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Composite Control of Linear Uncertain Systems Subject to Uncertainties

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Abstract

In this paper, a hybrid control scheme is designed to control an uncertain linear system subject to uncertainties and time-varying parameters. In this regard, a sliding mode control is considered to tackle uncertainties, while an optimal control scheme (Linear Quadratic Regulation) is designed to obtain the minimum control effort. In this approach, the upper bound of the uncertainties will be estimated with an adaptive law, which is derived by a proper Lyapunov function. One of the advantages of this method is the reduction of the computation process. The bound of uncertainty should not be known a priori to design OSMC, which can be known as the great merit of this novel approach. The efficiency of this novel method will be shown on a linear system in the presence of uncertainties. By comparison the simulation results of this method with SMC, it can be clearly demonstrated that it can control the linear systems not only robustly but also optimally.

Keywords: Composite Control, Non-Singular Terminal Sliding Mode Control, Optimal control, Adaptive Law, Uncertain system

INTRODUCTION

In the field of robust control, sliding mode control is highly appreciated due to its great robust behavior towards matched uncertainty (Edwards, 1998), which is in the range space of the input distribution matrix. Though the conventional SMC is highly robust, it requires a high control input as the gain of the controller needs to be higher than the upper bound of the uncertainty. But a high control gain is undesirable as it may saturate the actuator and the cost of the controller also becomes high. Therefore, an optimal sliding mode controller (Xu, 2006; Janardhanan, 2008; Basin, 2012) has been developed to tackle uncertain systems with a minimum expense of control energy. To develop an OSMC, the optimal controller is incorporated with the integral SMC. However, the OSMC suffers from the following drawbacks:

- The bound of the matched uncertain should be known a prior.
- It cannot tackle the mismatched uncertainty.

To design the switching control of sliding mode, an upper bound of the matched uncertainty should be known a priori. If the upper bound of the matched uncertainty is not known in advance, a popular choice is to choose high gain switching which increases the control energy. To know the upper bound of the matched uncertainty, different adaptive tuning methods (Huang, 2008; Wai, 2006; Hu, 2007) have been proposed in the literature. But many times the bound of the matched uncertainty is overestimated, which is not desirable. In (Plestan, 2010) Plestan et al. proposed an adaptive SMC to solve the problem of overestimation. Another drawback of the conventional SMC is that it cannot tackle the mismatched uncertainty. In recent literature, a few methods have been proposed to design sliding mode controllers for systems affected by mismatched uncertainties. Some such methods are the output feedback sliding mode controller (Choi, 2009; Siva, 2009) and backstepping sliding mode controller (Adhikary,2013) which have been designed to handle mismatched uncertain systems. For linear systems affected by the norm-bounded mismatched uncertainty, the sliding surface was designed by solving the linear inequality matrix in (Choi, 1998; Choi, 2003). Observer-based SMC has recently been proposed (Yang, 2013) to tackle the mismatched uncertainty affecting the linear system. In (Yang, 2013) a disturbance observer has been used to estimate the mismatched uncertainty and design the sliding surface based on the estimation.

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In this paper, two design approaches are proposed for uncertain linear systems using first-order optimal sliding mode controller. In the first design approach, a first-order optimal adaptive sliding mode controller (OASMC) is proposed for the uncertain linear system affected by the matched uncertainty whose upper bound is unknown. The optimal controller is designed for the nominal linear system by using the LQR technique. Then an integral sliding mode controller is combined with the optimal controller. As the upper bound of the matched uncertainty is not known, and adaptive law is used to estimate the upper bound of the uncertainty which is needed for designing the switching control. By designing a conventional SMC, the great performance of OASMC in comparison to the conventional SMC will be shown.

PROBLEM FORMULATION AND STABILIZATION ANALYSIS

In this section, a first-order optimal adaptive sliding mode controller (OASMC) is proposed for the linear system affected by the matched uncertainty whose upper bound is unknown. LQR technique is used to design the optimal controller for the nominal linear system and an integral sliding mode controller is combined with the optimal controller to impart robustness. An adaptive tuning law is also used to design the sliding mode controller. This proposed OASMC can be designed for both stabilization and tracking problems.

In stabilization, the system states are forced to converge to the equilibrium state. The proposed OASMC is designed to bring the system states to the equilibrium state using minimum control effort.

