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Electro-Mechanical Modelling and Load Sway Simulation of Container Cranes with Hoisting

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Abstract: In this work, a nonlinear model representing the dynamics of the load sway of a container crane is derived. The dynamics of the induction motors are also taken into consideration in addition to the simultaneous travelling, trolley and hoisting motions. The data of a small container crane is used to simulate the behavior of the load under an actual transportation plan with load hoisting. The container sway is found to be relatively high and may exceed the safe limits, especially for load hoisting and crane acceleration. Hence, the derived model can be implemented to achieve safe transportation plans. A feedback control scheme can be also developed on the basses of the derived model.

Key words: Container Cranes, Dynamic Modelling, Electro-Mechanical Systems, Load Hoisting, Simulation.

INTRODUCTION

Container cranes are widely used in ports, and are installed at quayside to handle heavy loads. The necessity to increase the travelling, trolley and hoisting speed generally induces undesirable sway of the container, and serious damage could occur during load transportation. Therefore, it is necessary to predict the sway angles of the load by an accurate dynamic model such that safe transportation planes can be selected according to the derived model. A satisfactory control scheme can be also proposed according to this model.

In the last decades, several investigators considered the modelling and control of container cranes (Sakawa,Y. and Shindo, Y., 1982; Kazuhiko Terashima, 2007). Most of existing approaches consist of a two-stage procedure: off-line trajectory/path planning, carried out in accordance with proper optimisation criteria, and on-line tracking by traditional controllers. Optimal control techniques have been widely used to address the path planning problem. Specific paths minimising traveling time, energy consumption or proper performance indexes linked to the swing angle and its derivative have been proposed in the literature. Nevertheless, due to model uncertainties and many other implementation factors, it often happens that the actual crane behavior significantly differs from the "optimal", desired, one. The usual goal is to achieve zero swing only at the end of the transport, and a two-stage control structure is often used: a "tracking" controller during the load transfer, and a "stabilizing" one to be switched on when a suitable vicinity of the destination point is achieved. Barmeshwar *et al.* (2000) proposed a nonlinear control strategy for the trolley crane system using Lyapunov method without considering the sway angle dynamics in the stability analysis. Fang et al. (2001) designed a proportional derivative (PD) controller to regulate the overhead crane system to the desired position with natural damping of sway oscillation. Liu et al. (2003) developed a fuzzy logic control with sliding mode control for an overhead crane system. Fang et al. (2001) developed a nonlinear coupling control law to stabilize a 3-DOF overhead crane by using Lasalle invariance theorem. However the parameters must be known in advance. Burg et al. (1996) used the variable transformation method to regulate the crane system. d'Andrea-Novel and Boustany (1991) proposed an adaptive feedback linearization method for mechanical systems of the overhead crane type. Karkoub and Zribi (2002) used the passivity property of mechanical system in the design of the nonlinear control of overhead crane. Ishide et al. (1991) construct a fuzzy back-propagation neural network based control. However their results have shown that the speed of the trolly is large at the desired destination.Yu et al. (1995) developed a nonlinear tracking for load position and velocity. However the results are shown for sway angle dynamics much faster than the cart motion dynamics. Another approach is developed

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by Kiss et al. (2000) in which an output feedback PD controller is used to stabilize a nonlinear crane system. Benhidjeb and Gissinger (1995) compared a fuzzy logic control system with Linear Quadratic Guassian control (LQG) for an overhead crane.Yi et al. (2003) developed a fuzzy controller for anti-swing and position control for an overhead crane based on Single Input Rule Modules (SIRMs) dynamically connected fuzzy interference model. Cheng and Chen (1996) combined a feedback linearization approach and a time delay control scheme. The time delay control is applied to complete the feedback linearizatrion for an uncertainty nonlinear system. However, the above work neglected the dynamics of the driving electrical motors, which could lead to considerable errors. Also, it is convenient to use the inputs of the driving motors as control variables.

In this work, a nonlinear dynamic model of a container crane is derived. The dynamics of the induction motors as well as the simultaneous travel, transverse and hoisting motions are taken into consideration. An actual transportation plan is used to simulate the load behavior of a small container crane. The effects of load hoisting and crane acceleration on the load sway have been explored. It is found that the sway is relatively high. The simulation results also illustrate the effectiveness of the derived model in the selection of appropriate transportation planes. This model can be also employed to apply an active vibration control, which is capable to suppress the sway of the load.

