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A Cutting Plane Method for Optimization with First Order Stochastic Dominance Constraints

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Abstract: Optimization problems with stochastic dominance constraints have been introduced in recent years. In this article we consider the optimization problem with first order stochastic dominance constraints in the case of discrete joint distributions. This problem can be formulated as a mixed integer program. Here we verify this problem and develop a cutting plane method to solve it more efficiently. Finally computational results are presented for several real-world instances of the portfolio optimization problem with a stochastic dominance constraint.

Keywords: stochastic dominance constraints, cutting plane method, optimization, portfolio optimization.

1. INTRODUCTION

The relation of the first order stochastic dominance is a fundamental concept in decision models to formalize preferences among random outcomes. This notation is originated from the theory of majorization (Hardy, G.H., 1934; Marshall, A.W. and I. Olkin, 1979) for the discrete case, was further extended to general distributions (Quirk, J.P. and R. Saposnik, 1962) and also introduced in statistics (Lehmann, E., 1955; Mann, H.B. and D.R. Whitney, 1974). Then, it has been applied and developed in economics and finance (Hadar, J. and W. Russell, 1969; Hanoch, G. and H. Levy, 1969; Rothschild, M. and J.E. Stiglitz, 1969; Whitmore, G.A., 1970) and now it has been widely used in this area (Mosler, K. and M. Scarsini, 1991; Levy, H., 1955).

Models involving constraints of stochastic dominance relations has been also introduced recently in (Dentcheva, D. and A. Ruszczyñski, 2003; Dentcheva, D. and A. Ruszczyñski, 2004a; Dentcheva, D. and A. Ruszczyñski, 2004b; Dentcheva, D. and A. Ruszczyñski, 2004c; Dentcheva, D. and A. Ruszczyñski, 2006) and (Noyan, N. and A. Ruszczyñski, 2008). Most studies in this field are dedicated to the first or second order stochastic dominance relations. Stochastic dominance relations have been used for some various applications. One of the frequently used applications of stochastic dominance problems is in portfolio optimization (Dentcheva, D. and A. Ruszczyñski, 2006). In such problems the aim is to choose a portfolio with the best expected return while its return rate stochastically dominates a certain benchmark return rate.

Stochastic programming under first order stochastic dominance constraints presents nonconvex feasible area which makes challenges to solve them. Therefore, it makes a motivation to develop methods to improve solving such problems. Noyan and Ruszczyñski (2008) study the problem in discrete case and suggest heuristics to get feasible solutions based on a mixed integer formulation. Ahmadi-Javid and Kazemzadeh (2009) present a safe approximation for this model. In this paper we present a cutting plane method to efficiently solve the problem.

To continue our discussion, we need to establish some notation and definitions used throughout the paper. The triple $(\Omega, \mathbf{F}, P)$ is a probability space, where Ω is the entire space, $\mathbf{F}$ is a σ - field of subsets of Ω and P is a probability measure defined on $\mathbf{F}$. The symbol $\mathbf{X}$ denotes the space of all random variables defined on $(\Omega, \mathbf{F})$ (i.e. all finite Borel measurable functions X from Ω to $\Re$). For the random variable $X \in \mathbf{X}$, $F_X(\eta) = \Pr(X \le \eta)$, $\eta \in \Re$, is the (right-continuous) cumulative distribution function (CDF). We say that for random variables $X, Y \in \mathbf{X}$, X dominates Y in the first order stochastic dominance, denoted by $X \succeq_{(1)} Y$, if $F_X(\eta) \le F_Y(\eta)$ for all $\eta \in \Re$.

Generally we can define stochastic optimization model with a first order stochastic dominance constraint as:

$$\begin{aligned} &\max \ f(X) \\ &\text{s.t.} \quad X \succeq_{(1)} Y,\ X \in \mathbf{C} \end{aligned} \tag{1}$$

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where $\mathbf{C} \subseteq \mathbf{X}$, $f : \mathbf{C} \to \Re$, and Y is a random variable. In practice, Y is an available reference outcome, and our intention is to have the new outcome X preferable over Y in the sense of the first order stochastic dominance.

