

## Ranking Trapezoidal Fuzzy Numbers by Using Hadi Method

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**Abstract:** One of the most applications of the ranking fuzzy numbers is in management, data analysis, stochastic, artificial intelligence, operations research and etc. In spite of many ranking methods, no one can rank fuzzy numbers with human intuition consistently in all cases. Shortcoming are found in some of the convenient methods for ordering fuzzy numbers such as the coefficient of variation (CV index), distance between fuzzy sets, centroid point and original point, and also weighted mean value. In this paper, we propose a new method entitled Hadi method for ordering the trapezoidal fuzzy numbers as one of the most important kind of fuzzy numbers to overcome the shortcomings of the previous methods. Finally, we shall examine the method with present two examples to illustrate the advantage of our method.

**Key words.** Linear order, ranking fuzzy numbers, trapezoidal fuzzy number.

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### INTRODUCTION

In the most decision making problems ranking fuzzy numbers plays a very important role. Various approaches have been proposed to compare the fuzzy numbers (Abbasbandy, S. and B. Asady, 2006; Asady, B. and A. Zendehnam, 2007; Chen, L.H. and H.W. Lu, 2001; Cheng, C.H., 1998; Chu, T.C. and C.T. Tsaoi, 2002; Kwang, H.L. and J.H. Lee, 1999; Lee, E.S. and R.L. Li, 1988). Some of these methods compared and reviewed in (Wang, X., E. Kerre, 2001). Lee *et al.* (1988) ranked fuzzy numbers based on two different criteria, namely, the fuzzy mean and the fuzzy spread of fuzzy numbers. They pointed that human intuition world favor a fuzzy number with the following characteristics: higher mean value and at the same time lower spread. However, when higher mean value and at the same time higher spread or lower mean value and at the same time lower spread exists, it is not easy to compare its ordering clearly. Therefore, Cheng (1998) proposed the coefficient of variance (CV index) to improve Lee and Li's ranking method (Lee *et al.* (1988). Chu (2002) pointed out the shortcomings of Cheng's method and suggested to rank fuzzy numbers with the area between the centroid point and the point of origin. This paper presents a new method for ranking the trapezoidal fuzzy numbers to overcome the shortcomings of the previous methods. Furthermore, the method will be examined by some illustrative examples the advantage of the method.

The paper has been organized in 5 sections. In Section 2, we give some basic definitions of fuzzy sets theory. In Section 3, we introduce a new ranking method to the trapezoidal fuzzy numbers. In Section 4, we use some illustrative examples to show the advantage of the proposed method. Finally, the conclusions are given in the last section.

#### 2. Preliminaries:

Here, we briefly give some fundamental definitions and concepts of fuzzy sets theory (taken from (Mahdavi-Amiri, N. and S.H. Nasseri, 2007)).

##### Definition 2.1:

If  $X$  is a collection of objects denoted generically by  $x$ , then a fuzzy set  $A$  in  $\mathbb{R}$  is defined to be a set of ordered pairs  $A = \{(x, \mu_A(x)) | x \in \mathbb{R}\}$ , where  $\mu_A(x)$  is called the membership function for

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the fuzzy set. The membership function maps each element of  $\mathbb{R}$  to a membership value between 0 and 1.

**Definition 2.2:**

The **support** of a fuzzy set  $A$  is the set of points  $x$  in  $X$  with  $\mu_A(x) > 0$ .

**Definition 2.3:**

The **core** of a fuzzy set is the set of points  $x$  in  $X$  with  $\mu_A(x) = 1$ .

**Definition 2.4:**

A fuzzy set  $A$  is called **normal** if its core is nonempty.

**Definition 2.5:**

The  $\alpha$ -cut set of a fuzzy set  $A$  is a crisp set defined by  $A_\alpha = \{x \in X, \mu_A(x) \geq \alpha\}$

**Definition 2.6:**

A fuzzy set  $A$  on  $X$  is **convex** if for any  $x, y \in X$  and any  $\lambda \in [0,1]$ , we have

$$\mu_A(\lambda x + (1-\lambda)y) \geq \min \{ \mu_A(x), \mu_A(y) \} .$$

**Definition 2.7:**

A **fuzzy number**  $A$  is a fuzzy set on the real line that satisfies the conditions of normality and convexity.

