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A Robust Optimization Approach to Supply Chain Cost Optimization

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Abstract

Problems under uncertainties require an efficient operation of supply chain planning. In this paper, we discuss and illustrate a four-layer supply chain network including suppliers, manufacturers, warehouses, and markets, which is designed to address the production and transportation of a single product. We use a robust optimization approach for the analysis of uncertainties associated with the primary input parameters including demand, supply, and cost parameters. Additionally, we provide a numerical example of a supply chain network and show how sensitive is the objective function to the conservatism degrees at different levels of changes in the decision parameters. It is found that the objective function is most sensitive to the forecast of uncertain demands.

Key words: Robust Optimization, Supply Chain Cost, Uncertainty

INTRODUCTION

The structure of supply chain has major effects on decisions of the businesses and may contribute to sustainable advantages. The decisions related to the structure of supply chain are categorized into three primary themes; strategic, tactical planning, and operational planning decisions. Strategic decisions include determining the optimal location of the suppliers, manufacturers, and warehouses, as wells as the capacity of the facilities. Tactical planning decisions include those of related to demand, production, and distribution. These decisions are inherently different in terms of timeline, goal, and nature. The costs associated with each decision in a supply chain must be taken into account in order to assure that the supply chain is cost-effective (Namdari et al., 2017). The mathematical formulation of the supply chain models have long been subject of academic studies. The primary objectives of supply chain models are maximizing service levels, minimizing transit times, and minimizing the total costs including those of locating the manufacturers and warehouses, transportation of goods and products between the layers of a supply chain, etc. Hence, it is important to present an appropriate mathematical model (Tohidi, et al. 2017), which results in the minimum costs. Supply chain management with consideration of conflicting interests of the participants in the supply chain is very important for academia and industry (Yue&You, 2014).

The classical approach in mathematical formulation is based on deterministic or known input data and does not take into account the variations in uncertain parameters (Soyster, 1973). The disadvantage of these models is that the optimal solution may not be feasible as the input data takes values different from the nominal values. Therefore, it is required to design robust approaches that take into account data uncertainty. The idea of robust optimization can be found in Ben-Tal and Nemirovski (1998) and Ghaoui et al. (1998). The drawback of the robust formulation is that it results in nonlinear models, which involve more inherent difficulty and computational complexities relative to the earlier linear models. Bertsimas and Sim (2004) proposed a new approach for robust linear optimization that protects against the violations of the constraints under a prespecified conservatism degree of coefficient of the uncertain parameters and results in a feasible solution. Additionally, they extended their approach to a probabilistic method, which guarantees feasible solution with high probability even if more than the prespecified conservatism degree in uncertain parameters are observed. Baron et al. (2011) used a robust approach and formulated the uncertainties in demand based on a mixed integer conic program model. Ben-Tal et al. (2004) proposed the Adjustable Robust Counterpart (ARC) methodology which is capable of adjusting the uncertain parameters. They applied dynamic programing to enhance the tractability of their proposed model and solve an infinite-dimensional problem. Klibi et al. (2010) presented a review on the robust optimization of supply chain network and examined the effect of uncertainties on the network. Delage and Ye (2010) and Wiesemann et al. (2014) investigated the idea of Distributionally Robust Optimization to examine models with uncertain parameters. Aghezzaf (2005) and Jin et al. (2014) tested model robustness measures for Supply Chain Network Design (SCND).

Related work can be categorized according to type of model, transportation, product, objective function, approach to solving the problem and data sets. The authors have proposed different objective functions for single and multiple products under certainty and uncertainty. The importance of presenting an accurate supply chain model and an appropriate mathematical approach to solving the model and determining the most significant variable affecting the supply chain costs have been addressed in limited research. Most authors have used the traditional methods to model the inherent uncertainties in the supply chain. In this paper, we present a mathematical formulation for robust supply chain network, which determines the strategic and tactical decisions and minimizes the total costs including the cost of locating the manufacturers and warehouses as well as the shipment costs. The model and case study presented in this paper has not previously been studied in the literature.

Mathematical Modeling of the Supply Chain Costs:

This section introduces a robust model when uncertainties are applied to cost of locating the manufacturers, cost of locating the warehouses, shipment costs, demand of the markets, and supply capacity of the suppliers. To specify the study scope, two assumptions and simplifications are postulated in the proposed model formulation as follows:

- The uncertain variables are assumed to have only positive deviations.
- The uncertainty budgets are assumed to take only integer values.

