Australian Journal of Basic and Applied Sciences, 4(7): 2081-2092, 2010
ISSN 1991-8178

Weight Optimisation of Structures under Displacment Limitations by Interpolating Constraints

[1]Khamlichi Abdellatif, [2]Abboud Nabil, [3]El Bakkali Larbi, [4]Limam Ali

[1]Equipe Modélisation et Simulation des Systèmes Mécaniques, FS-UAE/E28, Département de Physique, Faculté des Sciences de Tétouan, BP 2121, M'hannech, Tétouan, Maroc Phone: +21267795068, Fax:+21239350702,
[2]Equipe Modélisation et Simulation des Systèmes Mécaniques, FS-UAE/E28, Département de Physique, Faculté des Sciences de Tétouan, BP 2121, M'hannech, Tétouan, Maroc Phone: +212669238872, Fax:+21239350702,
[3]Equipe Modélisation et Simulation des Systèmes Mécaniques, FS-UAE/E28, Département de Physique, Faculté des Sciences de Tétouan, BP 2121, M'hannech, Tétouan, Maroc Phone: +212665309390, Fax:+21239350702,
[4]LGCIE, 23INSA Lyon, Bâtiment Coulomb 20, Avenue A Einstein 9662,1 Villeurbanne Cedex, France, Phone: +33 6 37 69 98 25, Fax: +33 4 72 43 85,

Abstract: Structural optimisation problems are very complex to solve in practice. Even in the simplest case where the shape of the structure is given and the optimisation is only about some discrete sizing variables, the associated mathematical program is very intricate to formulate since the problem lacks in general to have a closed analytical form. When an explicit formulation could be derived, the resulting problem is habitually non convex and non linear, some extra difficulties may then attend using the common numerical tools to solve the mathematical program. Weight minimisation of continuous beams under displacement limitations pertains to this last category of structural optimisation problems. In this work, a methodology based on performing interpolations of the intervening problem constraints is developed in order to formulate an approximate mathematical program. Constraint interpolations are obtained by using quadratic polynomials and finite element computations. By considering special embedded domains where these interpolations are sequentially performed, a series of approximate problems is then constructed. If a feasible initial domain is used, this sequence is shown to yield through numerical experiments, after only a few number of iterations, an arbitrary close estimate of the exact original problem solution.

Key words: Structural optimisation, Finite element method, Continuous beams, Three angles theorem, Interpolation, SQP algorithm

INTRODUCTION

Structural optimisation is among research fields where important efforts of investigation are currently being carried out. Various recent works have been devoted to this kind of problems which may take various forms (Begg and 2000; Boggs and Tolle, 1996; Gil *et al.,* 2004; Hansen and Horst, 2008; Khosravi *et al.,* 2008; Li *et al.,* 2009; Hansen and Horst, 2008). In general, one distinguishes between first level structural optimisation or topology optimisation and second level optimisation or sizing optimisation, (Hansen and Horst, 2008). Only this last category is considered in the following for which various numerical solution methods have been proposed in the literature (Boggs and Tolle, 1996) and (Perez and Behdinan, 2007).

When the structure shape is fixed, the objective of structural optimisation is to evaluate n sizing or design variables $x = \{x_1, x_2, ..., x_n\}$ where X_i is in general some mechanical or geometrical property such as: transverse dimensions of a bar, thickness of a plate, Young's modulus, etc…

Corresponding Author: Khamlichi Abdellatif, Equipe Modélisation et Simulation des Systèmes Mécaniques, FS-UAE/E28, Département de Physique, Faculté des Sciences de Tétouan, BP 2121, M'hannech, Tétouan, Maroc
Phone: +21267795068, Fax:+21239350702,
E-mail: khamlichi7@yahoo.es

Design variables must minimize the cost function which has the general form $f(x) = f(x_1, x_2, ..., x_n)$ where f is for instance the weight, the cost of mounting or any other function obtained by combining some of the structure cost criteria.

Design variables must also satisfy a set of m design constraints having the general form: $g_i(x) = g_i(x_1, x_2, ..., x_n) \leq 0$ with i=1,...,m. Practically most of the constraint equations refer to constraints imposed on displacements or stresses. These constraints are function of the intrinsic rigidity or resistance of materials used to build the structure, its geometry, the applied loads and the boundary conditions. The constraints respond to design criteria specifications as mentioned in universal structural codes. In the following attention will be focused only on the particular type of constraints holding limitations of structure's displacements.

Considering the case where the structure is subjected to static loads, the principle of virtual work gives a variationnal formulation and could be used to compute via finite element method displacements and stresses once the design variables are specified. Finding the optimal design variables with regards for example to the total weight minimisation of the structure is a major objective for designers. But, in practice this problem is very delicate to formulate and to solve. It is easy in fact to compute the structural response associated to the selected design variables, but the inverse problem which aims at finding the design variables enabling to get a given response is not obvious. Solving structural optimisation problems requires having the ability to get the functions f and g appearing in the symbolic form of the mathematical optimisation problem as function of the applied loads and material properties. But, we do not know in general how to write in closed and explicit forms stresses or displacements as function of loads and design variables.