**Definition 2.8:**

A fuzzy number  $\tilde{a}$  on  $\mathbb{R}$  is said to be a trapezoidal fuzzy number, if there exist real numbers  $a^L$  and  $a^U$ ,  $a^L \leq a^U$  and  $a^\alpha, a^\beta > 0$  such that

$$\tilde{a}(x) = \begin{cases} \frac{x}{a^\alpha} + \frac{a^\alpha - a^L}{a^\alpha}, & x \in [a^L - a^\alpha, a^L] \\ 1, & x \in [a^L, a^U] \\ \frac{-x}{a^\beta} + \frac{a^U - a^\beta}{a^\beta}, & x \in [a^U, a^U + a^\beta] \end{cases} \quad (2-1)$$

where,  $\tilde{a}(x)$  is the membership function of the fuzzy number  $\tilde{a}$ . Thus, we may denote a trapezoidal fuzzy number  $\tilde{a}$  by  $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ , where the support of  $\tilde{a}$  is  $(a^L - a^\alpha, a^U + a^\beta)$  and  $[a^L, a^U]$  is its core and we denote the set of all trapezoidal fuzzy numbers with  $F(\mathbb{R})$ .

**3. Construction of a Method to Compare the Trapezoidal Fuzzy Numbers:**

In this section we shall introduce a new ranking method for trapezoidal fuzzy numbers which is very simple and efficient.

For any trapezoidal fuzzy number  $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ , define

$$\underline{a} = a^m - \frac{1}{2}h_{\underline{a}}, \bar{a} = a^m + \frac{1}{2}h_{\bar{a}}, \quad (3-1)$$

where  $a_m = \frac{a^L + a^U}{2}$ , and  $h_{\underline{a}} = \frac{a^\alpha}{a^\alpha + a^\beta}, h_{\bar{a}} = \frac{a^\beta}{a^\alpha + a^\beta}$

Now assume that  $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ ,  $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$  be two trapezoidal fuzzy numbers. Let

$$\begin{aligned} \bar{R}(\tilde{a}, \tilde{b}) &= \bar{a} - \bar{b}, \\ \underline{R}(\tilde{a}, \tilde{b}) &= \underline{a} - \underline{b}, \end{aligned} \tag{3-2}$$

where  $\underline{a}, \bar{a}, \underline{b}, \bar{b}$  are defined in (3-1).

**Lemma 3.1:**

Assume  $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ ,  $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$  be two trapezoidal fuzzy numbers. Then, we have

$$\begin{aligned} \bar{R}(\tilde{a}, \tilde{b}) &= -\bar{R}(\tilde{b}, \tilde{a}) = \bar{R}(-\tilde{b}, -\tilde{a}) \\ \underline{R}(\tilde{a}, \tilde{b}) &= -\underline{R}(\tilde{b}, \tilde{a}) = \underline{R}(-\tilde{b}, -\tilde{a}) \end{aligned} \tag{3-3}$$

**Proof:**

It is straightforward from (3-2).

**Definition 3.1:**

Assume  $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ ,  $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$  be two trapezoidal fuzzy numbers and  $\underline{R}(\tilde{b}, \tilde{a}) \geq 0$ . Define the relations  $\prec$  and  $\approx$  on  $F(\mathbb{R})$  as given below:

- i)  $\tilde{a} \approx \tilde{b}$  if and only if  $\underline{R}(\tilde{a}, \tilde{b}) = \bar{R}(\tilde{a}, \tilde{b})$ .
- ii)  $\tilde{a} \prec \tilde{b}$  if and only if  $\underline{R}(\tilde{a}, \tilde{b}) > \bar{R}(\tilde{a}, \tilde{b})$ .

**Remark 3.1:**

We denote  $\tilde{a} \preceq \tilde{b}$  if and only if  $\tilde{a} \approx \tilde{b}$  or  $\tilde{a} \prec \tilde{b}$ . Then  $\tilde{a} \preceq \tilde{b}$  if and only if  $\underline{R}(\tilde{a}, \tilde{b}) \geq \bar{R}(\tilde{a}, \tilde{b})$ . Also  $\tilde{a} \prec \tilde{b}$  if and only if  $\tilde{b} \succ \tilde{a}$ .

We let  $\tilde{0} = (0,0,0,0)$  as a zero trapezoidal fuzzy numbers. Thus any  $\tilde{a}$  such that  $\tilde{a} \approx \tilde{0}$ , is a zero too.

**Lemma 3.2:**

Assume  $\tilde{a} \prec \tilde{b}$ , then  $-\tilde{a} \succ -\tilde{b}$

**Proof:**

Since  $\tilde{a} \prec \tilde{b}$ , we have:

$$\underline{R}(\tilde{b}, \tilde{a}) > \bar{R}(\tilde{a}, \tilde{b})$$

So by use of Lemma 3.1, we have

$$\underline{R}(-\tilde{a}, -\tilde{b}) > \bar{R}(-\tilde{b}, -\tilde{a})$$

Now from Definition 3.1, we obtain  $-\tilde{b} \prec -\tilde{a}$

The following Lemma is immediately at hand by using equations (3-2) and (3-3) and is omitted the proofs here.

