , what resulted, will be: $\mu_1 = \dots = \mu_m = 0$

So from (o)th, it can be easily obtained:

$$p_1 y_{1o} + \dots + p_s y_{so} \leq 0 .$$

Since $Y_o > 0$ , it is concluded: $p_1 = \cdots p_s = 0$ . Therefore, the regularity condition is satisfied.

3) If we assume $X_o > 0$ , $Y_o \ngtr 0$ , we will have what followed from (n+1)th and (n+2)th:

$$\mu_1 x_{1o} + \dots + \mu_m x_{mo} = 0.$$

Because of $X_o > 0$ , it is resulted: $\mu_1 = \dots = \mu_m = 0.$

Now, if we use the assumption in the inequality (1) to (n), the conclusion will be:

$$\begin{cases} p_1 y_{11} + \dots + p_s y_{s1} \leq 0 & \textit{non} - \textit{equation the first} \\ \vdots \qquad\qquad \vdots & \\ p_1 y_{1o} + \dots + p_s y_{so} \leq 0 & \textit{non} - \textit{equation o th} \\ \vdots \qquad\qquad \vdots & \\ p_1 y_{1n} + \dots + p_s y_{sn} \leq 0 & \textit{non} - \textit{equation } (n) \textit{ th} \end{cases}$$

Moreover by multiplying (-1) in the inequality (1) to (n) the result will be:

$$\begin{cases} -p_1 y_{11} - \cdots - p_s y_{s1} \geq 0 & non-equation\ the\ first \\ \vdots \qquad\qquad\qquad \vdots & \\ -p_1 y_{1o} - \cdots - p_s y_{so} \geq 0 & non-equation\ o\ th \\ \vdots \qquad\qquad\qquad \vdots & \\ -p_1 y_{1n} - \cdots - p_s y_{sn} \geq 0 & non-equation\ (n)\ th \end{cases}$$

After summing the both parts of non-equation the above system and non-equation (n+3)th, we have: $p_1(y_{1o} - y_{11} - \cdots - y_{1o} - \cdots - y_{1n}) + \cdots + p_s(y_{so} - y_{s1} - \cdots - y_{so} - \cdots - y_{sn}) \geq 0$ , or $p_1(-y_{11} - \cdots - y_{1n}) + \cdots + p_s(-y_{s1} - \cdots - y_{sn}) \geq 0$.

For what above said, we should have: $p_1 = \cdots p_s = 0$ .

It had been got a result before: $\mu_1 = \cdots \mu_m = 0$ . So: $(\mu_1, \ldots, \mu_m, p_1, \ldots, p_s) = 0 \in R^{m+s}$ .

It purports that the system has trivial conclusion. So, it is easily claimed that the regularity condition is established.

### *3.2. Regularity Condition in BCC Model:*

Regularity condition in BCC is now examined. The envelopment form of BCC model in input oriented is being considered as follows:

$$\begin{aligned} Min \quad & \theta \\ s.t \quad & X\lambda \leq \theta x_o \\ & Y\lambda \geq y_o \\ & 1\lambda = 1 \\ & \lambda \geq 0, \end{aligned} \tag{8}$$

The dual multiplier form of this linear program is expressed as:

$$\begin{aligned} Max \quad & UY_o - u_o \\ s.t \quad & UY_o - VX_o - u_o e \leq 0 \\ & VX_o = 1 \\ & U \geq 0, V \geq 0, u_o\, free\ in\ sign \end{aligned} \tag{9}$$

where e is a row vector with all elements equal to 1.
The standardized of model (8) will be:

$$\begin{aligned} Min \quad & \theta_1 - \theta_2 \\ s.t \quad & -X\lambda + \theta_1 X_o - \theta_2 X_o \geq 0 \\ & Y\lambda \geq y_o \\ & 1\lambda \geq 1 \\ & -1\lambda \geq -1 \\ & \lambda \geq 0, \theta_1 \geq 0, \theta_2 \geq 0 \end{aligned} \tag{10}$$

It is important to know that expressed model has any solution or not.

$Ad \geq 0$ , $Cd \leq 0$ , $d \geq 0$ , $d \neq 0$

Suppose that, $d = (d_1, d_2, d_3)$ in which : $d_1 \in R^n, d_2 \in R, d_3 \in R$ .

So the above system will be:

$$\begin{cases} -Xd_1 + X_o d_2 - X_o d_3 \geq 0 \\ Yd_1 \geq 0 \\ 1d_1 = 0 \\ d_1 \geq 0, d_2 \geq 0, d_3 \geq 0 \\ (d_1, d_2, d_3) \neq (0,0,0) \end{cases}$$

Because of $d_1 \geq 0, 1d_1 = 0$ , getting the result is that $d_1 = 0$ .