In the particular case of structural optimisation related to weight minimisation of a continuous beam under the limit service state where deformations are prescribed to not exceed some given thresholds, the situation is very well since we can write the optimisation problem under an explicit form by using for instance the three angles theorem, (Timoshenko, 1970). The obtained problem is however non convex and strongly non linear, (Ciarlet, 1998). It is not easy to solve with ordinary optimisation tools. Moreover, due to the fact that the design variables are dissimilar: geometrical properties, material characteristics, the problem may suffer ill conditioning and numerical methods may fail to track the correct solution.

There exists now no general methodology enabling for systematic formulation and solution of structural optimisation problems and each situation must be dealt with apart. In reference (Gil *et al.,* 2004) the authors have presented an approach which permits formulating and solving such problems by using an interpolation technique. This enables getting an approximate problem intended to be representative of the exact original one. So, solution of the approximate problem is expected to constitute an approximation of the exact problem solution. They have applied this technique in order to optimize a corbel and a reinforced tunnel. They have verified in the first case that the calculated optimum is close to the analytic solution. These authors have not performed analysis about the influence on the obtained numerical solution due to variations that may affect the domain used for operating interpolations in order to derive the approximate problem constraints. Analysis of convergence conditions of the interpolation process towards the exact problem solution has not also been done. In this work, attention will be focused on these two key points emerging within the framework of this methodology of approximating structural optimisation problems by performing interpolation of constraints.

The model problem considered in the present work is related to weight minimisation of a continuous beam which consists of three spans which are loaded each one with a concentrated force acting on the middle point of the span. Optimisation constraints are related to the mid-span point's displacements which must not exceed the code prescribed thresholds. The problem can be formulated in an exact way by means of the three angles theorem, (Timoshenko, 1970) and its solution can be achieved by using the SQP algorithm (*Sequential Quadratic Programming*), (Boggs and Tolle, 1996) which is readily programmed under the Matlab command *fmincon*.

In order to consider feasibility of solving this particular problem by using the technique of constraints interpolation, displacements are considered to be directly calculated by a finite element based model of the structure. Closed analytical forms of functions g are derived then by operating interpolations of the obtained displacements appearing in the problem constraints. The key advantage of this technique is that it yields a systematic way to derive an arbitrary approximate optimisation problem, once the process of interpolation is specified, even if the explicit forms of constraints are unknown. The only thing to undertake then is to assess how the approximate problem will yield the exact original problem solution. Here, a methodology consisting of defining a sequence of such approximate problems by operating interpolation on embedded domains is presented in this work. Quadratic polynomials interpolating mid-span displacements in terms of the design variables are used. Convergence is shown to occur towards the exact problem solution when some precautions are taken up regarding the choice of the initial domain of constraints interpolation.

Exact Solution of Weight Minimisation of a Continuous Beam under Displacements Constraints:

A continuous beam having three spans of respective lengths L_1, L_2 and L_3 is shown in figure 1.

The beam transverse section is assumed to be rectangular with uniform width b and variable height h_i, i=1,2,3 depending on the considered span. It is assumed that each span is subjected to a concentrated force applied at the mid-section of the span and having intensity. P_i i=1,2,3. The beam material is supposed to be linear elastic with Young's modulus E.

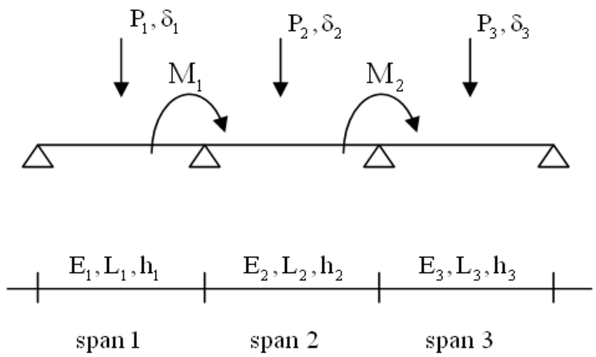

Fig. 1: A three spans simply supported beam

The considered mathematical problem which is dealt with in this work states as follows:
Find the optimal three design variables X=h_1, y=h_2 and z=h_3 *which minimise the total beam weight:*

$\rho b(L_1h_1 + L_2h_2 + L_3h_3)$ *, where* ρ *is the beam material density, by considering only the limit state of deformation for which the mid-span displacements are bounded by the admissible thresholds fixed according to a design code by the given values* δ_a^i *,* i=1,2,3.

It could be noticed that the cost function associated to this problem simplifies to $f(x,y,z)=L_1x+L_2y+L_3z$ because the quantity ρb is assumed to be constant.

In this optimisation problem, the constraints are related to the value of the mid-span displacement which must be less or equal to a given threshold value for each considered beam span. No other constraints, in particular those related to admissible stresses, are taken into account. This is supposed in order to get simple algebra intervening in the problem. One should notice that no extra technical difficulties appear if the more general case of structural optimisation problem, where constraints on both displacements and stresses are present, is considered.