2. Dynamical Modelling:

The dynamical model of the container crane shown in Fig. 1 will be derived. The crane travels parallel to the quayside on the railway while the trolley moves in the transverse direction carrying the container. The hoisting of the container takes place during transportation. The container, which can be assumed as a suspended load from point O, will be assumed as a rigid body. Crane motors are induction motors for their simplicity and reliability. In the following analysis, the travel, hoisting and trolley motors will be modelled using the d-q technique (Al-Fares, F.S., 2008). The kinetic and potential energies of the compound electro-mechanical system will be obtained, and the dynamic equations associated with the generalized coordinates will be derived, using the Lagrangian approach. This will be derived and presented in the following subsections.

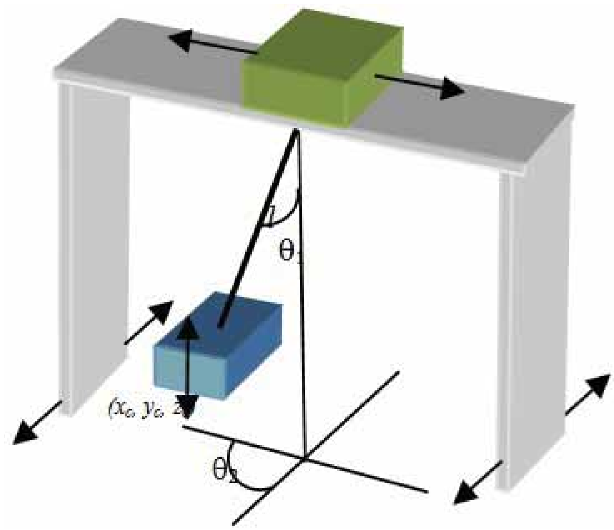

Fig. 1: Free body diagram for container crane

2.1 Lagrangian Function of the System:

The coordinates of the container x_c, y_c *and* z_c can be expressed in terms of the coordinates of the crane, *"u, v and w"*, the container sway angle, q_1 and q_2, and the length of the wire *"l"* as follows;

$$x_c = u + l\sin\theta_1\sin\theta_2,\ \ y_c = v + l\sin\theta_1\cos\theta_2,\ \ \text{and}\ \ z_c = w + l\cos\theta_1$$

The kinetic energy of the mechanical system represents the kinetic energy of the suspended container of mass M, and moment of inertia I_α and I_t , the trolley of mass m_2, the travelling girder of mass m_1 and the kinetic energy of the three driving motors of mass moment of inertia I_i and radis r_i. Hence, the kinetic energy can be written as follows;

$$K.E._m = \frac{1}{2}M\left(\dot{x}_c^2 + \dot{y}_c^2 + \dot{z}_c^2\right) + m_1\dot{u}^2 + \frac{1}{2}I_\alpha\dot{\theta}_2^2\cos^2\theta_1 + \frac{1}{2}m_2\left(\dot{u}^2 + \dot{v}^2\right)$$

$$+\frac{1}{2}I_t\left(\dot{\theta}_1^2 + \dot{\theta}_2^2\sin^2\theta_1\right) + \frac{1}{2}I_1\left(\frac{\dot{u}}{r_1}\right)^2 + \frac{1}{2}I_2\left(\frac{\dot{v}}{r_2}\right)^2 + \frac{1}{2}I_3\left(\frac{\dot{l}}{r_3}\right)^2 \quad (1)$$

The potential energy of the mechanical subsystem represents the potential energy of the suspended container, and the stored energy in the wire ropes of stiffness Q and twist angle φ. The following expression of the potential energy can be obtained;

$$P.E_m = \frac{1}{2}Q\phi^2 - Mgl\cos\theta_1 \quad (2)$$

The equivalent kinetic energy of the crane induction motors; namely the travelling motor "*r*" trolley motor "*t*", the girder motor "*g*" and the hoisting motor *"h"* can be expressed in the following equations,

$$K.E._t = \frac{1}{2}\begin{bmatrix} L_{ts}I_{ts}I_{ts}^* + L_{tr}I_{tr}I_{tr}^* \\ + L_{tm}\left(I_{ts}I_{tr}^* + I_{ts}^*I_{tr}\right)\end{bmatrix} \quad (3)$$

where L_{is} is stator inductance *s,* or rotor *r,* and I_{is} is stator current for the ith motor;

$$K.E._r = \frac{1}{2}\begin{bmatrix} L_{rs}I_{rs}I_{rs}^* + L_{rr}I_{rr}I_{rr}^* \\ + L_{rm}\left(I_{rs}I_{rr}^* + I_{rs}^*I_{rr}\right)\end{bmatrix} \quad (4)$$

$$K.E._h = \frac{1}{2}\begin{bmatrix} L_{hs}I_{hs}I_{hs}^* + L_{hr}I_{hr}I_{hr}^* \\ + L_{hm}\left(I_{hs}I_{hr}^* + I_{hs}^*I_{hr}\right)\end{bmatrix} \quad (5)$$