Here we present a method to solve optimization problem with first order stochastic dominance constraints more efficiently. The above formulation and the results derived in the following can also be generalized to the optimization models with several first order stochastic dominance constraints.

The paper is organized as follows. In Section 2, we explain the optimization problem with first order stochastic dominance constraint in the case of discrete joint distributions. A cutting plane method for solving this problem is developed in Section 3. The numerical results are presented in Section 4. Finally, concluding remarks are in Section 5.

2. REFORMULATION FOR DISCRETE DISTRIBUTIONS

The stochastic optimization problem with a first order stochastic dominance constraint for discrete random variables can be reformulated as a mixed integer linear program. Suppose that the entire space Ω has finitely many elementary events $\omega_1,...,\omega_T$ with probabilities $p_t = \Pr(\{\omega_t\})$. Consider that Y is a discrete random variable in $\mathbf{X}$ with finite support, and y_i are its realizations with probability of q_i, $i = 1,...,m$. Without loss of generality assume that $y_1 < y_2 < ... < y_m$. Since F_Y is a right-continuous step function, the first order stochastic dominance constraint in (1) is equivalent to $\Pr(X < y_i) \le \Pr(Y < y_i) = F_Y(y_{i-1})$, $i = 1,...,m$, so by defining $y_0 = -\infty$, $X \succeq_{(1)} Y$ can be rewritten as:

$$\Pr(X < y_i) \le F_Y(y_{i-1}),\ i = 1,...,m. \tag{2}$$

Thus, model (1) can be formulated as follows:

$$\begin{aligned} \max\ & f(X) \\ \text{s.t.}\ & \Pr(X < y_i) \le F_Y(y_{i-1}),\ i = 1,...,m \\ & X \in \mathbf{C}. \end{aligned} \tag{3}$$

We have $F_Y(y_{i-1}) = \sum_{k=1}^{i-1} q_k$; moreover, we can write $\Pr(X < y_i) = \sum_{t=1}^{T} p_t z_{it}$ by introducing binary variables $z_{it} \in \{0,1\}$ such that for $i = 1,...,m$ and $t = 1,...,T$,

$$z_{it} = \begin{cases} 1 & \text{if } y_i - X(\omega_t) > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Condition (4) can be expressed as linear mix integer constraints. To do this, it is required to define a big number $M \in \Re$ satisfying

$$M \ge \max_t \{y_m - X(\omega_t)\}. \tag{5}$$

Then, constraints $-M(1 - z_{it}) \le y_i - X(\omega_t) \le M z_{it}$, $i = 1,...,m$, $t = 1,...,T$ make z_{it} satisfy the condition (4). Therefore, model (3) can be reformulated as the following mixed integer program

$$\begin{aligned} \max\ & f(X) \\ \text{s.t.}\ & -M(1 - z_{it}) \le y_i - X(\omega_t) \le M z_{it},\ i = 1,...,m,\ t = 1,...,T \\ & \sum_{t=1}^{T} p_t z_{it} \le \sum_{k=1}^{i-1} q_k,\ i = 1,...,m \\ & z_{it} \in \{0,1\},\ i = 1,...,m,\ t = 1,...,T \\ & X \in \mathbf{C}. \end{aligned} \tag{6}$$

2.1. Comments on model (6)

To simplify model (6) we can omit constraints $-M(1-z_{it}) \le y_i - X(\omega_t)$, $i=1,...,m$, $t=1,...,T$ (see Ahmadi-Javid and Kazemzadeh (2009). Since these constraints involve big Ms, elimination of these constraints generally will help to solve the model more efficiently. In this case we reach to the following program:

$$
\begin{aligned}
\max\ & f(X) \\
\text{s.t.}\ & y_i - X(\omega_t) \le M z_{it},\ i=1,...,m,\ t=1,...,T \\
& \sum_{t=1}^{T} p_t z_{it} \le \sum_{k=1}^{i-1} q_k,\ i=1,...,m \\
& z_{it} \in \{0,1\},\ i=1,...,m,\ t=1,...,T \\
& X \in \mathbf{C}.
\end{aligned} \tag{7}
$$