Denoting M^1 and M^2 the flexural moments acting on the intermediate supports, figure 1, the three angles theorem enables to get explicit expressions of the mid-span displacements under the following form

$$\begin{cases} \delta_1 = \dfrac{P_1L_1^3}{48EI_1} + \dfrac{M_1L_1^2}{16EI_1} \\ \delta_2 = \dfrac{P_2L_2^3}{48EI_2} + \dfrac{M_1L_2^2}{16EI_2} + \dfrac{M_2L_2^2}{16EI_2} \\ \delta_3 = \dfrac{P_3L_3^3}{48EI_3} + \dfrac{M_2L_3^2}{16EI_3} \end{cases} \tag{1}$$

with

$$I_1 = \frac{b_1 h_1^3}{12} \quad , \quad I_2 = \frac{b_2 h_2^3}{12} \quad , \quad I_3 = \frac{b_3 h_3^3}{12}$$

$$\frac{-3}{8\Delta}\left\{\frac{2P_1L_1^2L_2}{I_1I_2} + \frac{2P_1L_1^2L_3}{I_1I_3} + \frac{P_2L_2^3}{I_2^2} + \frac{2P_2L_2^2L_3}{I_2I_3} - \frac{P_3L_2L_3^2}{I_2I_3}\right\}$$

$$M_2 = \frac{-3}{8\Delta}\left\{\frac{2P_1L_1^2L_2}{I_1I_2} + \frac{2P_3L_3^2L_1}{I_1I_3} + 2\frac{P_3L_3^2L_2}{I_2I_3} + \frac{P_2L_2^3}{I_2^2} - \frac{P_1L_1^2L_2}{I_1I_2}\right\}$$

$$\Delta = 4\left(\frac{L_1L_2}{I_1I_2} + \frac{L_1L_3}{I_1I_3} + \frac{L_2L_3}{I_2I_3}\right) + \frac{3L_2^2}{I_2^2}$$

By choosing the admissible displacement thresholds to be given by

$$\begin{cases} |\delta_1| \le \delta_a^1 = \dfrac{L_1}{250} \\ |\delta_2| \le \delta_a^2 = \dfrac{L_2}{250} \\ |\delta_3| \le \delta_a^3 = \dfrac{L_3}{250} \end{cases} \qquad (2)$$

the constraints write

$$\begin{cases} g_1(x,y,z) \le 0 \\ g_2(x,y,z) \le 0 \\ g_3(x,y,z) \le 0 \\ g_4(x,y,z) \le 0 \\ g_5(x,y,z) \le 0 \\ g_6(x,y,z) \le 0 \end{cases} \qquad (3)$$

with

$$\begin{aligned} g_1(x,y,z) &= \alpha_1^1 y^9 + \alpha_2^1 y^6 x^3 + \alpha_3^1 y^6 z^3 + \alpha_4^1 y^3 x^3 z^3 + \alpha_5^1 y^6 + \alpha_6^1 y^3 x^3 \\ &+ \alpha_7^1 y^3 z^3 + \alpha_8^1 x^3 z^3 + \alpha_9^1 x^6 z^3 + \alpha_{10}^1 x^6 y^3 + \alpha_{11}^1 y^3 z^6 + \alpha_{12}^1 x^3 z^6 \\ g_2(x,y,z) &= \alpha_1^2 y^9 + \alpha_2^2 y^6 x^3 + \alpha_3^2 y^6 z^3 + \alpha_4^2 y^3 x^3 z^3 + \alpha_5^2 y^6 + \alpha_6^2 y^3 x^3 \\ &+ \alpha_7^2 y^3 z^3 + \alpha_8^2 x^3 z^3 + \alpha_9^2 x^6 z^3 + \alpha_{10}^2 x^6 y^3 + \alpha_{11}^2 y^3 z^6 + \alpha_{12}^2 x^3 z^6 \\ g_3(x,y,z) &= \alpha_1^3 y^9 + \alpha_2^3 y^6 x^3 + \alpha_3^3 y^6 z^3 + \alpha_4^3 y^3 x^3 z^3 + \alpha_5^3 y^6 + \alpha_6^3 y^3 x^3 \\ &+ \alpha_7^3 y^3 z^3 + \alpha_8^3 x^3 z^3 + \alpha_9^3 x^6 z^3 + \alpha_{10}^3 x^6 y^3 + \alpha_{11}^3 y^3 z^6 + \alpha_{12}^3 x^3 z^6 \\ g_4(x,y,z) &= \alpha_1^4 y^9 + \alpha_2^4 y^6 x^3 + \alpha_3^4 y^6 z^3 + \alpha_4^4 y^3 x^3 z^3 + \alpha_5^4 y^6 + \alpha_6^4 y^3 x^3 \\ &+ \alpha_7^4 y^3 z^3 + \alpha_8^4 x^3 z^3 + \alpha_9^4 x^6 z^3 + \alpha_{10}^4 x^6 y^3 + \alpha_{11}^4 y^3 z^6 + \alpha_{12}^4 x^3 z^6 \end{aligned} \qquad (4)$$