**Lemma 3.3:**

Assume  $\tilde{a}, \tilde{b}, \tilde{c} \in F(\mathbb{R})$ . Then

- i)  $\tilde{a} \approx \tilde{a}$ , for every  $\tilde{a}$  (reflexivity),
- ii) If  $\tilde{a} \approx \tilde{b}$ , then  $\tilde{b} \approx \tilde{a}$  (symmetry),
- iii) If  $\tilde{a} \approx \tilde{b}$  and  $\tilde{b} \approx \tilde{c}$ , then  $\tilde{a} \approx \tilde{c}$  (transitivity).

**Remark 3.2:**

In fact, the above Lemma shows that the relation  $\approx$  is an equivalence relation on  $F(\mathbb{R})$ .

Moreover, if  $\tilde{a}$  is an element of  $F(\mathbb{R})$ , the fuzzy subset of  $F(\mathbb{R})$  defined by  $[\tilde{a}] = \{\tilde{b} \in F(\mathbb{R}) \mid \tilde{a} \approx \tilde{b}\}$  is called the equivalence fuzzy set of  $\tilde{a}$ . The equivalent fuzzy set of  $\tilde{a}$  is thus the set of all elements which are equivalent to  $\tilde{a}$ .

We now discuss the topic of order relations and denote this subject which is necessary for future works. The reader will find it helpful to keep in mind that a partial order relation is valid as we shall prove it later.

**Lemma 3.4.** The relation  $\preceq$  is a partial order on  $F(\mathbb{R})$ .

**Proof.** Assume that  $\tilde{a}, \tilde{b}, \tilde{c} \in F(\mathbb{R})$ . It is enough to prove the below triple properties.

- i)  $\tilde{a} \preceq \tilde{a}$ , for every  $\tilde{a}$  (reflexivity),
- ii) If  $\tilde{a} \preceq \tilde{b}$  and  $\tilde{b} \preceq \tilde{a}$  then  $\tilde{a} \approx \tilde{b}$  (symmetry),
- iii) If  $\tilde{a} \preceq \tilde{b}$  and  $\tilde{b} \preceq \tilde{c}$ , then  $\tilde{a} \preceq \tilde{c}$  (transitivity).

The reflexivity property is valid, because

$$\tilde{a} \preceq \tilde{a} \Leftrightarrow \underline{R}(\tilde{a}, \tilde{a}) \geq \overline{R}(\tilde{a}, \tilde{a}) \Leftrightarrow \underline{a} - \underline{a} \geq \overline{a} - \overline{a}$$

For symmetry property, assume that  $\tilde{a} \preceq \tilde{b}$  and  $\tilde{b} \preceq \tilde{a}$ , then

$$\begin{cases} \tilde{a} \preceq \tilde{b} \Leftrightarrow \underline{R}(\tilde{b}, \tilde{a}) \geq \overline{R}(\tilde{a}, \tilde{b}) \Leftrightarrow \underline{b} - \underline{a} \geq \overline{a} - \overline{b} \\ \tilde{b} \preceq \tilde{a} \Leftrightarrow \underline{R}(\tilde{a}, \tilde{b}) \geq \overline{R}(\tilde{b}, \tilde{a}) \Leftrightarrow \underline{a} - \underline{b} \geq \overline{b} - \overline{a} \end{cases}$$

or,

$$\begin{cases} \tilde{a} \preceq \tilde{b} \Leftrightarrow \underline{b} - \underline{a} \geq \overline{a} - \overline{b} \\ \tilde{b} \preceq \tilde{a} \Leftrightarrow \underline{b} - \underline{a} \geq \overline{a} - \overline{b} \end{cases}$$

Now since a natural partial order exists on  $\mathbb{R}$ , therefore it follows that  $\underline{b} - \underline{a} = \overline{a} - \overline{b}$ , or  $\underline{R}(\tilde{b}, \tilde{a}) = \overline{R}(\tilde{a}, \tilde{b})$ . Hence we obtain  $\tilde{a} \approx \tilde{b}$ .

