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Compromise Solutions for Rough Multiple Objective Decision Making Problems

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Abstract

The technique for order preference by similarity to ideal solution (TOPSIS) is introduced to develop a methodology to find compromise solutions for the Multiple Objective Decision Making (MODM) Problems with Rough intervals parameters in the objective functions (RMODM) of Mixed-type. Anew algorithm is presented for the proposed TOPSIS approach for solving these types of mathematical programming problems. Also, an illustrative numerical example is solved and compared the solution of proposed algorithm with the ideal solutions.

Key words: Compromise Programming, Rough Programming, TOPSIS method, Multiple Objective Programming, Rough Intervals, and Rough Parameters.

INTRODUCTION

Compromise programming (CP) was initially proposed by Zeleny (1973) and subsequently used by many researchers. (Zeleny, M., 1973). Yu (1973) and Zeleny (1974) define the ideal solution as any solution that would simultaneously optimize each individual objective, (Yu, P.L., 1973; Zeleny, M., 1974).

Rough set theory is considered from the excellent mathematical tool for dealing with the description of vague objects, Rough set methodology has been introduced as new handling of analysis of vague concepts classificatory by Pawlak (1991). Any Vague concept is introduced by pair of precise concepts called "Lower & Upper" approximations, (Hamzehee, A., 2014; Osman, M.S., 2011; Youness, E., 2006).

Linear optimization problem which is considered where some or all of its coefficients in the objective function and/or constraints are rough intervals is introduced by Hamzehee *et al*. (2014).

Several algorithms for solving different kinds of large scale multiple objective optimization problems using TOPSIS approach are presented in (Abou-El-Enien, T.H.M., 2013). TOPSIS method assumes that any DM seeks a solution which has the shortest distance from positive ideal solution (PIS), and the farthest distance from the negative ideal solution (NIS), (Lai, Y.J., 1994).

In the following sections, the formulation of RMODM problems is given in section (2). Also, transformation of RMODM problem into deterministic RMODM problems (by use of TOPSIS method and "Lower & Upper" approximations method) is introduced in section (2). Anew algorithm for solving deterministic RMODM problems is proposed in section (3). For the sake of illustration, we present an example for the extended TOPSIS method and compared the solution of proposed algorithm with the ideal solutions in section (4).

Formulation of the problem:

Consider the following Linear Multiple Objective Decision Making (LMODM) Problem, (Hwang, C.L., and A.S.M. Masud, 1979; Yu, P.L., 1985; Zeleny, M., 1982), with rough parameters in the objective functions [RLMODM]:

$Maximize/Minimize\ (f_1(X_1,X_2,\dots,X_n,),\dots\dots,f_k(X_1,X_2,\dots,X_n))$
$subject\ to$ (1)
$X\in\mathbb{M}=\{X\in\mathbb{R}^n: DX\le b\}$
where
$f_i=\sum_{j=1}^{n}c_{ij}\,([Q_{ij}^{LL},Q_j^{HL}],[Q_{ij}^{LH},Q_{ij}^{HH}])x_j,\ i=1,2,\dots.,k,$ (2)
m : the number of constraints,
n : the number of variables,
k : the number of objective functions,
c_{ij} : realconstants coefficients of the objective functions , $(i=1,2,\dots.,k, j=1,\dots.,n)$.
b: an$m$-dimensional column vector of right-hand sides of constraints
D : an $(m\times n)$coefficient matrix,
$\mathbb{R}$: the set of all real numbers,
X : an n-dimensional column vector of variables,
$\mathbb{N}$ = *{1,2,.....,n}*,
$\mathbb{R}^n=\{X=(x_1,x_2,\dots.,x_n)^T: x_i\in\mathbb{R}, i\in\mathbb{N}\}$,
$[Q_{ij}^{LL},Q_{ij}^{HL}],[Q_{ij}^{LH},Q_{ij}^{HH}]$are rough interval coefficients of the objective functions , $(i=1,2,\dots.,k, j=1,\dots.,n)$.

Using the upper and lower approximation method, (Hamzehee, A., 2014), the multiple objective decision making problems with Rough parameters in the

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objective functions (RMODM) can be transformed to the following four deterministic LMODM problems:

$\underline{P^{LL}}$: (Lower interval - Lower interval coefficients)

$Maximize/Minimize F_i^{LL} = \sum_{j=1}^{n} c_{ij}\, Q_{ij}^{LL} x_j,\ i = 1,2, \ldots, k,$
subject to (3)
$X \in \mathbb{M}$

$\underline{P^{HL}}$:(Lower interval - Upper interval coefficients)

$Maximize/Minimize F_i^{HL} = \sum_{j=1}^{n} c_{ij}\, Q_{ij}^{HL} x_j,\ i = 1,2, \ldots, k,$
subject to (4)
$X \in \mathbb{M}$

$\underline{P^{LH}}$: (Upper interval - Lower interval coefficients)

$Maximize/Minimize F_i^{LL} = \sum_{j=1}^{n} c_{ij}\, Q_{ij}^{LH} x_j,\ i = 1,2, \ldots, k,$
subject to (5)
$X \in \mathbb{M}$

$\underline{P^{HH}}$:(Upper interval - Upper interval coefficients)

$Maximize/Minimize F_i^{HL} = \sum_{j=1}^{n} c_{ij}\, Q_{ij}^{HH} x_j,\ i = 1,2, \ldots, k,$
subject to (6)
$X \in \mathbb{M}$

TOPSIS Approach for RLMODM:

A modified version of TOPSIS method, (Abou-El-Enien, T.H.M., 2013; Lai, Y.J., 1994), is introduced to find compromise solutions, (Hwang, C.L., and A.S.M. Masud, 1979; Zeleny, M., 1982), for the RLMODM problems. Modified equations for the distance function equation from the positive ideal solution (PIS) and the distance function equation from the negative ideal solution (NIS) are introduced.