$$g_5(x,y,z) = \alpha_1^5 y^9 + \alpha_2^5 y^6 x^3 + \alpha_3^5 y^6 z^3 + \alpha_4^5 y^3 x^3 z^3 + \alpha_5^5 y^6 + \alpha_6^5 y^3 x^3$$
$$+\alpha_7^5 y^3 z^3 + \alpha_8^5 x^3 z^3 + \alpha_9^5 x^6 z^3 + \alpha_{10}^5 x^6 y^3 + \alpha_{11}^5 y^3 z^6 + \alpha_{12}^5 x^3 z^6$$

$$g_6(x,y,z) = \alpha_1^6 y^9 + \alpha_2^6 y^6 x^3 + \alpha_3^6 y^6 z^3 + \alpha_4^6 y^3 x^3 z^3 + \alpha_5^6 y^6 + \alpha_6^6 y^3 x^3$$
$$+\alpha_7^6 y^3 z^3 + \alpha_8^6 x^3 z^3 + \alpha_9^6 x^6 z^3 + \alpha_{10}^6 x^6 y^3 + \alpha_{11}^6 y^3 z^6 + \alpha_{12}^6 x^3 z^6$$

where constants $\alpha_i^j;\ i = 1,...,12;\ j = 1,...,6$ are given in appendix 1.

One can verify that the constraints (3) are non linear and non convex. The non linear mathematical program describing the actual structural optimisation problem writes

$$\begin{cases} \text{Find } (h_1,h_2,h_3) \in R^3 \text{ such as} \\ f(h_1,h_2,h_3) = \inf f(x,y,z) \\ g_1(x,y,z) \le 0 \\ g_2(x,y,z) \le 0 \\ g_3(x,y,z) \le 0 \\ g_4(x,y,z) \le 0 \\ g_5(x,y,z) \le 0 \\ g_6(x,y,z) \le 0 \\ -x \le 0 \\ -y \le 0 \\ -z \le 0 \end{cases} \qquad (5)$$

The command *fmincon* of Matlab enables to solve straightforwardly problem (5). The obtained solution is designated in the following as the exact solution because it is related to the exact form of the optimisation problem without any further approximations applied on functions f or g. Depending on the considered initialisation, there exists however some risk because the problem is non convex to obtain for the actual problem via the command *fmincon* a local minimum rather than the global minimum. To reduce this risk various initialisations are performed in order to locate carefully the global minimum.

Approximate Solution of the Optimisation Problem by the Technique of Interpolated Constraints:

It is not always possible to give explicit representation of the structural optimisation problem under a closed form like in equation (5). In the following a general approach is presented with the aim to deal in a systematic way with any structural optimisation problem. A sequence of approximate optimisation problems having solutions that are intended to converge towards the exact problem solution is constructed. Each approximate problem is built via an interpolation procedure using finite element computational results obtained on a discrete set of interpolation points. The interpolation field domains are built in an embedded manner such as they constitute a decreasing sequence regarding the inclusion order and the sequence tends towards a point defining a neighbourhood of the exact solution as the order of terms increases. In the subsequent the interpolation is introduced for a fixed domain before considering in section 4 the recursive process of interpolations that are performed on embedded domains and studying convergence of the approximate problems sequence towards the exact solution.

For a given span of the continuous beam denoted by index n, the interpolating polynomial for the beam mid-span displacement is assumed to be a complete quadratic polynomial in terms of the unknowns: $x=h_1$, $y=h_2$ et $z=h_3$. The interpolating polynomial contains a constant and nine monomials corresponding to:

$x, y, z, xy, yz, zx, x^2, y^2, z^2$. This polynomial writes

$$P_n(x,y,z) = a_1^n + a_2^n x + a_3^n y + a_4^n z + a_5^n xy + a_6^n yz + a_7^n zx + a_8^n x^2 + a_9^n y^2 + a_{10}^n z^2 \qquad (6)$$

The interpolation coefficients $a_i^n, i = 1,...,10$ of the interpolating polynomial associated to span n must satisfy the following equation

$$a_1^n + a_2^n x + a_3^n y + a_4^n z + a_5^n xy + a_6^n yz + a_7^n zx + a_8^n x^2 + a_9^n y^2 + a_{10}^n z^2 = \delta^n \tag{7}$$

where δ^n is the mid-span displacement of beam span number n as it is evaluated by finite element method. Once coefficients $a_i^n, i = 1,...,10$ are calculated by inverting in a least square sense a system of equations having the form of equation (7), the constraints $\tilde{g}_n(x, y, z) \le 0$, $n = 1,...,6$ write

$$\tilde{g}_n(x, y, z) = P_n(x, y, z) - \delta_a^n \le 0 \text{ and } \tilde{g}_{n+1}(x, y, z) = -P_n(x, y, z) - \delta_a^n \le 0 \tag{8}$$