Now, the kinetic and potential energy of the system can be expressed in the eqs. (1-5), and the Lagrangian function can be obtained as follows;

$$L = K.E._m + K.E._t + K.E._r + K.E._h - P.E._m$$

which can be written as;

$$L = \frac{1}{2}\left[L_{ts}I_{ts}I_{ts}^* + L_{tr}I_{tr}I_{tr}^* + L_{tm}\left(I_{ts}I_{tr}^* + I_{ts}^*I_{tr}\right)\right] + \frac{1}{2}m_1\dot{u}^2 - \frac{1}{2}Q\phi^2 + Mgl\cos\theta_1$$

$$+\frac{1}{2}\left[L_{rs}I_{rs}I_{rs}^* + L_{rr}I_{rr}I_{rr}^* + L_{rm}\left(I_{rs}I_{rr}^* + I_{rs}^*I_{rr}\right)\right] + \frac{1}{2}M\left(\dot{x}_c^2 + \dot{y}_c^2 + \dot{z}_c^2\right) + \frac{1}{2}m_2\left(\dot{u}^2 + \dot{v}^2\right)$$

$$+\frac{1}{2}\left[L_{hs}I_{hs}I_{hs}^* + L_{hr}I_{hr}I_{hr}^* + L_{hm}\left(I_{hs}I_{hr}^* + I_{hs}^*I_{hr}\right)\right]$$

$$+\frac{1}{2}I_a\dot{\theta}_2^2\cos^2\theta_1+\frac{1}{2}I_t\left(\dot{\theta}_1^2+\dot{\theta}_2^2\sin^2\theta_1\right)$$
$$+\frac{1}{2}I_1\left(\frac{\dot{u}}{r_1}\right)^2+\frac{1}{2}I_2\left(\frac{\dot{v}}{r_2}\right)^2+\frac{1}{2}I_3\left(\frac{\dot{l}}{r_3}\right)^2 \quad (6)$$

The above Lagrangian equation will be used to derive the dynamic differential equation of the system.

2.2 Modelling of the Electrical Sub-System:

In the following, the Lagrangian function derived in eq. (6) will be differentiated to obtain the dynamic equation governing the transients of one of the crane induction motors. These motors are the travelling motor "*r*" trolley motor "*t*", and the hoisting motor *"h"* and the subscript "*i*" will refer to one of them. First, differentiate with respect to the stator current, the following equation can be obtained;

$$\frac{dL}{dI_{ir}}=L_{ir}I_{ir}+L_{im}I_{is}=\psi_{ir} \quad (7)$$

The right hand side of eq. (7) is the stator flux. The above equation can be differentiated with respect to time to obtain the following equation;

$$\frac{d}{dt}\left\{\frac{dL}{dI_{is}}\right\}=V_{is}-I_{is}R_{is}-j\omega_{ik}\psi_{is}=\frac{d}{dt}\{\psi_{is}\} \quad (8)$$

Regarding the rotor side, the above steps can be repeated to obtain the following equations;

$$\frac{dL}{dI_{ir}}=L_{ir}I_{ir}+L_{im}I_{is}=\psi_{ir} \quad (9)$$

$$\frac{d}{dt}\left\{\frac{dL}{dI_{ir}}\right\}=V_{ir}-I_{ir}R_{ir}-j\left(\omega_{ik}-\omega_i\right)\psi_{ir}=\frac{d}{dt}\{\psi_{ir}\} \quad (10)$$

Equation (8) and eq. (10) represent the state equation of the *i*th motor, where the flux vectors are considered as states. However, it is convenient to use the primitive machine or the direct-quadrature phase quantities, d-q representation. In the following, the above motor vector quantities are represented by its d-q components;

$$\begin{aligned}\psi_{is}&=\psi_{isd}+j\psi_{isq}, &\psi_{ir}&=\psi_{ird}+j\psi_{irq}\\ I_{is}&=I_{isd}+jI_{isq}, &I_{ir}&=I_{ird}+jI_{irq}\\ V_{is}&=V_{isd}+jV_{isq}, &V_{ir}&=V_{ird}+jV_{irq}\end{aligned} \quad (11)$$

