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Calculation of Flow Past A Sphere in the Vicinity of A Ground Using A Direct Boundary Element Method

[1]M. Mushtaq, [2]N. A. Shah, [3]G. Muhammad
[1]Assistant Professor, Department of Mathematics, U.E.T. Lahore, Pakistan
[2]Professor, Department of Mathematics, U.E.T. Lahore, Pakistan
[3]Ph.D Scholar, Department of Mathematics, U.E.T. Lahore, Pakistan

Abstract: The aim of this paper is to calculate flow past a sphere in the vicinity of a ground by using a direct boundary element method. The procedure is very simple, accurate, efficient and time saving which needs less computational efforts.. This method can even be applied to higher order boundary elements. The validity of this method is best checked by the closed analytical and computed results.

Key words: Direct boundary element method, Flow past, Sphere, Ground.

INTRODUCTION

From the time of fluid flow modeling, it had been struggled to find the solution of a complicated system of partial differential equations (PDE) for the fluid flows which needed more efficient numerical methods. With the passage of time, many numerical techniques such as finite difference method, finite element method, finite volume method and boundary element method etc. came into beings which made possible the calculation of practical flows. Due to discovery of new algorithms and faster computers, these methods were evolved in all areas in the past. These methods are CPU time and storage hungry. The boundary element method has been superceded due to its efficiency and accuracy. It has many advantages, one of the advantages is that with boundary elements one has to discretize the entire surface of the body, whereas with domain methods it is essential to discretize the entire region of the flow field. The most important characteristics of boundary element method are the much smaller system of equations and considerable reduction in data which is prerequisite to run a computer program efficiently. Furthermore, this method is well–suited to problems with an infinite domain. From above discussion, it is concluded that boundary element method is a time saving, accurate and efficient numerical technique as compared to other numerical techniques which can be classified into direct boundary element method and indirect boundary element method. The direct method takes the form of a statement which provides the values of the unknown variables at any field point in terms of the complete set of all the boundary data.

Flow past a sphere:

Let a sphere of unit radius with center at origin be near the ground in a uniform stream of velocity U in the positive direction of z–axis as shown in figure 1.

The inviscid flow over the ground, which is considered to be a plane, remains everywhere tangential to its surface. This flow is simulated by the so–called 'mirror image' principle. A mirror image of the sphere is imagined to be present below the ground plane. The flow field by a uniform stream parallel to the ground plane is disturbed by the sphere and its image is therefore symmetrical with respect to the ground plane. The plane of symmetry is a stream surface and represents the ground. An axisymmetric flow is most conveniently formulated in cylindrical polar coordinates. The cylindrical polar coordinates are taken as (r,θ , z).
The velocity potential and stream function for a sphere of radius a moving in the negative direction of z– axis with velocity U can be calculated as

$$\varphi = \frac{-1}{2} U \frac{a^3}{\gamma^2} \cos\theta , \qquad \psi = \frac{1}{2} U \frac{a^3}{\gamma^2} \sin^2\theta \tag{1}$$

Corresponding Author: M. Mushtaq, Assistant Professor, Department of Mathematics, U.E.T. Lahore, Pakistan

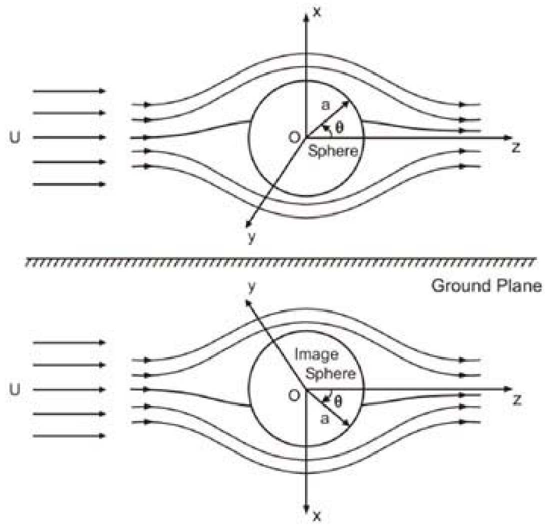


Fig. 1:

Also the velocity potential and stream function for a uniform stream moving with velocity U in the positive direction of z–axis are given by

$$\varphi = -U r \cos\theta \,, \; \psi = -\frac{1}{2} U r^2 \sin^2\theta \qquad (2)$$