Algorithm (I):

Step 1:

Use the "Lower & Upper" approximations method to transform the RLMODM Problem (1) into the four deterministic LMODM problems (3)-(6).

Step 2:

I- Find the $PIS(F_t^{*LL})$, $PIS(F_t^{*HL})$, $PIS(F_t^{*LH})$, $PIS(F_t^{*HH})$, $NIS(F_v^{-LL})$, $NIS(F_v^{-HL})$, $NIS(F_v^{-LH})$ and $NIS(F_v^{-HH})$ which are (Abou-El-Enien, T.H.M.**,** 2013; Hamzehee, A., 2014; Lai, Y.J., 1994):

$$F^{*LL} = \underset{X\in\mathbb{M}}{Maximize(or\ Minimize)} F_t^{LL}(X)\big(or\ \ F_v^{LL}(X)\big), \forall\ t\ (\ and\ v)$$
$$F^{-LL} = \underset{X\in\mathbb{M}}{Minimize(or\ Maximize)} F_t^{LL}(X)\big(or\ \ F_v^{LL}(X)\big), \forall\ t\ (\ and\ v)$$
$$F^{*HL} = \underset{X\in\mathbb{M}}{Maximize(or\ Minimize)} F_t^{HL}(X)\big(or\ \ F_v^{HL}(X)\big), \forall\ t\ (\ and\ v)$$
$$F^{-HL} = \underset{X\in\mathbb{M}}{Minimize(or\ Maximize)} F_t^{HL}(X)\big(or\ \ F_v^{HL}(X)\big), \forall\ t\ (\ and\ v)$$
$$F^{*LH} = \underset{X\in\mathbb{M}}{Maximize(or\ Minimize)} F_t^{LH}(X)\big(or\ \ F_v^{LH}(X)\big), \forall\ t\ (\ and\ v)$$
$$F^{-LH} = \underset{X\in\mathbb{M}}{Minimize(or\ Maximize)} F_t^{LH}(X)\big(or\ \ F_v^{LH}(X)\big), \forall\ t\ (\ and\ v)$$
$$F^{*HH} = \underset{X\in\mathbb{M}}{Maximize(or\ Minimize)} F_t^{HH}(X)\big(or\ \ F_v^{HH}(X)\big), \forall\ t\ (\ and\ v)$$
$$F^{-HH} = \underset{X\in\mathbb{M}}{Minimize(or\ Maximize)} F_t^{HH}(X)\big(or\ \ F_v^{HH}(X)\big), \forall\ t\ (\ and\ v)$$

where

$F_t(X)$: Objective function for Maximization ,$t \in k_1 \subset k,$
$F_v(X)$: Objective function for Minimization ,$v \in k_2 \subset k,$
$k = k_1 \cup k_2,$

II- Construct PIS and NIS payoff tables for the four deterministic LMODM problems (3)-(6).

Step 3:

Let $w_i = w_i{}^*, i = 1,2, \ldots, k,$ where $\sum_{i=1}^{k} w_i = 1$and$= p^*, p^* \in \{1,2, \ldots, \infty\}$.

Step 4:

I- Construct distance functions$d_p^{PIS^{LL}}$and $d_p^{NIS^{LL}}$ for problem (3) as following, (Abou-El-Enien, T.H.M., 2013; Lai, Y.J., 1994):

$$d_p^{PIS^{LL}} = \left(\sum_{t\in k_1} w_t^p \left(\frac{F_t^{*LL} - F_t^{LL}(X)}{F_t^{*LL} - F_t^{-LL}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{LL}(X) - F_v^{*LL}}{F_v^{-LL} - F_v^{*LL}}\right)^p\right)^{1/p} \quad (7.1)$$

and

$$d_p^{NIS^{LL}} = \left(\sum_{t\in k_1} w_t^p \left(\frac{F_t^{LL}(X) - F_t^{-LL}}{F_t^{*LL} - F_t^{-LL}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{-LL} - F_v^{LL}(X)}{F_v^{-LL} - F_v^{*LL}}\right)^p\right)^{1/p} \quad (7.2)$$