The approximate mathematical program writes then

$$\begin{cases} \text{Find } (h_1, h_2, h_3) \in R^3 \text{ such as} \\ f(h_1, h_2, h_3) = \inf f(x, y, z) \\ \tilde{g}_1(x, y, z) \le 0 \\ \tilde{g}_2(x, y, z) \le 0 \\ \tilde{g}_3(x, y, z) \le 0 \\ \tilde{g}_4(x, y, z) \le 0 \\ \tilde{g}_5(x, y, z) \le 0 \\ \tilde{g}_6(x, y, z) \le 0 \\ -x \le 0 \\ -y \le 0 \\ -z \le 0 \end{cases} \tag{9}$$

The approximate problem (9) can be solved as in the case of the exact problem (5) by using the Matlab command *fmincon*.

The interpolation polynomials for mid-span displacements depend on the particular choice of discrete design values $x=h_1$, $y=h_2$ and $z=h_3$ used during calculation of the polynomials coefficients. A particular choice is associated to a triplet (x_i, y_j, z_k) with $i, j, k \in \{1, 2, 3\}$. Due to the fact that the exact solution lays always in a polyhedron, one can retain for the triplets (x_i, y_j, z_k) the summits and the mid points of the segments defined by the summits of a particular polyhedron contained in $\mathbb{R}^3$ and having the simple form of a parallelepiped, figures 2 and 3. One can have then at its disposition 27 equations of the same form than equation (7) after performing 27 computations of mid-span displacements by means of the finite element method.

Due to the fact that only ten coefficients $a_i^n, i = 1,...,10$ are to be evaluated, inversion of the 27×10 system of equations must be performed in the least square sense.

Recursive Algorithm Proposed to Compute the Optimal Solution:

When written under the form of equation (9), the structural optimisation problem associated to the weight minimisation of a continuous beam under displacement constraints is not valid if the domain of parameters does not satisfy the simplified theory of beams requirements. Accordingly, beam thickness must not be very thin or very thick. If this is not the case another structural theory must be applied to formulate the problem (shell theory in the first case and complete 3D theory in the second case). Regarding these new constraints on beam design variables, the initial largest domain which contains the solution can be chosen under the following form

$$x_{min} = \frac{L_1}{100} \le x \le x_{max} = \frac{L_1}{5}$$
$$y_{min} = \frac{L_2}{100} \le y \le y_{max} = \frac{L_2}{5} \qquad (10)$$
$$z_{min} = \frac{L_3}{100} \le z \le z_{max} = \frac{L_3}{5}$$

This domain is a parallelepiped in $\mathbb{R}^3$ space, figure 2.

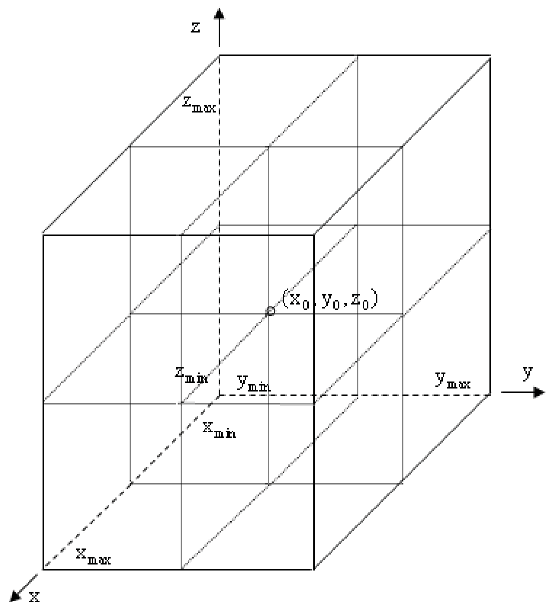

Fig. 2: An initial parallelepiped divided into eight regular parallelepipeds

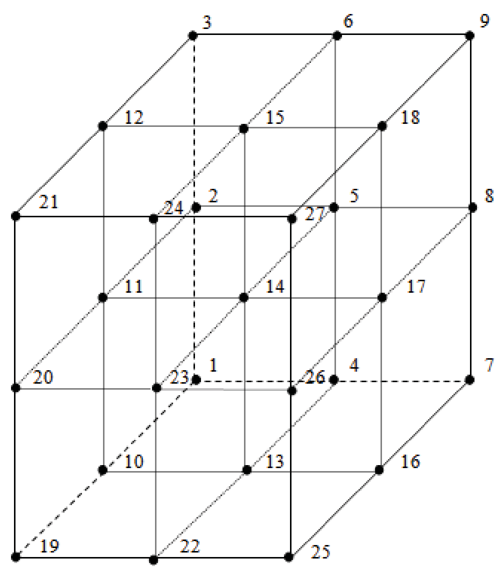

Fig. 3: Nodes used for writing equations on the interpolation coefficients

Solution of equations (9) and (10) by means of Matlab command *fmincon* when the initialisation is performed by the parallelepiped centre: $\left(x_0, y_0, z_0\right)$ does not yield at the first iteration the exact solution because interpolation is considered over a large domain and a closed representation of problem constraints is not yet achieved. For the approximate problem to be effectively close to the exact problem, the interpolation must be performed on a small domain around the exact solution.