The robust model of the supply chain network can be written as follows:

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$$Min\tilde{Z} = \sum_a t_a^T y_a + \sum_b t_b^W z_b + \sum_g \sum_a c_{ga}^{ST} x_{ga} + \sum_a \sum_b c_{ab}^{TW} u_{ab} + \sum_b \sum_q c_{bq}^{WM} v_{bq}$$

$$+ \max_{\Omega^{t_1}\Omega^{t_2}\Omega^{c_1}\Omega^{c_2}\Omega^{c_3}} \left\{ \sum_{q \in S^{t_1}} \hat{t}_a^T y_a + \sum_{q \in S^{t_2}} \hat{t}_b^W y_b + \sum_{(g,a) \in S^{c_1}} \hat{c}_{ga}^{ST} x_{ga} + \sum_{(a,b) \in S^{c_2}} \hat{c}_{ab}^{TW} x_{ab} + \sum_{(b,q) \in S^{c_3}} \hat{c}_{bq}^{WM} x_{bq} \right\} + \sum_q L_q A_q$$

$$\Omega^{t_1} = \{S^{t_1} | S^{t_1} \subseteq Q^{t_1}, |S^{t_1}| \le \Gamma^{t_1}\}$$

$$\Omega^{t_2} = \{S^{t_2} | S^{t_2} \subseteq Q^{t_2}, |S^{t_2}| \le \Gamma^{t_2}\}$$

$$\Omega^{c_1} = \{S^{c_1} | S^{c_1} \subseteq Q^{c_1}, |S^{c_1}| \le \Gamma^{c_1}\}$$

$$\Omega^{c_2} = \{S^{c_2} | S^{c_2} \subseteq Q^{c_2}, |S^{c_2}| \le \Gamma^{c_2}\}$$

$$\Omega^{c_3} = \{S^{c_3} | S^{c_3} \subseteq Q^{c_3}, |S^{c_3}| \le \Gamma^{c_3}\}$$

$$Q^{t_1} = \{a | \hat{t}_a^T > 0\}, \Gamma^{t_1} \in [0, |Q^{t_1}|]$$

$$Q^{t_2} = \{b | \hat{t}_b^W > 0\}, \Gamma^{t_2} \in [0, |Q^{t_2}|]$$

$$Q^{c_1} = \{(g,a) | \hat{c}_{ga}^{ST} > 0\}, \Gamma^{c_1} \in [0, |Q^{c_1}|]$$

$$Q^{c_2} = \{(a,b) | \hat{c}_{ab}^{TW} > 0\}, \Gamma^{c_2} \in [0, |Q^{c_2}|]$$

$$Q^{c_3} = \{(b,q) | \hat{c}_{bq}^{WM} > 0\}, \Gamma^{c_3} \in [0, |Q^{c_3}|] \quad (1)$$

Uncertainty in t_a^T, t_b^W, c_{ga}^{ST}, c_{ab}^{TW}, and c_{bq}^{WM}:

Following the robust optimization procedure, linear protection functions are derived for the uncertain coefficients and the linear dual forms for the uncertain parameters are formulated. The nonlinear model can be transformed into the following linear model:

$$Min\tilde{Z} = \sum_a t_a^T y_a + \sum_b t_b^W z_b + \sum_g \sum_a c_{ga}^{ST} x_{ga} + \sum_a \sum_b c_{ab}^{TW} u_{ab} + \sum_b \sum_q c_{bq}^{WM} v_{bq} + \lambda^{t_1}\Gamma^{t_1} + \sum_{g \in Q^t} \mu_g^{t_1} + \lambda^{t_2}\Gamma^{t_2} + \sum_{b \in Q^t} \mu_b^{t_2} + \lambda^{c_1}\Gamma^{c_1} + \sum_{(g,a) \in Q^{c_1}} \mu_{ga}^{c_1} + \lambda^{c_2}\Gamma^{c_2} + \sum_{(a,b) \in Q^{c_2}} \mu_{ab}^{c_2} + \lambda^{c_3}\Gamma^{c_3} + \sum_{(b,q) \in Q^{c_3}} \mu_{bq}^{c_3} + \sum_q L_q A_q \quad (2)$$

The objective function is subject to the following constraints in its variables.