Therefore the velocity potential and stream function for the streaming motion past a fixed sphere in the positive direction of z–axis take the forms

$$\varphi = -U\gamma \cos\theta \frac{1}{2} U \frac{a^3}{\gamma^2} \cos\theta = -U\left(\gamma + \frac{a^3}{2\gamma^2}\right) \cos\theta \qquad (3)$$

$$\text{and } \psi = -\frac{1}{2} U \gamma^2 \sin^2\theta + \frac{1}{2} U \frac{a^3}{\gamma} \sin^2\theta = -\frac{1}{2} U\left(\gamma 2 \frac{a^3}{\gamma}\right) \sin^2\theta \qquad (4)$$

The velocity components at any point (r ,θ) are given by

$$v_r = \frac{\delta\varphi}{\delta r} = U\left(1 - \frac{a^3}{r^3}\right)\cos\theta$$

$$v\theta = -\frac{1}{r}\frac{\delta\varphi}{\delta r} = -U\left(1 + \frac{a^3}{2r^3}\right)\sin\theta$$

$$v_r = \frac{1}{r^2 \sin\theta}\frac{\delta\Psi}{\delta\theta} = -\frac{1}{r^2\sin\theta}\left[-U\left(r^2 - \frac{a^2}{r}\right)\sin\theta\cos\theta\right]$$

$$= U\left(1 - \frac{a^3}{r^3}\right)\cos\theta$$

$$v\theta = \frac{1}{r\sin\theta}\frac{\delta\Psi}{\delta r} = \frac{1}{r\sin\theta}\left[-\frac{1}{2} U\left(2r + \frac{a^3}{r^2}\right)\sin^2\theta\right]$$

$$= -U\left(1 + \frac{a^3}{\ }\right)\sin\theta$$

$2r^3$

The speed at any point in the flow field is given by

$$V = \sqrt{v_r^2 + v_\theta^2}$$

$$= \sqrt{\left[U\left(1-\frac{a^3}{r^3}\right)\sin\theta\right]^2 + \left[-U\left(1+\frac{a^3}{2r^3}\right)\sin\theta\right]^2}$$

Therefore the speed any point on the sphere itself is given by

$$V = \sqrt{\phi + U^2\left(1+\frac{1}{2}\right)^2 \sin^2\theta}$$

$$= \sqrt{\frac{9}{4}U^2 \sin^2\theta}$$

$$= \frac{3}{2} U\sin\theta \qquad (5)$$

Now the pressure distribution at any point of the flow field can be calculated by using the Bernoulli's equation between two points i.e.

$$\frac{p}{\rho} + \frac{1}{2}v^2 = \frac{p_\infty}{\rho} + \frac{1}{2}U^2$$

or $$p = p_\infty + \frac{1}{2}\rho\,(U^2 - V^2) \qquad (6)$$

where p_∞ is the pressure at infinity

Equation (6) takes the form while using equation (5) .

$$p = p_\infty + \frac{1}{2}\rho U^2\left(1 - \frac{9}{4}\sin^2\theta\right) \qquad (7)$$

Now the pressure is maximum and minimum at the points where $\theta = 0$ or π and $\theta = \pm\frac{\pi}{2}$ respectively

Thus $p_{max} = p_\infty + \frac{1}{2}\rho U^2$ and $p_{min} = p_\infty - \frac{5}{8}\rho U^2$

Therefore from equation (7), the pressure coefficient C p on the boundary of the sphere is given by

$$C_p = 1 - \frac{9}{4}\sin^2\theta \qquad (8)$$

From equations (5) and (8)

$C_p = 1 - V^2$ taking $U = 1$

Boundary conditions:

The boundary conditions to be satisfied over the surface of a sphere is

$$\frac{\partial\varphi\ \text{sphere}}{\partial n} = U(\hat{n}.\hat{k})\frac{z}{\sqrt{x^2+y^2+z^2}} \qquad (9)$$

where φ_{sphere} is the perturbation velocity potential and $\hat{n}$ is the unit normal draw outward from the surface of the sphere.

The equation of the surface of a sphere is $x^2 + y^2 + z^2 = 1$
where the radius a of the sphere is taken to be 1.

Thus equation (9) becomes

$$\frac{\partial \varphi_{sphere}}{\partial n} = U\, z = z \text{ , taking } U = 1$$

Discretization of elements:

The direct boundary element method is applied to calculate the potential flow solution around a sphere when it is lying very close to the ground for which the analytical solution is available.