To enhance accuracy of the approximate problem it is needed to reduce the interpolation domain and to perform interpolation of constraints in the neighbourhood of the exact solution. It is here proposed to achieve this task by dividing the initial parallelepiped defined by equation (10) into eight regular parallelepipeds, figure 2. Solution of equation (9) with the new actualised constraints over each parallelepiped enables to compute the minimum of minima of the cost function f over the eight parallelepipeds and to identify the domain where this value is attained. It is not sure here again that the calculated minimum after this second iteration will be enough closer to the exact solution because may be here also the interpolating domain is still too large. But, it is straightforwardly to continue the iterative process by dividing the actual parallelepiped to eight new small parallelepipeds. The process can work like this till the instance where the interpolating domain is sufficiently small and the obtained approximate solution is very close to the exact solution.

A program reproducing this recursive algorithm was developed under Matlab environment. Convergence towards the exact solution was shown heuristically to occur after only a few number of iterations.

Parametric Study:

Computations are performed in the following for geometrical and material properties that are given by : Young's modulus $E=2\times10^{11}$, beam width $b=5\times10^{-2}$ m, L_1=5m, L_2=10m, L_3=15m. The applied loads are: $P_1=10^3$N,$P_2=2\times10^3$N, $P =3\times10^3$N,

The exact solution is obtained by solving the exact mathematical program (5). As the problem is non convex, the obtained minimum, while using Matlab command *fmincon* and an arbitrary initialisation, corresponds in general to only a local minimum which may be different from the global searched one. Various initialisations of the algorithm have been performed in order to guarantee that convergence is occurring towards the global minimum. The exact solution of the problem is then given by: x=0.05m;y=0.1m and z=0.15m. According to the recursive algorithm the numerical solution is identical.

In order to get a better idea about the convergence performance of the recursive iterative process independently from the actual problem data, the numerical solution is compared to the exact solution through a parametric study which are performed on geometrical dimensions and load intensities.

With the geometrical and material data considered above, we assume that the applied loads satisfy to the following relations: P_1 =P_2, P_3 =2P, P_3 =3P, where $P \in \left[10^3, 5\times10^3\right]$ N . Figure 4 gives, as function of the applied load intensity P, a comparison between the exact solution and the computed numerical solution by the recursive algorithm.

With the material data and loadings considered above, we assume that the span lengths satisfy the following relations: $L_1 = L$, $L_2 = 2L$, $L_3 = 3L$ wher e $L \in [0.5, 2.5]$ m . Figure 5 presents, in terms of the length L, a comparison between the exact solution and the computed numerical one via the recursive algorithm.

One can notice that curves of both figures 4 et 5 predict a good behaviour of the numerical recursive algorithm which is shown to yield always the exact optimal solution.

Conclusions:

It has been shown that the structural optimisation problem related to weight minimisation of a continuous beam, having uniform width, and loaded by a set of three concentrated loads applied at mid-sections of beam spans where the constraints are associated to mid-span displacements yields a non linear and non convex problem.

A method enabling to formulate a sequence of approximate problems whose solutions track the structural optimum has been developed. A recursive algorithm enabling to solve these problems has been presented.

Large numerical experiments have demonstrated that the proposed methodology yields accurate approximation to the exact optimisation problem solution. Parametric studies have shown that convergence occurs for various values of parameters if the initial interpolation domain is carefully chosen to include the problem solution.

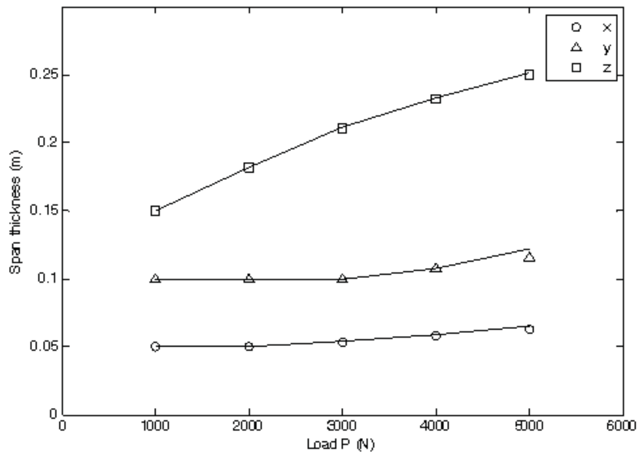

Fig. 4: Variations of optimal thicknesses as function of the load intensity P (—) approximate numerical solution, (O, Δ, □) exact solution