A linear uncertain system is defined as

$$\begin{aligned} &\dot{x}_i(t) = x_{i+1}(t), \;\; i = 1,2,\cdots,n-1 \\ &\dot{x}_n(t) = a_1x_1(t) + a_2x_2(t) + \cdots + a_nx_n(t) + b_1u(t) + \Delta a x(t) + \Delta b_1 u(t) + \omega_1(t) \qquad (1) \\ &y(t) = x_1(t) \end{aligned}$$

where $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$ is the state vector, $a_1, a_2, \cdots, a_n, b_1$ are known integers and the perturbation in the system is defined as $\Delta a = \begin{bmatrix} \Delta a_1 & \Delta a_2 & \cdots & \Delta a_n \end{bmatrix}$ and Δb_1. The disturbance of the system is defined as $\omega_1(t)$. Uncertain part of the system is bounded and satisfies the matched condition but the bound of the uncertainty is unknown. The output of the system is denoted by $y(t)$. The linear uncertain system (1) can be written as

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t) + \Delta Ax(t) + \Delta Bu(t) + \omega(t) \\ &y(t) = x_1(t) \qquad (2) \end{aligned}$$

Where $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ddots & 1 \\ a_1 & a_2 & \cdots & a_n \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_1 \end{bmatrix}$. Uncertainties affecting the system are defined as $\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ddots & 0 \\ \Delta a_1 & \Delta a_2 & \cdots & \Delta a_n \end{bmatrix}$ $\Delta B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \Delta b_1 \end{bmatrix}$, $\omega(t) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \omega_1(t) \end{bmatrix}$.

The objective is to design an optimal sliding mode controller for the above uncertain system to achieve stabilization and tracking at the expense of minimum control input. An adaptive tuning law is used to design a sliding mode controller with an unknown upper bound of the matched uncertainty.

OPTIMAL CONTROLLER DESIGN

In the calibration, the model equations, the kinematic parameters, and the estimators as shown in Figure 1(a) are replaced with the neural network as shown in Figure 2(a). The calibration method using the neural network is feasible and flexible. So it is called the adaptive calibration.

The optimal sliding mode control design is divided into two parts. In the first part, optimal control for the nominal system is designed and in the second part, the sliding surface as well as the switching control are designed.Hence the control input $u(t)$ in

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equation (2) is obtained as $u(t) = u_1(t) + u_2(t)$ where $u_1(t)$ is the optimal control applied to the nominal system and $u_2(t)$ is the sliding mode control to tackle uncertainties. The optimal control law for the nominal system is designed by using a conventional linear quadratic regulator (LQR) technique as discussed below.
Neglecting the uncertainties, the state equation of system (2) becomes

$$\dot{x}(t) = Ax(t) + Bu_1(t) \qquad (3)$$

The performance index to be minimized for the optimal control is defined as

$$J = \int_0^\infty \left(e(t)^T Q e(t) + u_1(t)^T R u_1(t) \right) dt \qquad (4)$$

where $Q \in \Re^{n \times n}$ and $R \in \Re$ are positive definite weighing matrices. The optimal control law $u_1(t)$ is given by

$$u_1(t) = -R^{-1} B^T P e(t) = -Ke(t) \qquad (5)$$

Where $K = R^{-1} B^T P$ and P is a symmetric positive definite matrix which is the solution of the algebraic Riccati equation

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \qquad (6)$$

ADAPTIVE SLIDING MODE CONTROLLER DESIGN

After designing the optimal controller, the uncertainty system (2) can be defined as

$$\dot{x}(t) = Ax(t) - BKx(t) + Bu_2(t) + \Delta Ax(t) + \Delta Bu(t) + \omega(t) \qquad (7)$$

To combine the optimal controller with a sliding mode controller (SMC), an integral sliding surface $s(t)$ is defined as

$$s(t) = Cx(t) - Cx(0) - \int_0^t C\left[Ax(\tau) - BK\, x(\tau) \right] d\tau = 0 \qquad (8)$$

where $C \in \Re^{1 \times n}$, $x(0)$ is the initial state and CB is considered as nonsingular. Here the upper bound of the system uncertainty is unknown. So, the controller gain is designed using the adaptive law proposed by Plestan et al. (Plestan, 2008). The adaptation law by Plestan et al. (Plestan, 2008) guarantees a real sliding mode. The gain $\Psi(t)$ is found by using the following tuning law (Plestan, 2008),

$$\dot{\Psi}(t) = \begin{cases} \Xi |s(t)| sign\left(|s(t)| - \varepsilon\right) & if \;\; \Psi(t) > \nu \\ \nu & if \;\; \Psi(t) \le \nu \end{cases} \qquad (9)$$

where $\Xi > 0$, $\varepsilon > 0$ and $\nu > 0$ are very small with $\Psi(0) > 0$. Parameter ν is introduced to get only positive values for Ψ.
Hence, the control law $u_2(t)$ is designed as

$$u_2(t) = -(CB)^{-1} \Psi(t) sign\left(s(t)\right) \qquad (10)$$

The gain $\Psi(t)$ has an upper bound meaning that there exists a positive constant $\hat{\Psi}$ so that $\Psi(t) \le \hat{\Psi},\ \forall t > 0$.