According to the definition of z_{it}, if $z_{it}=0$ then $y_i \le X(\omega_t)$. In addition, because of $y_1 \le y_2 \le ... \le y_{i-1} \le y_i$, we can say $y_1 \le y_2 \le ... \le y_{i-1} \le X(\omega_t)$. Therefore, we obtain $z_{1t} = ... = z_{i-1,t} = 0$. On the other hand, if $z_{it}=1$ then $y_i > X(\omega_t)$ and similar to the previous deduction we have $z_{i+1,t} = ... = z_{mt} = 1$. As a result, the following inequalities are valid for model (7):

$$
z_{it} \le z_{i+1,t},\ i=1,...,m,\ t=1,...,T\ . \tag{8}
$$

By adding these inequalities to model (7), model (9) is obtained:

$$
\begin{aligned}
\max\ & f(X) \\
\text{s.t.}\ & y_i - X(\omega_t) \le M z_{it},\ i=1,...,m,\ t=1,...,T \\
& \sum_{t=1}^{T} p_t z_{it} \le \sum_{k=1}^{i-1} q_k,\ i=1,...,m \\
& z_{it} \le z_{i+1,t},\ i=1,...,m-1,\ t=1,...,T \\
& z_{it} \in \{0,1\},\ i=1,...,m,\ t=1,...,T \\
& X \in \mathbf{C}.
\end{aligned} \tag{9}
$$

Because of the additional valid inequalities, model (9) may be able to reach the optimum solution faster than model (7). This will be shown by numerical results presented in Section 4.

In practice we usually use the following formulation instead of the rather abstract formulation (7) to conveniently model and efficiently solve real world problems using the existing rich knowledge in numerical optimization,

$$
\begin{aligned}
\max\ & f(X_x) \\
\text{s.t.}\ & y_i - X_x(\omega_t) \le M z_{it},\ i=1,...,m,\ t=1,...,T \\
& \sum_{t=1}^{T} p_t z_{it} \le \sum_{k=1}^{i-1} q_k,\ i=1,...,m \\
& z_{it} \le z_{i+1,t},\ i=1,...,m-1,\ t=1,...,T \\
& z_{it} \in \{0,1\},\ i=1,...,m,\ t=1,...,T \\
& x \in \mathbf{S},
\end{aligned} \tag{10}
$$

where $\mathbf{S} \subseteq \Re^N$ and X_x is a random variable which depends on x.

For example, in portfolio optimization $X_x \equiv R(x) = R_1x_1 + R_2x_2 + ... + R_Nx_N$ where $R_1, R_2, ..., R_N$ are random return rates of assets $1,2,...,N$. The aim is to invest a certain capital in these assets in order to obtain some desirable characteristics of the total return rate on the investment. Denoting by $x_1, x_2, ..., x_N$ the fractions of the initial capital invested in assets $1,2,...,N$, we have:

$\mathbf{S} = \left\{x \in \Re^N : x_1 + x_2 + ... + x_N = 1, x_j \geq 0, j = 1,2,...,N\right\}$.

If we consider that the return rates have a discrete joint distribution with realizations r_{jt}, $t = 1,2,...,T$, $j = 1,2,...,N$, attained with probabilities p_t, $t = 1,2,...,T$, the objective function is the expected return rate, $f(X_x) \equiv \mathrm{E}[R(x)] = \sum_{t=1}^{T} p_t \sum_{j=1}^{N} x_j r_{jt}$, and Y is a discrete random variable with realizations y_i, and probabilities q_i, $i = 1,...,m$, the portfolio optimization problem becomes:

$$\begin{aligned}
\max\ & \sum_{t=1}^{T} p_t \sum_{j=1}^{N} x_j r_{jt} \\
\text{s.t.}\ & y_i - \sum_{j=1}^{N} x_j r_{jt} \leq M z_{it},\ i = 1,...,m,\ t = 1,...,T \\
& \sum_{t=1}^{T} p_t z_{it} \leq \sum_{k=1}^{i-1} q_k,\ i = 1,...,m \\
& \sum_{j=1}^{N} x_j = 1 \\
& z_{it} \leq z_{i+1,t},\ i = 1,...,m-1,\ t = 1,...,T \\
& x_j \geq 0,\ j = 1,...,N \\
& z_{it} \in \{0,1\},\ i = 1,...,m,\ t = 1,...,T.
\end{aligned} \tag{11}$$