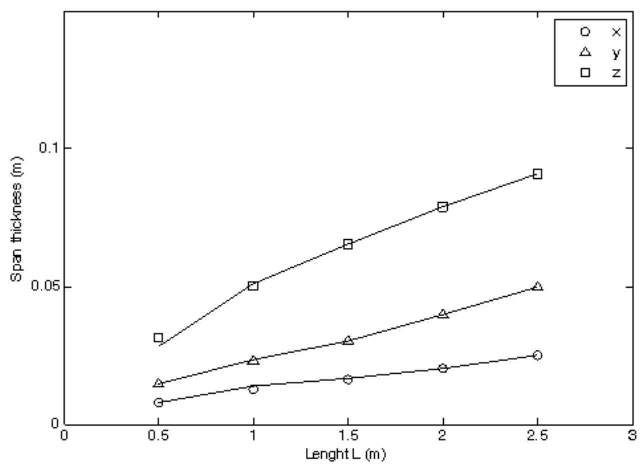

Fig. 5: Variations of optimal thicknesses as function of length (—) approximate numerical solution,(O, Δ, □) exact solution

The field of application of the proposed methodology is not limited to only the model problem considered in this work, it can be easily extended to various other problems of structural optimisation where the intervening constraints could be approximated by interpolating polynomials after achieving finite element computations or may be experimental measurements.

Appendix 1: Expressions of coefficients α_i^j

$$\alpha_1^1 = 0$$

$$\alpha_2^1 = -(4/3375)EL_1^2b^3L_3$$

$$\alpha_3^1 = 0$$

$$\alpha_4^1 = -(4/3375)EL_1^2b^3L_2$$

$$\alpha_5^1 = (7/216)P_1L_1^4L_3b^2$$

$$\alpha_6^1 = (2/27)P_1L_1^3L_2L_3b^2 - (1/24)P_2L_1^2L_2^2L_3b^2 + (1/48)L_1^2P_3L_3^2L_2b^2$$

$$\alpha_7^1 = (7/216)P_1L_1^4L_2b^2$$

$$\alpha_8^1 = (1/18)P_1L_1^3L_2^2b^2 - (1/48)L_1^2P_2L_2^3b^2$$

$$\alpha_9^1 = (-1/1125)EL_1b^3L_2^2$$

$$\alpha_{10}^1 = (-4/3375)EL_1b^3L_2L_3$$

$$\alpha_{11}^1 = 0$$

$$\alpha_{12}^1 = 0$$

;

$$\alpha_1^2 = (-4/3375)EL_2b^3L_1L_3$$

$$\alpha_2^2 = (-4/3375)EL_2^2b^3L_3$$

$$\alpha_3^2 = -\ 4/3375\ E\ b^3L_2^2L_1$$

$$\alpha_4^2 = -(1/1125)EL_2^3b^3$$

$$\alpha_5^2 = (2/27)P_2L_2^3L_1L_3b^2 - (1/24)P_1L_2^2L_1^2L_3b^2 - (1/24)L_2^2P_3L_3^2L_1b^2$$

$$\alpha_6^2 = (7/216)P_2L_2^4L_3b^2 - (1/48)L_2^3P_3L_3^2b^2$$

$$\alpha_7^2 = (7/216)P_2L_2^4L_1b^2 - (1/48)L_2^3P_1L_1^2b^2$$

$$\alpha_8^2 = (1/72)P_2L_2^5b^2$$

$$\alpha_9^2 = 0$$

$$\alpha_{10}^2 = 0$$

$$\alpha_{11}^2 = 0$$

$$\alpha_{12}^2 = 0$$

;

$$\alpha_1^3 = 0$$

$$\alpha_2^3 = 0$$

$$\alpha_3^3 = (-4/3375)EL_3^2b^3L_1$$

$$\alpha_4^3 = (-4/3375)EL_3^2b^3L_2$$

$$\alpha_5^3 = (7/216)P_3L_3^4b^2L_1$$

$$\alpha_6^3 = (7/216)P_3L_3^4L_2b^2$$

$$\alpha_7^3 = (2/27)P_3L_3^3L_1L_2b^2 + (1/48)L_3^2P_1L_1^2L_2b^2 - (1/24)L_3^2P_2L_2^2L_1b^2$$

$$\alpha_8^3 = (1/18)P_3L_3^3L_2^2b^2 - (1/48)L_3^2P_2L_2^3b^2$$

$\alpha_9^3 = 0$

$\alpha_{10}^3 = 0$

$\alpha_{11}^3 = (-4/3375)EL_3b^3L_1L_2$

$\alpha_{12}^3 = (-1/1125)EL_3b^3L_2^2$
;

$\alpha_1^4 = 0$

$\alpha_2^4 = -4/3375\ L_1^2E\ b^3L_3$

$\alpha_3^4 = 0$

$\alpha_4^4 = -4/3375\ L_1^2Eb^3L_2$

$\alpha_5^4 = -7/216\ P_1\ L_1^4L_3b^2$

$\alpha_6^4 = -2/27\ P_1L_1^3L_2L_3b^2 + 1/24\ L_1^2P_2L_2^2L_3b^2 - 1/48\ L_1^2P_3L_3^2L_2b^2$