Finally, for transitivity property, assume that  $\tilde{a} \preceq \tilde{b}$  and  $\tilde{b} \preceq \tilde{c}$ . So, since  $\tilde{a} \preceq \tilde{b}$  we have:

$$\underline{R}(\tilde{b}, \tilde{a}) \geq \overline{R}(\tilde{a}, \tilde{b})$$

or equivalently,

$$\underline{b} - \underline{a} \geq \bar{a} - \bar{b} \tag{3-7}$$

Also from  $\tilde{b} \preceq \tilde{c}$ , we have:

$$\underline{R}(\tilde{c}, \tilde{b}) \geq \bar{R}(\tilde{b}, \tilde{c}) ,$$

or equivalently,

$$\underline{c} - \underline{b} \geq \bar{b} - \bar{c} \tag{3-8}$$

From (3-7) and (3-8), we obtain

$$\underline{c} - \underline{a} \geq \bar{a} - \bar{c} \tag{3-9}$$

or

$$\underline{R}(\tilde{c}, \tilde{a}) \geq \bar{R}(\tilde{a}, \tilde{c})$$

It follows that  $\tilde{a} \preceq \tilde{c}$

**Remark 3.3:**

We emphasize that the relation  $\preceq$  is a linear order on  $F(\mathbb{R})$  too, because any two elements in  $F(\mathbb{R})$  are comparable by this relation.

**Lemma 3.5:** If  $\tilde{a} \preceq \tilde{b}$  and  $\tilde{c} \preceq \tilde{d}$ , then  $\tilde{a} + \tilde{c} \preceq \tilde{b} + \tilde{d}$ .

**Proof:**

The proof is straightforward by using equation (3-2).

Now we can introduce our method entitled "Hadi method" to order all trapezoidal fuzzy numbers based on the above discussion and results.

**Algorithm 3.1.** For two trapezoidal fuzzy numbers  $\tilde{a}$  and  $\tilde{b}$ , assume that  $\tilde{b} \geq \tilde{a}$ .

Compute:  $\bar{R}(\tilde{a}, \tilde{b}) = \bar{a} - \bar{b}$  and  $\underline{R}(\tilde{b}, \tilde{a}) = \underline{b} - \underline{a}$  (it is obvious that  $\underline{R}(\tilde{b}, \tilde{a}) \geq 0$ , by this fact that  $\underline{b} \geq \underline{a}$ ).

Let  $R = \underline{R}(\tilde{b}, \tilde{a}) - \bar{R}(\tilde{a}, \tilde{b})$ . Then

if  $R = 0$ , then  $\tilde{a} \approx \tilde{b}$ ,

if  $R > 0$ , then  $\tilde{a} \prec \tilde{b}$ , else  $\tilde{b} \prec \tilde{a}$ .

**4. Numerical Examples:**

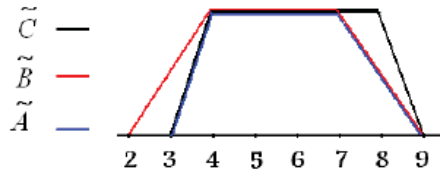
Here, we illustrate the proposed method in the last section by two examples.

**Example 4.1:**

Consider the following trapezoidal fuzzy numbers as follows:

$$\tilde{A} = (4, 7, 1, 2), \tilde{B} = (4, 7, 2, 2), \tilde{C} = (4, 8, 1, 1)$$

The result of our method for ranking of the above trapezoidal fuzzy numbers is:  $\tilde{A} \prec \tilde{B} \prec \tilde{C}$ . In Fig. 4.2, we easily can verify it.



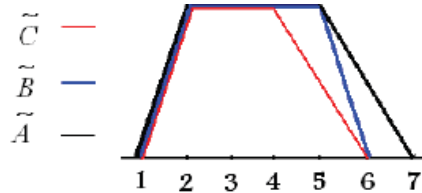
**Fig. 4.2:**

**Example 4.2:**

Consider the following trapezoidal fuzzy numbers as follows:

$$\tilde{A} = (2, 5, 1, 2), \tilde{B} = (2, 5, 1, 1), \tilde{C} = (2, 4, 1, 2)$$

The result of our method for ranking of the above trapezoidal fuzzy numbers is:  $\tilde{C} \prec \tilde{B} \prec \tilde{A}$ . In Fig. 4.3, we easily can verify it.



**Fig. 4.3:**

**5. Conclusion:**

In spite of many ranking approaches, no one can rank fuzzy numbers with human intuition consistently in all cases. Shortcomings are found in ranking fuzzy numbers with some convenient methods such as Asady, Chen method, Cheng distance and Chu-Tsao methods. We proposed a new method for ranking trapezoidal fuzzy numbers to overcome shortcomings and gave some comparative examples to illustrate the advantage of our method. Furthermore, the mentioned method will be useful for solving fuzzy linear programming problems by using ranking functions (Mahdavi-Amiri, N. and S.H. Nasseri, 2007; Mahdavi-Amiri, *et al.*, 2006).

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