$$\mu_{ga}^{c_1} + \lambda^{c_1} \ge \hat{c}_{ga}^{ST} x_{ga} \quad \forall (g,a) \in Q^{c_1} \quad (3)$$

$$\mu_{ab}^{c_2} + \lambda^{c_2} \ge \hat{c}_{ab}^{TW} u_{ab} \quad \forall (a,b) \in Q^{c_2} \quad (4)$$

$$\mu_{bq}^{c_3} + \lambda^{c_3} \ge \hat{c}_{bq}^{WM} v_{bq} \quad \forall (b,q) \in Q^{c_3} \quad (5)$$

$$\mu_a^{t_1} + \lambda^{t_1} \ge \hat{t}_a^T y_a \quad \forall a \in Q^{t_1} \quad (6)$$

$$\mu_b^{t_2} + \lambda^{t_2} \ge \hat{t}_b^W z_b \quad \forall b \in Q^{t_2} \quad (7)$$

$$\mu_{ga}^{c_1} \ge 0 \quad \forall (g,a) \in Q^{c_1} \quad (8)$$

$$\mu_{ab}^{c_2} \ge 0 \quad \forall (a,b) \in Q^{c_2} \quad (9)$$

$$\mu_{bq}^{c_3} \ge 0 \quad \forall (b,q) \in Q^{c_3} \quad (10)$$

$$\mu_a^{t_1} \ge 0 \quad \forall a \in Q^{t_1}, \ \mu_b^{t_2} \ge 0 \quad \forall b \in Q^{t_2} \quad (11) – (12)$$

$$\lambda^{c_1} \ge 0, \lambda^{c_2} \ge 0, \lambda^{c_3} \ge 0, \lambda^{t_1} \ge 0, \lambda^{t_2} \ge 0 \quad (13) – (14)$$

Uncertainty in d_q, s_g, and L_q:

We define the auxiliary variable H_q and add constraint (44) to the model:

$$L_q A_q \le H_q \quad \forall q \quad (15)$$

$$d + \frac{\Gamma^m}{m}\hat{d} - \sum_b v_{bq} \le A_q \quad \forall q \quad (16)$$

$$\sum_a x_{ga} - s + \frac{\Gamma^l}{l}\hat{s} \le S_g \quad \forall g \quad (17)$$

$$L_q\left(d + \frac{\Gamma^m}{m}\hat{d} - \sum_b v_{bq}\right) + \lambda_q^p \Gamma_q^p + \mu_q^p \le H_q \quad \forall q \quad (18)$$

$$\mu_q^p \ge 0 \quad \forall q, \ \lambda_q^p \ge 0 \quad \forall q \quad (19) - (20)$$

The model can be transformed to the following counterpart:

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$$Min\tilde{Z} = \sum_a t_a^T y_a + \sum_b t_b^W z_b + \sum_g \sum_a c_{ga}^{ST} x_{ga} + \sum_a \sum_b c_{ab}^{TW} u_{ab} + \sum_b \sum_q c_{bq}^{WM} v_{bq} + \lambda^{t_1}\Gamma^{t_1} + \sum_{g\in Q^t} \mu_g^{t_1} + \lambda^{t_2}\Gamma^{t_2} + \sum_{b\in Q^t} \mu_b^{t_2} + \lambda^{c_1}\Gamma^{c_1} + \sum_{(g,a)\in Q^{c_1}} \mu_{ga}^{c_1} + \lambda^{c_2}\Gamma^{c_2} + \sum_{(a,b)\in Q^{c_2}} \mu_{ab}^{c_2} + \lambda^{c_3}\Gamma^{c_3} + \sum_{(b,q)\in Q^{c_3}} \mu_{bq}^{c_3} + \sum_q H_q \quad (21)$$

Case Study Description:

In this section we present a case study a supply chain including suppliers, manufacturers, warehouses, and markets and the model is solved under certain assumptions and constraints. All nominal values are generated numbers. There are nine potential suppliers (A1 – A9), five potential manufacturers (L1 – L5), eight potential warehouses (Q1 – Q8), and sixteen potential markets (G1 – G16). The values of the primary parameters are presented in Exhibit 1.

Exhibit 1: Nominal values of the parameters.

Parameters	Values
s_g	4,356,000 kg
d_q	2,360,000 kg
k_a	7,325,000 kg
w_b	8,125,000 kg
t_a^T	$1,200,000
t_b^W	$2,800,000
P_q	$1,250

Costs of shipping raw materials from suppliers to manufacturers, shipping products from manufacturers to warehouses, and shipping products from warehouses to markets are summarized in Exhibit 2, Exhibit 3, and Exhibit 4, respectively.