3. PROPOSED CUTTING PLANE METHOD

According to model (9), optimization with first order stochastic dominance constraints in discrete distribution case can be reformulated as a mixed integer program. This causes difficulties in solving such problem. It can be found that the most serious difficulty in model (9) refers to its first constraint involving the big number M. By using this fact we present a cutting plane method to solve model (9) more efficiently. The algorithm is described in three steps.

Step 1. Solve the problem:

$$\begin{aligned}
\max\ & f(X) \\
\text{s.t.}\ & y_1 - X(\omega_1) \leq M z_{1,1} \\
& \sum_{t=1}^{T} p_t z_{it} \leq \sum_{k=1}^{i-1} q_k,\ i = 1,...,m \\
& z_{it} \leq z_{i+1,t},\ i = 1,...,m-1,\ t = 1,...,T \\
& z_{it} \in \{0,1\},\ i = 1,...,m-1,\ t = 1,...,T.
\end{aligned} \tag{12}$$

Step 2. Check the following constraints for the solution obtained in Step 1.

$$y_i - X(\omega_t) \leq M z_{it},\ i = 1,...,m,\ t = 1,...,T\ . \tag{13}$$

If all of the constraints are satisfied, the current solution is the optimum solution of model (9), and then the algorithm is finished. However, if there is a constraint which is violated, select a violated constraint as follows:
In the set of violated constraints with the lowest index i, select the one with the lowest t.
Then, add this constraint to the last problem that we have solved in the algorithm.

Step 3. Solve the new problem, and then go to Step 2.

If in Step 2 we add more violated constraints to the problem instead of one violated constraint, the algorithm may reach the optimum solution more quickly; however, it depends on the number of constraints added in each iteration. According to numerical results, we can say that in most instances it seems if the number of violated constraints added to the problem in each iteration is almost equivalent to 10% of the number of the realizations, average CPU time needed to solve the problem is the shortest time. This result will be shown in the next section.

The process of our method is given in the following algorithm. The number of violated constraints added to the problem in each iteration, i.e. the *number of cuts*, is denoted by NC. This parameter must be defined before using the algorithm. Also, nc is defined as a variable to control that the number of cuts actually added to the problem in each iteration, which does not exceed NC. We define w_t as an indicator to show that in each iteration whether a violated constraint with index t has been added to the problem or not. We use w_t in 7th line of the algorithm in order to ensure that constraints with the same t do not be added to the problem in an iteration.

1 Initially set problem P as model (12), and solve problem P
2 **do while** for the solution of problem P, there is a pair of $i \in \{1,...,m\}$ and $t \in \{1,...,T\}$ for which constraint $y_i - X(\omega_t) \le Mz_{it}$ is violated
3   Set $w_t = 0$ for $t = 1,...,T$ and $nc = 0$
4   **for** $i = 1$ **to** m **do**
5   **for** $t = 1$ **to** T **do**
6   **if** constraint $y_i - X(\omega_t) \le Mz_{it}$ is violated and $w_t = 0$ and $nc < NC$ **then**
7   Set $w_t = 1$ and $nc = nc + 1$
8   Add constraint $y_i - X(\omega_t) \le Mz_{it}$ to problem P
9   **end**
10   **if** $nc = NC$ **then** go to line *L*
11   **end**
12   **end**
13   *L*: Solve problem P
14 **loop**