If $Cd \leq 0$ :

$Cd = (0,1,-1)(d_1, d_2, d_3) = d_2 - d_3 \leq 0 \Rightarrow d_2 \leq d_3$ .

As a result:

$$\begin{cases} d_1 = 0 \\ d_2 \leq d_3 \end{cases}$$

The rest is like the CCR model.

For infecting the regularity condition of co-efficient of BCC model in input oriented, we should scrutinize the following system. As told before, in the BCC dual form (envelopment model) we have:

$$A = \begin{pmatrix} -x_{11} & \cdots & -x_{1o} & \cdots & -x_{1n} & x_{1o} & -x_{1o} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -x_{m1} & \cdots & -x_{mo} & \cdots & -x_{mn} & x_{mo} & -x_{mo} \\ y_{11} & \cdots & y_{1o} & \cdots & y_{1n} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{s1} & \cdots & y_{so} & \cdots & y_{sn} & 0 & 0 \\ 1 & \cdots & 1 & \cdots & 1 & 0 & 0 \\ -1 & \cdots & -1 & \cdots & -1 & 0 & 0 \end{pmatrix},$$

$$\mu = (\mu_1, \ldots, \mu_m, p_1, \ldots p_s, w_1, w_2 \ )$$

$b = (0, \ldots, 0, y_{1o}, \ldots, y_{so}, 1, -1)$

The conditions of the system should be established. So, we have:

$$\begin{cases} -\mu_1 x_{11} - \cdots - \mu_m x_{m1} + p_1 y_{11} + \cdots + p_s y_{s1} + w_1 - w_2 \leq 0 & non-equation\ the\ first \\ \vdots \qquad\qquad\qquad \vdots & \vdots \\ -\mu_1 x_{1o} - \cdots - \mu_m x_{mo} + p_1 y_{1o} + \cdots + p_s y_{so} + w_1 - w_2 \leq 0 & non-equation\ o\ th \\ \vdots \qquad\qquad\qquad \vdots & \vdots \\ \mu_1 x_{1n} + \cdots + \mu_m x_{mn} + p_1 y_{1n} + \cdots + p_s y_{sn} + w_1 - w_2 \leq 0 & non-equation\ (n)\ th \\ \mu_1 x_{1o} + \cdots + \mu_m x_{mo} \leq 0 & non-equation\ (n+1)\ th \\ -\mu_1 x_{1o} - \cdots - \mu_m x_{mo} \leq 0 & non-equation\ (n+2)\ th \end{cases}$$

It is got the result from non-equation (n+1)th and (n+2)th:

$$\mu_1 x_{1o} + \cdots + \mu_m x_{mo} = 0$$

Two cases should be considered:

1) If $X_j \geq 0$ , $X_j \neq 0$ , $Y_j \geq 0$ , and $Y_j \neq 0$ .

The rest is like of the CCR model.

2) Suppose $X_o > 0, Y_o > 0$ , So we have: $\mu_1 = \cdots = \mu_m = 0$

By considering the condition $\mu b \geq 0$ and $(o)_{th}$ we obtain:

$$\begin{cases} p_1 y_{1o} + \cdots + p_s y_{so} + w_1 - w_2 \leq 0 \\ p_1 y_{1o} + \cdots + p_s y_{so} + w_1 - w_2 \geq 0 \end{cases}$$

So $p_1 y_{1o} + \cdots + p_s y_{so} + w_1 - w_2 = 0.$

By supposing $w_1 = w_2 \neq 0$ : $p_1 y_{1o} + \cdots + p_s y_{so} = 0$ and because of $Y_o > 0$ ,

$$p_1 = \cdots = p_s = 0.$$

Since $w_1 = w_2 \neq 0$ , the regularity condition is not satisfied.

If $w_1 = w_2 = 0$ :

$(\mu_1, \ldots, \mu_m, p_1, \ldots, p_s, w_1, w_2) = 0 \in R^{m+s+2}$ and so, regularity condition exists.

As an important point, if $w_1 \neq w_2$ , deducing is not possible.

***4. Conclusion:***

As expressed, there is a model in lieu of any linear programming model, that called the dual form. Principles of duality appear in various branches of mathematics, physics, and statistics.

In linear programming, duality theory turns out to be of great practical use. Also, duality in linear programming admits an elegant economic interpretation. One of the most important characteristics in accordance with primal/dual form of a problem is regularity condition.

In this paper, the regularity conditions in a general form and analyzing the conditions in DEA basic models such as CCR and BCC are stated and scrutinized. Also we conclude that the examination of the conditions in CCR and BCC models, have some similarities in some aspects.

It seems that regularity conditions analyzing in DEA basic models such as CCR and BCC, because of setting forth for discussion in necessary and sufficient conditions for bounded and unbounded of optimum results of primal and dual problems, are valuable.

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