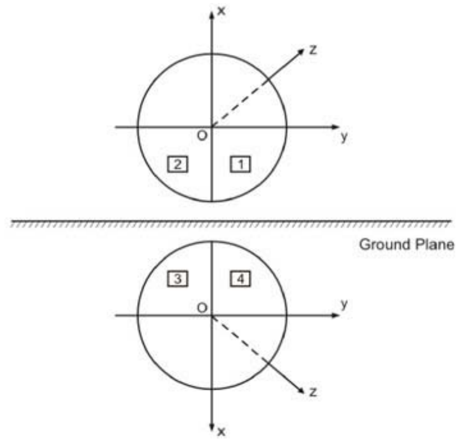


Fig. 2:

Suppose the side (1) is the side at which the values of the potential are to be evaluated at the fixed points 'i'. At a given point, the components of

$$\hat{H}_{ij} = \iint_{S_j - i} \frac{\partial}{\partial n}\left(\frac{1}{4\pi r}\right) ds \text{ and } G_{ji} = \iint_{S_j} \left(\frac{1}{4\pi r}\right) ds$$

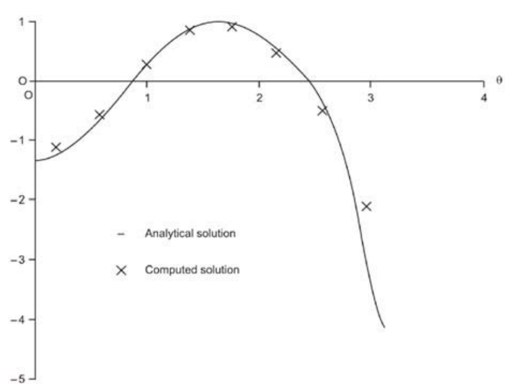
integrals due to an element on side (1) are evaluated first. The y–coordinates of all nodes of this element are then changed (side (2)) and the components from this reflected element are calculated. The x–coordinates of all nodes of this element are then changed (side (3)) and the components from this element are evaluated. Position (4) is then reached by changing the y–coordinates again and evaluating the components due to the element. The integral components from the corresponding elements on all the four sides are then summed to calculate the total integral values at the point on side (1) due to one particular element on side (1). The process is repeated for all the elements on side (1) and then for all the fixed points. The pressure distribution over the surface of a sphere of radius '1' unit with ground clearance of 0.1 units has been calculated using the above procedure. The pressure coefficient over the surface of the sphere is calculated for 96 and 384 boundary element.

To check the accuracy of computed results, these results are compared with the analytical results in Shah, N.A . The analytical solutions are based upon a truncated series of images. and the pressure distribution over the boundary of the sphere can be obtained for 384 to get more accuracy in comparison.

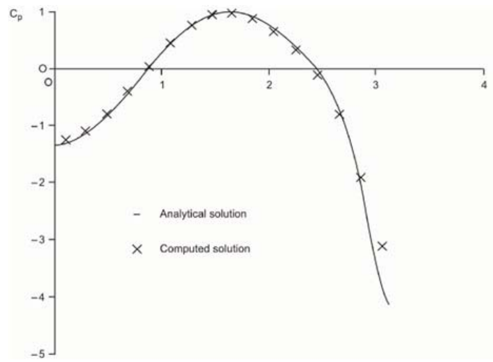
The table of 96 boundary elements for ground clearance of 0.1 units is only given, but the table of 384 boundary elements is large in size and so they are not given in this research paper.