It is to be noted that the voltage on the rotor is zero; i.e. $V_{ir}=0$

Substituting the above relationships into eq. (8) and eq. (10), the following state equations can be written;

$$\dot{\psi}_{isd}=-\alpha_{1i}\psi_{isd}+\omega_{ik}\psi_{isq}+\alpha_{2i}\psi_{ird}+V_{isd}$$

$$\dot{\psi}_{isq} = -\omega_{1i}\psi_{isd} \quad -\alpha_{ik}\psi_{isq} + \quad \alpha_{2i}\psi_{irq} \quad +V_{isd} \tag{12}$$

$$\dot{\psi}_{ird} = \alpha_{4i}\psi_{isd} + \left\{\omega_{ik} - \omega_i\right\}\psi_{irq} + \alpha_{3i}\psi_{ird}$$

$$\dot{\psi}_{irq} = -\left\{\omega_{ik} - \omega_i\right\}\psi_{isd} + \quad \alpha_{4i}\psi_{irq} + \quad \alpha_{3i}\psi_{ird}$$

The above equations represent the state equations of the ith motor, and can be written in matrix form as follows;

$$\dot{X}_i = G_i X_i + V_i \ , \qquad i = r, t, h \tag{13}$$

where; $X_i^T = \begin{bmatrix} \psi_{isd} & \psi_{isq} & \psi_{ird} & \psi_{irq} \end{bmatrix}$, (14)

$V_i^T = \begin{bmatrix} V_{isd} & V_{isq} & 0 & 0 \end{bmatrix}$, and

$$G_i = \begin{bmatrix} -\alpha_{1i} & \alpha_{ik} & \alpha_{2i} & 0 \\ -\alpha_{ik} & -\alpha_{1i} & 0 & \alpha_{2i} \\ \alpha_{4i} & \left(\alpha_{ik} - \alpha_i\right) & -\alpha_{3i} & 0 \\ -\left(\omega_{ik} - \omega_i\right) & \alpha_{4i} & 0 & -\alpha_{3i} \end{bmatrix}$$

The above state eq. (13), contains four states. The dynamics of the rotor and the motor torque are included in the following fifth equation;

$$F_i = \left\{\frac{\alpha_{5i}}{r_i}\right\}\left[\psi_{isd}\psi_{irq} - \psi_{ird}\psi_{isq}\right] \tag{15}$$

2.3 Modelling of the Mechanical Sub-system:

Lagrange's equation will be used to obtain the differential equations associated with the generalized coordinates. Five generalized coordinates are associated with the mechanical side of the compound electromechanical system, while twelve coordinates are associated with the electrical part. The details of the state equations of the induction motors are given in section 2.2, while the derivation of the state equations for the mechanical side are given in the present section. The following five equations are associated with the mechanical generalized coordinates;

$$\begin{aligned} &A_{11}\ddot{u} + A_{13}\ddot{l} + A_{14}\ddot{\theta}_1 + C_{135}\dot{l}\dot{\theta}_2 + C_{134}\dot{l}\dot{\theta}_1 \\ &+C_{145}\dot{\theta}_1\dot{\theta}_2 + C_{155}\dot{\theta}_2^2 + C_{144}\dot{\theta}_1^{\,2} + A_{15}\ddot{\theta}_2 = F_r \end{aligned} \tag{16}$$

$$\begin{aligned} &A_{22}\ddot{v} + A_{24}\ddot{\theta}_1 + A_{25}\ddot{\theta}_2 + C_{235}\dot{l}\dot{\theta}_2 + C_{234}\dot{l}\dot{\theta}_1 \\ &+C_{245}\dot{\theta}_1\dot{\theta}_2 + C_{255}\dot{\theta}_2^2 + C_{244}\dot{\theta}_1^{\,2} + A_{23}\ddot{l} = F_t \end{aligned} \tag{17}$$

$$A_{31}\ddot{u} + A_{32}\ddot{v} + A_{33}\ddot{l} + C_{355}\dot{\theta}_2^2 + C_{344}\dot{\theta}_1^{\,2} = Mg - F_h \tag{18}$$

$$A_{41}\ddot{u} + A_{42}\ddot{v} + A_{44}\ddot{\theta}_1 + C_{434}\dot{l}\dot{\theta}_1 + C_{455}\dot{\theta}_2^2 + D_{44} = 0 \qquad (19)$$