Exhibit 2: Costs of shipping raw materials from suppliers to manufacturers.

	L1	L2	L3	L4	L5
A1	$1,580	$660	$854	$1,735	$1,274
A2	$1,233	$926	$466	$1,163	$1,398
A3	$826	$374	$1,471	$389	$1,272
A4	$1,070	$1,654	$884	$652	$976
A5	$903	$1,718	$662	$830	$1,121
A6	$414	$1,036	$906	$1,532	$744
A7	$660	$1,034	$444	$323	$1,417
A8	$485	$806	$498	$364	$583
A9	$576	$1,650	$1,714	$553	$1,330

Exhibit 3: Costs of shipping products from manufacturers to warehouses.

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
L1	$338	$1,309	$498	$938	$1,321	$370	$353	$504
L2	$579	$1,109	$761	$592	$1,239	$491	$393	$1,301
L3	$913	$733	$764	$1,155	$815	$712	$322	$659
L4	$1,115	$667	$1,163	$793	$909	$399	$396	$340
L5	$205	$681	$1,134	$556	$863	$1,198	$666	$1,277

Exhibit 4. Costs of shipping products from warehouses to markets

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12	G13	G14	G15	G16
Q1	$1,178	$883	$388	$605	$535	$388	$957	$919	$734	$884	$206	$814	$530	$791	$318	$565
Q2	$583	$344	$981	$1,160	$504	$469	$887	$650	$301	$651	$1,000	$981	$1,015	$421	$639	$703
Q3	$222	$229	$132	$702	$1,187	$848	$1,094	$628	$364	$618	$1,000	$599	$984	$575	$473	$1,137
Q4	$384	$426	$1,122	$673	$141	$250	$1,080	$1,096	$1,076	$165	$895	$576	$166	$117	$1,147	$559
Q5	$550	$450	$904	$354	$1,074	$894	$467	$771	$131	$850	$265	$1,008	$539	$1,183	$1,113	$1,182
Q6	$754	$567	$637	$638	$1,105	$217	$869	$780	$639	$146	$826	$191	$680	$284	$157	$431

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Q7	$388	$659	$736	$787	$976	$819	$317	$1,046	$284	$178	$670	$246	$558	$216	$912	$871
Q8	$763	$194	$361	$847	$208	$644	$133	$986	$1,177	$674	$1,171	$290	$823	$510	$396	$833

Results and Findings:

Conservatism degrees of parameter coefficients associated with supply and demand, as well as locating manufacturers and warehouses are [0,9], [0,16], [0,5], and [0,8]. Conservatism degrees of coefficients associated with shipment costs parameters are [0,45], [0,40], and [0,128]. Coefficients of raw material consumption in manufacturers and product consumption in warehouses are 0.75 and 1.1, respectively,

We determine the sensitivity of the optimal costs to 5, 12, 18, 25, and 30 percent variabilities in each uncertain parameter. We also compare the objective function for different conservatism degrees of coefficients. The sensitivity of the objective function to variation in uncertain parameters as well as the conservatism degrees are illustrated in Exhibit 5 – Exhibit 11. The optimal costs are determined using the robust formulation except with the case that the conservatism degree is zero, which is determined using the deterministic model. Several items can be understood from the numerical results depicted in Exhibit 5 – Exhibit 11.

The highest values of conservatism degrees of demand and supply parameters result in the worst objective value (Exhibit 5 and Exhibit 6). This is not the case for location and shipment costs. In the case of uncertainties in the location and shipment costs, the worst objective values are associated with the conservatism degrees smaller than their maximum values (Exhibit 7 – Exhibit 11). These conservatism degrees are as follows: $\Gamma^{t_1} = 3$, $\Gamma^{t_2} = 5$, $\Gamma^{c_1} = 18$, $\Gamma^{c_2} = 15$, and $\Gamma^{c_3} = 30$, where the maximum values are $Q^{t_1} = 5$, $Q^{t_2} = 8$, $Q^{c_1} = 45$, $Q^{c_2} = 40$, and $Q^{c_3} = 128$.