STABILITY ANALYSIS OF THE SLIDING SURFACE

Let us consider the Lyapunov function $V_1(t)$ as

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$$V_1(t)=\frac{1}{2}s(t)^2+\frac{1}{2\gamma}\left(\Psi(t)-\hat{\Psi}\right)^2 \quad where\ \gamma>0$$

$$\begin{aligned}\dot{V}_1(t)&=s(t)\dot{s}(t)+\frac{1}{2\gamma}\left(\Psi(t)-\hat{\Psi}\right)\dot{\Psi}(t)\\&=s(t)\left[CBu_2(t)+C\left(\Delta Ax(t)+\Delta Bu(t)+\omega(t)\right)\right]+\frac{1}{2\gamma}\left(\Psi(t)-\hat{\Psi}\right)\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)\\&=s(t)\left(-\Psi(t)sign\left(s(t)+\zeta\left(x(t),u(t),\omega(t)\right)\right)+\frac{1}{\gamma}\left(\Psi(t)-\hat{\Psi}\right)\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)\right)\end{aligned} \tag{11}$$

where $\zeta\left(x(t),u(t),\omega(t)\right)=C\left(\Delta Ax(t)+\Delta Bu(t)+\omega(t)\right)$

$$\begin{aligned}\dot{V}_1(t)&\le-\Psi(t)|s(t)|+\zeta\left(x(t),u(t),\omega(t)\right)|s(t)|+\frac{1}{\gamma}\left(\psi(t)-\hat{\psi}\right)\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)\\&\le-\Psi(t)|s(t)|+\zeta\left(x(t),u(t),\omega(t)\right)|s(t)|+\hat{\psi}|s(t)|-\hat{\psi}|s(t)|\\&\quad+\frac{1}{\gamma}\left(\psi(t)-\hat{\psi}\right)\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)\\&\le\zeta\left(x(t),u(t),\omega(t)-\hat{\psi}\right)|s(t)|+\left(\psi(t)-\hat{\psi}\right)\left(-|s(t)|+\frac{1}{\gamma}\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)\right)\\&\le\zeta\left(x(t),u(t),\omega(t)-\hat{\psi}\right)|s(t)|+\left(\psi(t)-\hat{\psi}\right)\left(-|s(t)|+\frac{1}{\gamma}\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)\right)+\beta_\psi|\psi(t)-\hat{\psi}|\\&\quad-\beta_\psi|\psi(t)-\hat{\psi}|\end{aligned} \tag{12}$$

in which $\beta_\psi>0$

$$\begin{aligned}\dot{V}_1(t)&\le-\left(-\zeta\left(x(t),u(t),\omega(t)\right)+\hat{\psi}\right)|s(t)|-\beta_\psi|\psi(t)-\hat{\psi}|\\&\quad-|\psi-\hat{\psi}|\left(-|s(t)|+\frac{1}{\gamma}\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)-\beta_\psi\right)\\&\le-\beta_s|s(t)|-\beta_\psi|\psi(t)-\hat{\psi}|-\Gamma\end{aligned} \tag{13}$$

where

$$\beta_s=\left(-\zeta\left(x(t),u(t),\omega(t)\right)+\hat{\psi}\right)>0 \tag{14}$$

and

$$\Gamma=|\psi(t)-\hat{\psi}|\left(-|s(t)|+\frac{1}{\gamma}\Xi|s(t)|sign\left(|s(t)|-\varepsilon\right)-\beta_\psi\right) \tag{15}$$