$$A_{51}\ddot{u} + A_{52}\ddot{v} + A_{55}\ddot{\theta}_2 + C_{535}\dot{l}\dot{\theta}_2 + C_{545}\dot{\theta}_2\dot{\theta}_1 + D_{55}\theta_2 = -Q\phi \qquad (20)$$

where "A_{ij}" represents the elements of the effective inertia matrix "*A*", which can be found in Appendix. On the other side, the element "C_{ijk}" represents the coefficient of the dynamic force (centripetal or coriolis) at coordinate "*i*" due to the velocities at "*j*" and "*k*". These elements can be obtained from the $C(q,\dot{q})$, which can be found also in Appendix. Regarding the gravity loading matrix *D*, every diagonal element "D_{ii}" represents the gravity loading at joint "*i*". All the elements of this matrix are zero except "$D_{44} = Mgl\sin\theta_1$". The above equations can be written as follows;

$$A(q)\ddot{q} + C(q,\dot{q}) + D(q) = F_g \qquad (21)$$

3. System Simulation:

Crane accelerations $\ddot{u}$, $\ddot{v}$ and $\ddot{l}$ re decided according to the transportation plan. Therefore, the motor inputs can be calculated by using the nonlinear electromechanical model. The above equations will be used to simulate the response of the considered crane for a specified transportation plan.

3.1. First Transportation Plan:

The proposed plan is to move the crane with a constant acceleration of 0.3 m/sec^2, and the trolley with a constant acceleration of 0.15 m/sec^2 for eight seconds. Then, the crane and trolley moves with a constant velocity for another eight seconds. Finally, the crane moves with a constant deceleration of 0.3 m/sec^2, and the trolley moves with a constant deceleration of 0.15 m/sec^2 for anoth eight seconds. Since in practice the suspension ropes are not wound while the crane is in motion for saftey considerations (Kim, YS. 2004), no-hoisting of the load will be considered in this case. It is clear that the sway q_1 angle is increasing. It can be also noticed that the frequency of oscillation of q_1 is about 0.7 Hz, which can be predicted from the system data. From Fig.2, it can be seen that q_1 reaches 0.05 radians within 24 seconds. This indicates that the contianer vibrates with an amplitude of 25 cm. In the next subsection, the hoisting of the load will be considered.

3.2. Second Transportation Plan (Considering Load Hoisting):

In this transportation plane, the crane motion and trolly motion is the same as the first transportation plan. The hoisting of the load is carried out with a constant acceleration of 0.2 m/sec^2 for one second and moves with a constant velocity for 22 seconds. Finally, the lowering of the load is carried out with with a constant acceleration of 0.2 m/sec^2. It is clear from Fig. 2 that the load sway angle q_1 is higher and exceeds 0.07 radians which may be dangerous. In the third example the effect of hoisting of load with high acceleration for both crane and trolley will be considered.

Third Transportation Plan (Load Hoisting with High Crane Acceleration);

In this transportation plane, the hoisting scheme is the same as the second transportation plan. Regarding the crane and the trolley the acceleration is twice that of the first case. The crane moves with a constant acceleration of 0.6 m/sec^2, and the trolley with a constant acceleration of 0.3 m/sec^2 for three seconds. Then, the crane and trolley moves with a constant velocity for another 18 seconds. Finally, the girder moves with a constant deceleration of 0.6 m/sec^2, and the trolley moves with a constant deceleration of 0.3 m/sec^2 for anoth three seconds. It is clear from Fig. 2 that the sway angle q_1 is very high and exceeds 0.35 radians with 24 seconds. This state much be avioded to achieve safe transportation.

Table 1: Data of the Considered Crane.

m_1 =1000000 kg., m_1=10000 kg., M= 7500 kg., I_a= 600 $kg.m^2$, I_t = 600 kg. m^2, l = 5 m., Q = 150 N.m/rad.