4. COMPUTATIONAL RESULTS

To verify the computational efficiency of presented models (6), (7) and (9) and the cutting plane method, we consider a portfolio optimization problem with finitely many assets formulated in (11). We define benchmark return, Y, based on the average of return rates of assets in each realization, i.e. for each realization, y_i denoting the i th realization of the benchmark return is attained as:

$$y_i = \frac{1}{N}\sum_{j=1}^{N} r_{ji},\ i = 1,...,m .$$

We have exploited weekly returns of 435 stocks (N = 435) in the S&P 500 to construct test instances. These weekly return data are obtained by using closing stock price on Friday (or the last working day) of each week from January 2004 through March 2007. Each weekly return is considered as an outcome that occurs with equal probability. We construct test instances for years 2005, 2006, and 2007 with various number of outcomes realizations, T, which takes 20, 30, 40, 50, and 60. For these three years we take the weekly data immediately preceding March 11 of 2005, March 10 of 2006 and March 9 of 2007. For example, the instance for year 2007, with T = 50 is obtained by taking the weekly returns from March 31, 2006 through March 9, 2007. We have solved all problem instances on a computer with 5600 MHZ dual-core processor and 1024 MB of RAM by using Lingo 8.0.

4.1. Comparison of models (6), (7) and (9)

Table 1 provides CPU time needed to solve model (9) in comparison with models (6) and (7). The three formulations were not able to solve instance 2006 with $T = 40$ in 10800 seconds (3 hours), so this instance is excluded. As the results show and also we explain in Section 2, reducing some constraints of model (6), which results model (7), and also adding constraints (8) has caused a considerable reduction in CPU time for solving this problem.

Table 1. Reduction in CPU time obtained by model (9) in comparison with models (6) and (7).

Year	T	CPU Time (s)			Reduction (%) in CPU Time for Solving Model (9) in Comparison with:	
		Model (6)	Model (7)	Model (9)	Model (6)	Model (7)
2005	20	232	32	24	89.66	25
	30	1445	561	176	87.82	68.63
	40	>10800	>10800	3904	> 63.85	> 63.85
2006	20	363	89	58	84.02	34.83
	30	1895	1652	505	73.35	69.43
2007	20	181	13	10	94.48	23.08
	30	131	1421	78	40.46	94.51
	40	>10800	>10800	4039	> 62.6	> 62.6

Table 2. NC^* for cutting plane method.

T	Year	NC^*	Average of NC^*
20	2005	2	2.33
	2006	3	
	2007	2	
30	2005	4	3.67
	2006	4	
	2007	3	
40	2005	6	4.33
	2006	3	
	2007	4	
50	2005	5	5.67
	2006	6	
	2007	6	
60	2005	6	5.67
	2006	5	
	2007	6	

4.2. Best number of cuts

In Table 2, we have solved instances with various sizes of outcome realizations by considering various NC, and then for each instance we have determined the best number of cuts NC^* by which elapsed CPU time needed for solving the instance is minimum. In addition, the averages of NC^* for instances with equal size are given in this table. As it can be seen in Table 2, it seems that the best number of cuts NC^* is almost equivalent to 10% of the number of realizations for almost everyone of the instances. Therefore, we apply this rule for the cutting plane method in the sequel.

4.3. Performance of cutting plane method

Table 3 shows of CPU time needed to solve problems with various sizes by the cutting plane method in comparison with the formulation presented in (Noyan, N. and A. Ruszczyñski, 2008). The results show the remarkable reduction in CPU time for solving the problems by using the cutting plane method relative to the other method. In particular, for some problem instances with 40, 50 and 60 realizations, the other formulations cannot solve the problems in 10800 seconds (3 hours) while our new method solves them in some minutes.

4.4. Example

Here we present an example to show how we can use the proposed cutting plane method. Table 4 gives the weekly return rates of the first eight weeks of 2007 for ten stocks in the S&P 500. In Figure 1 the CDFs of the benchmark and solutions obtained in each iteration are depicted. Here we set $NC = 4$.