$\alpha_7^4 = -7/216\ P_1\ L_1^4L_2b^2$

$\alpha_8^4 = -1/18\ P_1L_1^3L_2^2b^2 + 1/48\ L_1^2P_2L_2^3b^2$

$\alpha_9^4 = -1/1125\ L_1\ E\ b^3L_2^2$

$\alpha_{10}^4 = -4/3375\ L_1Eb^3L_2L_3$

$\alpha_{11}^4 = 0$

$\alpha_{12}^4 = 0$
;

$\alpha_1^5 = -4/3375\ E\ b^3L_2L_1L_3$

$\alpha_2^5 = -4/3375\ Eb^3L_2^2L_3$

$\alpha_3^5 = -4/3375\ Eb^3L_2^2L_1$

$\alpha_4^5 = -1/1125\ E\ b^3L_2^3$

$\alpha_5^5 = -2/27\ P_2L_2^3L_1L_3b^2 + 1/24\ L_2^2P_1L_1^2L_3b^2 + 1/24\ L_2^2P_3L_3^2L_1b^2$

$\alpha_6^5 = 1/48\ L_2^3P_3L_3^2b^2 - 7/216\ P_2\ L_2^4L_3b^2$

$\alpha_7^5 = -7/216\ P_2L_2^4L_1b^2 + 1/48\ L_2^3P_1L_1^2b^2$

$\alpha_8^5 = -1/72\ P_2L_2^5b^2$

$\alpha_9^5 = 0$

$\alpha_{10}^5 = 0$

$\alpha_{11}^5 = 0$

$\alpha_{12}^5 = 0$
;

$\alpha_1^6 = 0$

$\alpha_2^6 = -\ 1/1125\ E\ b^3 L_3 L_2^2$

$\alpha_3^6 = -\ 4/3375\ E\ b^3 L_3^2 L_1$

$\alpha_4^6 = -\ 4/3375\ E\ b^3 L_3^2 L_2$

$\alpha_5^6 = -\ 7/216\ P_3 L_3^4 L_1 b^2$

$\alpha_6^6 = -\ 7/216\ P_3 L_3^4 L_2 b^2$

$\alpha_7^6 = -\ 2/27\ P_3\ L_3^3 L_1 L_2 b^2 + 1/24\ L_3^2 P_2 L_2^2 L_1 b^2 -\ 1/48\ L_3^2 P_1 L_1^2 L_2 b^2$

$\alpha_8^6 = -\ 1/18\ P_3 L_3^3 L_2^2 b^2 + 1/48\ L_3^2 P_2 L_2^3 b^2$

$\alpha_9^6 = 0$

$\alpha_{10}^6 = 0$

$\alpha_{11}^6 = -\ 4/3375\ E\ b^3 L_3 L_1 L_2$

$\alpha_{12}^6 = 0$

REFERENCES

Begg, D.W. and X. Liu, 2000. On simultaneous optimization of smart structures - Part II: Algorithms and examples. Comput. Methods Appl. Mech. Engrg., 184: 25-37.

Boggs, P.T. and J.W. Tolle, 1996. Sequential Quadratic Programming. Acta Numerica, 1-52.

Ciarlet, P.G., 1998. Introduction à l'analyse numérique matricielle et à l'optimisation. In French, Masson, Paris, France.

Gil L., A. Andreu and E. Blanco, 2004. Integration methodology of different software tools for constrained optimization. Computers & Structures, 82: 1639-1647.

Hansen, L.U. and P. Horst, 2008. Multilevel optimization in aircraft structural design evaluation. Computers and Structures, 86: 104-118.

Khosravi, P., R. Ganesan and R. Sedaghati, 2008. Optimization of thin-walled structures with geometric nonlinearity for maximum critical buckling load using optimality criteria. Thin-Walled Structures, 46: 1319-1328.

Li L.J., Z.B. Huang and F. Liu, 2009. A heuristic particle swarm optimization method for truss structures with discrete variables. Computers and Structures, 87: 435-443.

Papadrakakis, M., N.D. Lagaros and V. Plevris, 2005. Design optimization of steel structures considering uncertainties. Engineering Structures, 27: 1408-1418.

Perez, R.E. and K. Behdinan, 2007. Particle swarm approach for structural design optimization. Computers & Structures, 85(19-20): 1579-1588.

Prendes Gero, M.B., A. Bello Garcia, J.J. Coz Diaz, 2006. Design optimization of 3D steel structures: Genetic algorithms vs. classical techniques. Journal of Constructional Steel Research, 62: 1303-1309.

Timoshenko, S.P., 1970. Résistance des matériaux, Tome1. In French, Dunod. Paris, France.

Tong, W.H. and G.R. Liu, 2001. An optimization procedure for truss structures with discrete design variables and dynamic constraints. Computers and Structures, 79: 155-162.