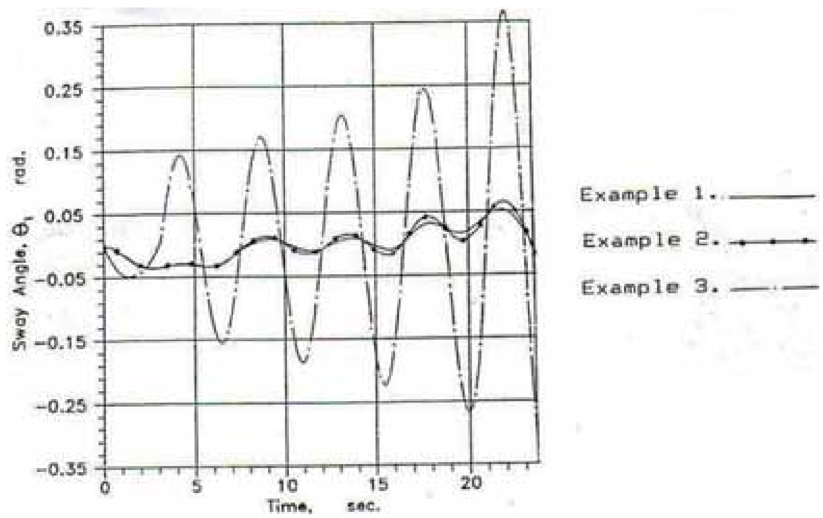

Fig. 2: Simulation Results for the different three examples.

6. Conclusions:

A nonlinear dynamical model of the electromechanical system of container crane has been developed to represent the load sway dynamics, using the Lagrangian approach. This dynamical model considers the induced sway of the suspended load due to the simultaneous motions of the girder, trolley and the hoisting of the load. The dynamics of the induction motors are also taken into consideration in addition to the afromentioned motions. The considered sway is induced not only in the plane of motion, but also in the plane determined by the ropes and the vertical axis through the suspension point.

The derived model is adopted to simulate the response of an actual container crane. The data of the crane is used to simulate the behaviour of the load under an actual transportation plan. Three simulation examples are then presented to illustrate the system response for different transportation planes. The container sway is found to exhibit persistent oscillations especially when the load hoisting is considered. The high acceleration rates for the crane and trolley also increase the load sway. The results show the effectiveness of the derived model in selecting appropriate transportation planes.

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Appendix System Matrices

In this Appendix, the following abbreviations will be used;

$S\theta_1 = \sin\theta_1$, $S\theta_2 = \sin\theta_2$, $C\theta_1 = \cos\theta_1$ and $C\theta_2 = \cos\theta_2$

$$A(q) = \begin{bmatrix} M_u & 0 & M\,S\theta_1 S\theta_2 & Ml C\theta_1 S\theta_2 & Ml S\theta_1 C\theta_2 \\ 0 & M_v & M S\theta_1 C\theta_2 & Ml C\theta_1 C\theta_2 & -Ml S\theta_1 S\theta_2 \\ M S\theta_1 S\theta_2 & M S\theta_1 C\theta_2 & M_l & 0 & 0 \\ Ml C\theta_1 S\theta_2 & Ml C\theta_1 C\theta_2 & 0 & I_{\theta 1} & 0 \\ Ml S\theta_1 C\theta_2 & -Ml S\theta_1 S\theta_2 & 0 & 0 & I_{\theta 2} \end{bmatrix}$$

$$C = M\begin{bmatrix} 2\dot{l}\dot{\theta}_2 S\theta_1 C\theta_2 + 2\dot{l}\dot{\theta}_1 C\theta_1 S\theta_2 + 2l\dot{\theta}_1\dot{\theta}_2 C\theta_1 C\theta_2 - l\dot{\theta}_2^2 S\theta_1 S\theta_2 - l\dot{\theta}_1^2 S\theta_1 S\theta_2 \\ -2\dot{l}\dot{\theta}_2 S\theta_1 S\theta_2 + 2\dot{l}\dot{\theta}_1 C\theta_1 C\theta_2 - 2l\dot{\theta}_1\dot{\theta}_2 C\theta_1 S\theta_2 - l\dot{\theta}_2^2 S\theta_1 C\theta_2 - l\dot{\theta}_1^2 S\theta_1 C\theta_2 \\ l\left(\dot{\theta}_2^2 \sin\theta_1 + \dot{\theta}_1^2\right) \\ 2l\dot{l}\dot{\theta}_1 + l^2\dot{\theta}_2^2 \sin\theta_1 \\ 2l\dot{l}\dot{\theta}_2(\sin\theta_1)^2 + \left[\left(2Ml^2 - 2I_1 + 2I_2\right)/M\right]\dot{\theta}_2\dot{\theta}_1 \sin\theta_1 \end{bmatrix}$$

$$F_E^T = [E_r X_r, \quad E_t X_t, \quad -E_h X_h]$$