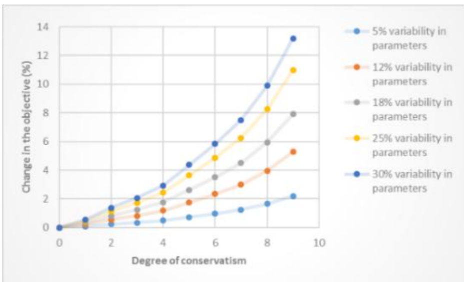

Exhibit 5: Sensitivity of the robust model to variations in supply.

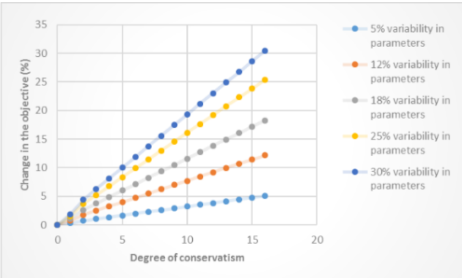

Exhibit 6: Sensitivity of the robust model to variations in demand.

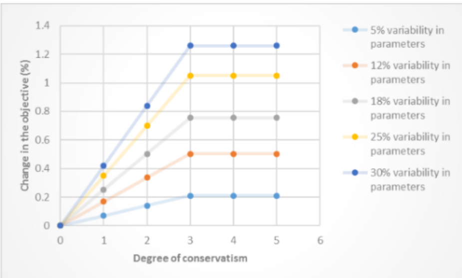

Exhibit 7: Sensitivity of the robust model to variations in cost of locating manufacturers.

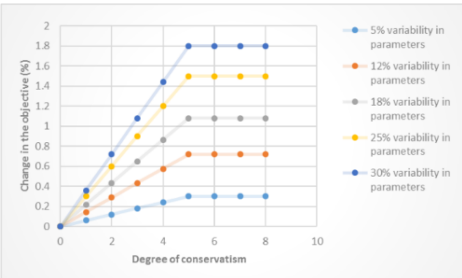

Exhibit 8: Sensitivity of the robust model to variations in cost of locating warehouses.

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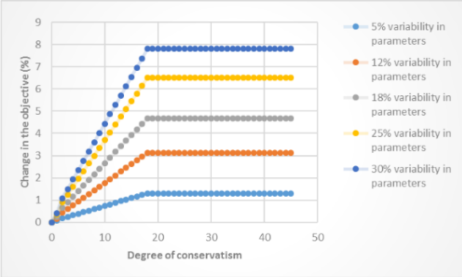

Exhibit 9. Sensitivity of the robust model to variations in cost of shipment from suppliers to manufacturers.

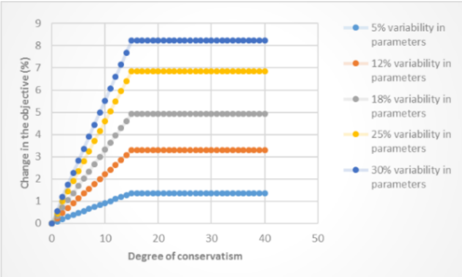

Exhibit 10: Sensitivity of the robust model to variations in cost of shipment from manufacturers to warehouses.

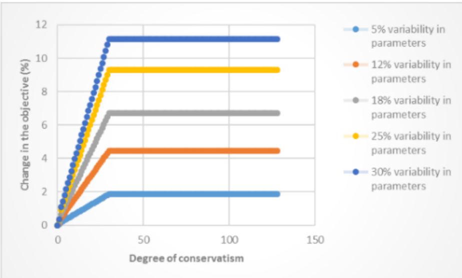

Exhibit 11: Sensitivity of the robust model to variations in cost of shipment from warehouses to markets.

Conclusion:

The objective of the robust model is to minimize the supply chain costs. The model is designed to incorporate the penalty costs in case demands of a certain market is not satisfied. A generated numerical example of supply chain network is provided. The objective function is solved following the robust optimization methods discussed in the literature and the optimal solutions are obtained when the decision constraints are satisfied. The optimal solutions for different conservatism degrees and variabilities in uncertain parameters are determined using the robust optimization model. It is shown how sensitive is the objective function to the changes in conservatism degrees. Changes in the optimal solutions are determined for a range of variabilities in the primary input parameters. It is found that the greatest conservatism degrees of demand and supply parameters result in the worst objective in terms of the supply chain costs. For location and shipment costs, the worst objectives are the results of conservatism degrees smaller than the maximum degrees. The results reveal that variation in demand causes the greatest changes in the optimal value of the objective function. The second most significant parameter is supply capacity.

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