Iteration 1:

Solve the following program (Problem *P*):

$$\max \sum_{t=1}^{8}\frac{1}{8}\sum_{j=1}^{10}x_j r_{jt}$$

$$\text{s.t.}\quad y_1-\sum_{j=1}^{N}x_j r_{j,1}\le 50\times z_{1,t}$$

$$\sum_{t=1}^{8}\frac{1}{8}z_{it}\le\sum_{k=1}^{i-1}q_k \qquad i=1,\dots,8$$

$$z_{it}\le z_{i+1,t} \qquad i=1,\dots,8-1,\quad t=1,\dots,8$$

$$z_{it}\in\{0,1\} \qquad i=1,\dots,8-1,\quad t=1,\dots,8.$$

Iteration 2:

Add the following constraints and solve the model again.

$$y_2-\sum_{j=1}^{10}x_j r_{j,2}\le 50\times z_{2,1},\ y_2-\sum_{j=1}^{10}x_j r_{j,2}\le 50\times z_{2,5}$$

$$y_2-\sum_{j=1}^{10}x_j r_{j,2}\le 50\times z_{2,8},\ y_6-\sum_{j=1}^{10}x_j r_{j,6}\le 50\times z_{6,6}$$

Iteration 3:

Add the following constraints and solve the model again.

$$y_3-\sum_{j=1}^{10}x_j r_{j,3}\le 50\times z_{3,5},\ y_3-\sum_{j=1}^{10}x_j r_{j,3}\le 50\times z_{3,8}$$

$$y_4-\sum_{j=1}^{10}x_j r_{j,4}\le 50\times z_{4,6},\ y_8-\sum_{j=1}^{10}x_j r_{j,8}\le 50\times z_{8,2}$$

Iteration 4:

Add the following constraints and solve the model again.

$$y_1-\sum_{j=1}^{10}x_j r_{j,1}\le 50\times z_{1,5},\ y_3-\sum_{j=1}^{10}x_j r_{j,3}\le 50\times z_{3,1}$$

$$y_6-\sum_{j=1}^{10}x_j r_{j,6}\le 50\times z_{6,7}$$

Iteration 5:

Add the following constraints and solve the model again.

$$y_3-\sum_{j=1}^{10}x_j r_{j,3}\le 50\times z_{3,6},\ y_4-\sum_{j=1}^{10}x_j r_{j,4}\le 50\times z_{4,5}$$

$$y_7-\sum_{j=1}^{10}x_j r_{j,7}\le 50\times z_{7,4}$$

We can see from Figure 1 the solution obtained in Iteration 5 is the optimal solution which dominates the benchmark in the sense of first order stochastic dominance.