Suppose $s(t)\neq0$. Form the dynamic of $\psi(t)$ and for bounded uncertainty $\zeta\left(x(t),u(t),\omega(t)\right)$, it follows that $\psi(t)$ is increasing and there exists a time t^* such that

$$\hat{\psi}=\psi(t^*)>\zeta\left(x(t^*),u(t^*),\omega(t^*)\right) \tag{16}$$

Hence,

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$$\begin{aligned}\dot{V}_1(t) &\le -\beta_s|s(t)| - \beta_\psi|\psi(t)-\hat{\psi}| - \Gamma \\ &\le -\min\left\{\beta_s\sqrt{2}, \beta_\psi\sqrt{2\gamma}\right\}\left(\frac{|s(t)|}{\sqrt{2}} + \frac{|\psi(t)-\hat{\psi}|}{\sqrt{2\gamma}}\right) - \Gamma \\ &\le -\hat{\beta}V_1^{\frac{1}{2}} - \Gamma\end{aligned} \tag{17}$$

where $\hat{\beta} = \sqrt{2}\min\left\{\beta_s, \beta_\psi\sqrt{\gamma}\right\}$.

Case 1. Suppose $|s(t)| > \varepsilon$. Then from (15), it is found that Γ is positive if $-|s(t)| + \frac{1}{\gamma}\Xi|s(t)| - \beta_\Psi > 0$ or $\gamma < \frac{\Xi|s(t)|}{|s(t)| + \beta_\Psi}$. It is always possible to choose γ such that the inequality condition $\gamma < \frac{\Xi|s(t)|}{|s(t)| + \beta_\Psi}$ is satisfied. Hence, it can be written as

$$\begin{aligned}\dot{V}_1(t) &\le -\hat{\beta}V_1^{\frac{1}{2}} - \Gamma \\ &\le -\hat{\beta}V_1^{\frac{1}{2}}\end{aligned} \tag{18}$$

Therefore, finite time convergence of $s(t)$ to a domain $|s(t)| \le \varepsilon$ is guaranteed.

Case 2. Suppose $|s(t)| \le \varepsilon$. So, Γ can be negative. It signifies that $\dot{V}_1(t)$ would be sign indefinite and it is impossible to conclude about the stability. Therefore, $|s(t)|$ can increase over ε. As soon as $|s(t)|$ becomes greater than ε, $\dot{V}_1(t) \le -\hat{\beta}V_1^{\frac{1}{2}}$ and $V_1(t)$ starts decreasing.

SIMULATION RESULTS

Let us consider the following mathematical model of a mass-spring-damper system.

$$\begin{aligned}x(t) &= \begin{bmatrix} 0 & 1 \\ -5 & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0.5\sin t \end{bmatrix} \\ y(t) &= x_1(t)\end{aligned} \tag{19}$$

Here, the state vector $x(t) = [x_1(t) \quad x_2(t)]^T$ where $x_1(t)$ is the position of the mass and $x_2(t)$ is the velocity of the mass. The output tracking problem for this system is considered and the desired output for tracking is chosen as $x_{d1}(t) = 0.5\sin(\sqrt{5}t)$. The control input $u(t)$ is the force applied to the system. System matrix $A = \begin{bmatrix} 0 & 1 \\ -5 & -0.5 \end{bmatrix}$, input matrix $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and the uncertain part of the system is defined as $\zeta(x(t), u(t), \omega(t)) = \begin{bmatrix} 0 \\ 0.5\sin t \end{bmatrix}$. The initial condition is $x(0) = [1 \quad 2]^T$.

The proposed first-order optimal adaptive sliding mode controller (OASMC) is now used for stabilization of the above linear uncertain system. For the stabilization problem, the weighing matrices Q and R in (4) are chosen as follows,

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = 1$$

The state feedback gain matrix K in (5) for the given system is found as,

$$K = [0.0990 \quad 0.7033]$$

In the first order OASMC (8), the value of C is chosen as $[2 \quad 1]$. Parameters in the gain adaptation law (9) are chosen as $\Psi(0) = 0.9,\ \Xi = 0.5,\ \varepsilon = 0.8,\ \nu = 0.08$.

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The states x_1 and x_2 obtained by using the proposed OASMC is shown in Figure (1). The control inputs obtained by using this controller is shown in Figure (2).