TABLE

ELEMENT	XM	YM	ZM	VELOCITY	CP
1	-.177E+00	-.934E+00	.177E+00	.17630E+01	-.21083E+01
2	-.522E+00	-.798E+00	.1 57E+00	.12268E+01	-.50499E+00
3	-.798E+00	-.522E+00	.1 57E+00	.72857E+00	.469 1 9E+00
4	-.934E+00	-.1 77E+00	.1 77E+00	.30244E+00	.90853E+00
5	-.934E+00	.177E+00	.177E+00	.38064E+00	.85511E+00
6	-.798E+00	.522E+00	.1 57E+00	.84685E+00	.28285E+00
7	-.522E+00	.798E+00	.157E+00	.12495E+01	-.56121E+00
8	-.177E+00	.934E+00	.177E+00	.14526E+01	-.11101E+01
9	.177E+00	.934E+00	.177E+00	.14526E+01	-.11101E+01
10	.522E+00	.798E+00	.157E+00	.12495E+01	-.56121E+00
11	.798E+00	.522E+00	.1 57E+00	.84685E+00	.28285E+00
12	.934E+00	.177E+00	.177E+00	.38064E+00	.85511E+00
13	.934E+00	-.1 77E+00	.1 77E+00	.30244E+00	.90853E+00
14	.798E+00	-.522E+00	.1 57E+00	.72857E+00	.4691 9E+00
15	.522E+00	-.798E+00	.1 57E+00	.12268E+01	-.50499E+00
16	.177E+00	-.934E+00	.177E+00	.17630E+01	-.21083E+01
17	-.157E+00	-.798E+00	.522E+00	.16631E+01	-.17658E+01
18	-.470E+00	-.703E+00	.470E+00	.13578E+01	-.84371E+00
19	-.703E+00	-.470E+00	.470E+00	.931 67E+00	.131 99E+00
20	-.798E+00	-.157E+00	.522E+00	.83573E+00	.30155E+00
21	-.798E+00	.1 57E+00	.522E+00	.84829E+00	.28040E+00
22	-.703E+00	.470E+00	.470E+00	.96421E+00	.70294E+00
23	-.470E+00	.703E+00	.470E+00	.13237E+01	-.75219E+00
24	-.157E+00	.798E+00	.522E+00	.14536E+01	-.11130E+01
25	.157E+00	.798E+00	.522E+00	.14536E+01	-.11130E+01
26	.470E+00	.703E+00	.470E+00	.13237E+01	-.75219E+00
27	.703E+00	.470E+00	.470E+00	.96421E+00	.70294E+00
28	.798E+00	.1 57E+00	.522E+00	.84829E+00	.28040E+00
29	.798E+00	-.157E+00	.522E+00	.83573E+00	.30155E+00
30	.703E+00	-.470E+00	.470E+00	.931 67E+00	.131 99E+00
31	.470E+00	-.703E+00	.470E+00	.13578E+01	-.84371E+00
32	.157E+00	-.798E+00	.522E+00	.16631E+01	-.17658E+01
33	-.1 57E+00	-.522E+00	.798E+00	.1 5499E+01	-. 14022E+01
34	-.470E+00	-.470E+00	.703E+00	.13667E+01	-.86795E+00
35	-.522E+00	-.157E+00	.798E+00	.12749E+01	-.62527E+00
36	-.522E+00	.1 57E+00	.798E+00	.12635E+01	-.59632E+00
37	-.470E+00	.470E+00	.703E+00	.13285E+01	-.76497E+00
38	-.157E+00	.522E+00	.798E+00	.14601E+01	-.11319E+01
39	.157E+00	.522E+00	.798E+00	.14601E+01	-.11319E+01
40	.470E+00	.470E+00	.703E+00	.13285E+01	-.76497E+00
41	.522E+00	.1 57E+00	.798E+00	.12635E+01	-.59632E+00
42	.522E+00	-.1 57E+00	.798E+00	. 12749E+01	-.62527E+00
43	.470E+00	-.470E+00	.703E+00	.13667E+01	-.86795E+00
44	.1 57E+00	-.522E+00	.798E+00	.1 5499E+01	-. 14022E+01
45	-. 177E+00	-. 177E+00	.934E+00	.14954E+01	-.12361E+01
46	-.177E+00	.177E+00	.934E+00	.14739E+01	-.11722E+01
47	.177E+00	.177E+00	.934E+00	.14739E+01	-.11722E+01
48	.177E+00	-.177E+00	.934E+00	.14954E+01	-.12361E+01



GRAPH. 1: Comparison of analytical and computed results for pressure coefficients over the boundary of a sphere with ground clearance 0.1 units for 96 boundary elements.



GRAPH. 2: Comparison of analytical and computed results for pressure coefficients over the boundary of a sphere with ground clearance 0.1 units for 384 boundary elements.

Conclusion:

A direct boundary element method is applied for the calculation of potential flow around a sphere in the vicinity of the ground. The computed results obtained by this method are compared with the analytical results for the flow past a sphere near the ground. It is found that the computed results are good in agreement with the analytical results at all points except the points near the line joining the centers of the sphere and image sphere.

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