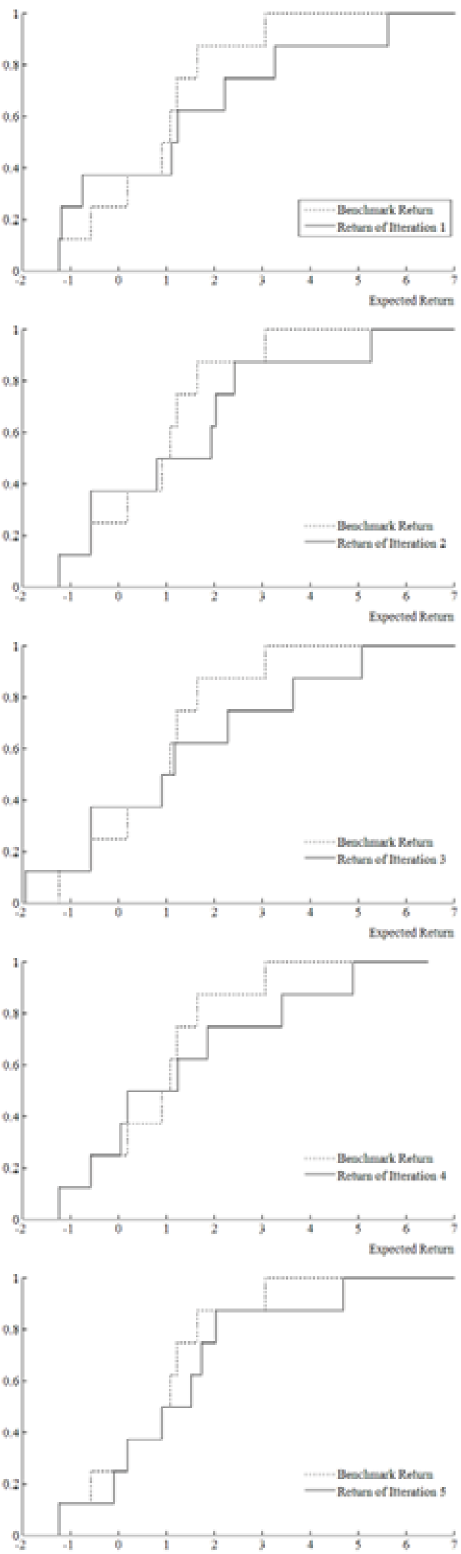

Figure 1. CDFs of benchmark and solutions obtained in iterations of proposed cutting plane method applied for example given in Section 4.4.

Table 3. Reduction in CPU time obtained by proposed cutting plane method relative to formulation given by Noyan and Ruszczyñski (2008).

Year	T	CPU Time (s)		Reduction (%)
		Noyan & Ruszczyński	Cutting Plane Method	
2005	20	24	2	91.67
	30	176	4	97.73
	40	3904	21	99.46
	50	>10800	93	> 99.14
	60	>10800	75	> 99.31
2006	20	58	16	72.41
	30	505	7	98.61
	40	>10800	1399	> 87.05
	50	>10800	1522	> 85.91
	60	>10800	1192	> 88.96
2007	20	10	2	80
	30	78	3	96.15
	40	4039	28	99.31
	50	>10800	617	> 94.29
	60	>10800	28	> 99.74

Table 4. Weekly return rates (in %) of first eight weeks of 2007 for ten stocks in S&P 500.

Stock name	Week							
	1	2	3	4	5	6	7	8
AAPL	0.247525	11.25221	-6.46798	-3.52542	-0.73788	-1.74631	1.873424	4.998232
AMD	-3.14496	-7.35667	-2.90252	-8.51664	-3.26757	-5.03505	0.268456	-1.67336
COP	-6.29283	-5.32756	0.189155	1.29012	3.883194	-0.44857	-0.64584	1.587302
DELL	4.264647	1.75841	-6.01052	-5.15588	-0.88496	0.85034	2.824621	-0.82008
GM	-1.57326	1.698302	2.586771	4.404724	0.152858	9.157509	1.621924	-5.72372
IBM	0.278753	1.966437	-3.19063	1.335002	1.759984	-0.32366	0.446474	-1.27286
INTC	4.220457	4.859457	-5.90641	-1.40029	3.428012	-0.42614	0.951022	-2.21385
MSFT	-0.73925	5.28975	-0.32154	-1.64516	-1.31191	-4.02127	-0.48477	0.556715
NKE	-0.20268	0.944354	0.181068	-4.74947	5.260384	3.385078	2.605832	1.642749
WMT	2.610966	1.25106	0.691099	-1.33112	0.86425	-0.22989	1.068287	2.238342
Benchmark	3.068364	1.641394	1.226099	1.081435	0.91971	0.199291	-0.57775	-1.22490

5. CONCLUTION

Optimization problems with first order stochastic dominance constraints in the case of discrete distribution can be formulated as mixed integer programs. In such problems the random variable of the desirable outcome must dominate an available random variable called benchmark in the sense of first order stochastic dominance. In this paper we have presented a cutting plane method to solve such mixed integer programs more efficiently.

To verify the computational efficiency of proposed method, we consider a portfolio optimization problem with finitely many assets whose return rates are described by a discrete joint distribution. We have exploited weekly returns of 435 stocks in the S&P 500 to construct test instances. The results show remarkable reduction in CPU time for solving the instances by using the cutting plane method relative to the mixed integer program presented by Noyan and Ruszczyński (2008). In particular, for some instances with 40, 50 and 60 realizations, the formulation of Noyan and Ruszczyński (2008) cannot solve the problems in 3 hours while our new method solves them in a few minutes.

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