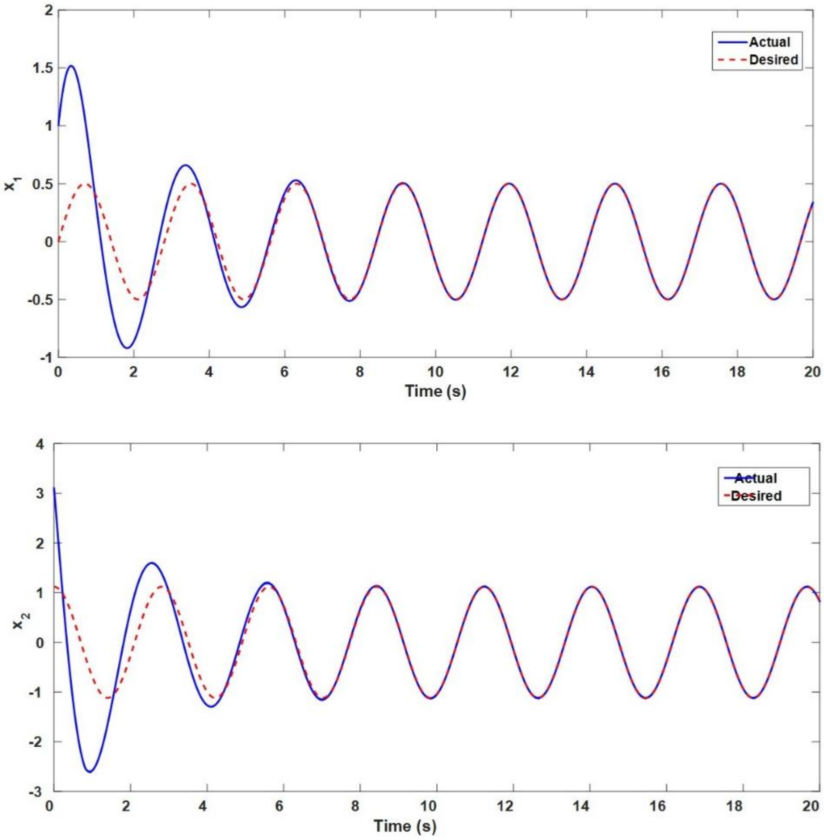

Figure 1. State $x_1(t)$ and $x_2(t)$ obtained by applying the proposed OASMC for tracking the linear uncertain system

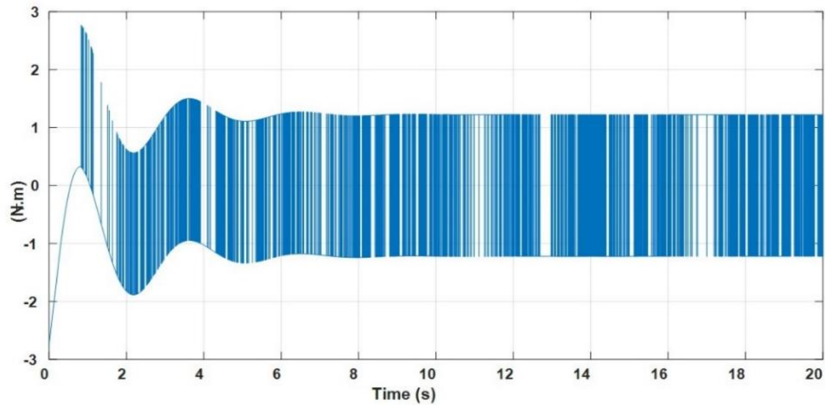

Figure 2. Control input obtained by applying the proposed OASMC for tracking the linear uncertain system

By comparing these figures with results which have been achieved in Ref. (Laghrouche, 2007), it is clearly evident that the proposed OASMC uses lesser control input compared to the other two controllers while offering similar convergence speed of the state. Table (1) compare the control energies by computing the second norm of the control until 20 sec. for tracking problem application. It is clear from this table that the proposed OASMC is able to reduce the control effort for comparable performance standard.

Table 1: Comparison of control energy of conventional SMC, integral SMC proposed by Laghrouche et al. (Laghrouche, 2007) and the proposed OASMC for the tracking problem

Method	**Control Energy**
Conventional SMC	33.6
Integral SMC proposed by Laghrouche et. al. (Laghrouche, 2007)	25.56
Proposed OASMC	14.02

CONCLUSION

In this paper, a first-order optimal adaptive sliding mode controller (OASMC) was designed for the linear system affected by matched uncertainty with an unknown upper bound. The optimal controller was designed by using the LQR technique for the nominal linear system and then an integral sliding mode controller is combined. An adaptive gain tuning method was used to tackle the unknown upper bound of the matched uncertainty for designing the OASMC. The proposed OASMC was applied for stabilization problems. Compared to conventional SMCs and the integral SMC, the proposed OASMC spends lowered control energy while maintaining similar performance standard. From simulation results, it was observed that the proposed optimal SMC requires lesser control effort while offering almost similar performance as that of the conventional SMC.

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