II- Construct distance functions $d_p^{PIS^{HL}}$and $d_p^{NIS^{HL}}$ for problem (4) as following :

$$d_p^{PIS^{HL}} = \left(\sum_{t\in k_1} w_t^p \left(\frac{F_t^{*HL} - F_t^{HL}(X)}{F_t^{*HL} - F_t^{-HL}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{HL}(X) - F_v^{*HL}}{F_v^{-HL} - F_v^{*HL}}\right)^p\right)^{1/p} \quad (8.1)$$

and

$$d_p^{NIS^{HL}} = \left(\sum_{t\in k_1} w_t^p \left(\frac{F_t^{HL}(X) - F_t^{-HL}}{F_t^{*HL} - F_t^{-HL}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{-HL} - F_v^{HL}(X)}{F_v^{-HL} - F_v^{*HL}}\right)^p\right)^{1/p} \quad (8.2)$$

III- Construct distance functions $d_p^{PIS^{LH}}$and $d_p^{NIS^{LH}}$ for problem (5) as following:

$$d_p^{PIS^{LH}} = \left(\sum_{t\in k_1} w_t^p \left(\frac{F_t^{*LH} - F_t^{LH}(X)}{F_t^{*LH} - F_t^{-LH}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{LH}(X) - F_v^{*LH}}{F_v^{-LH} - F_v^{*LH}}\right)^p\right)^{1/p} \quad (9.1)$$

and

$$d_p^{NIS^{\mathrm{LH}}} = \left(\sum_{t\in k_1} w_t^p \left(\frac{F_t^{\mathrm{LH}}(X)-F_t^{-\mathrm{LH}}}{F_t^{*\mathrm{LH}}-F_t^{-\mathrm{LH}}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{-\mathrm{LH}}-F_v^{\mathrm{LH}}(X)}{F_v^{-\mathrm{LH}}-F_v^{*\mathrm{LH}}}\right)^p\right)^{1/p} \tag{9.2}$$

IV- Construct distance functions $d_p^{PIS^{\mathrm{HH}}}$ and $d_p^{NIS^{\mathrm{HH}}}$ for problem (6) as following :

$$d_p^{PIS^{\mathrm{HH}}} = \left(\sum_{t\in k_1} w_{i_{max}}^p \left(\frac{F_t^{*\mathrm{HH}}-F_t^{\mathrm{HH}}(X)}{F_t^{*\mathrm{HH}}-F_t^{-\mathrm{HH}}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{\mathrm{HH}}(X)-F_v^{*\mathrm{HH}}}{F_v^{-\mathrm{HH}}-F_v^{*\mathrm{HH}}}\right)^p\right)^{1/p} \tag{10.1}$$

and

$$d_p^{NIS^{\mathrm{HH}}} = \left(\sum_{t\in k_1} w_t^p \left(\frac{F_t^{\mathrm{HH}}(X)-F_t^{-\mathrm{HH}}}{F_t^{*\mathrm{HH}}-F_t^{-\mathrm{HH}}}\right)^p + \sum_{v\in k_2} w_v^p \left(\frac{F_v^{-\mathrm{HH}}-F_v^{\mathrm{HH}}(X)}{F_v^{-\mathrm{HH}}-F_v^{*\mathrm{HH}}}\right)^p\right)^{1/p} \tag{10.2}$$

Step 5:

I- Construct the following bi-objective problem with two commensurable (but conflicting) objectives, (Abou-El-Enien, T.H.M., 2013; Lai, Y.J., 1994), using the distance functions $d_p^{PIS^{\mathrm{LL}}}$ and $d_p^{NIS^{\mathrm{LL}}}$:

$Minimize\ d_p^{PIS^{\mathrm{LL}}}(X)$
$Maximize\ d_p^{NIS^{\mathrm{LL}}}(X)$
subject to (11)
$X \in \mathbb{M}$
where $p = 1,2, \ldots\ldots, \infty$.

II- Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{\mathrm{HL}}}$ and $d_p^{NIS^{\mathrm{HL}}}$:

$Minimize\ d_p^{PIS^{\mathrm{HL}}}(X)$
$Maximize\ d_p^{NIS^{\mathrm{HL}}}(X)$
subject to (12)
$X \in \mathbb{M}$
where $p = 1,2, \ldots\ldots, \infty$.

III- Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{\mathrm{LH}}}$ and $d_p^{NIS^{\mathrm{LH}}}$:

$Minimize\ d_p^{PIS^{\mathrm{LH}}}(X)$
$Maximize\ d_p^{NIS^{\mathrm{LH}}}(X)$
subject to (13)
$X \in \mathbb{M}$
where $p = 1,2, \ldots\ldots, \infty$.

IV- Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{\mathrm{HH}}}$ and $d_p^{NIS^{\mathrm{HH}}}$:

$Minimize\ d_p^{PIS^{\mathrm{HH}}}(X)$
$Maximize\ d_p^{NIS^{\mathrm{HH}}}(X)$
subject to (14)
$X \in \mathbb{M}$
where $p = 1,2, \ldots\ldots, \infty$.

Step 6:

I- Construct PIS Payoff table for problem (11) :
At $p = 1$, use the simplex method or the interior point method,
At $p \geq 2$, use the generalized reduced gradient method,
and obtain

$$d_p^{-\mathrm{LL}} = \left(\left(d_p^{PIS^{\mathrm{LL}}}\right)^-, \left(d_p^{NIS^{\mathrm{LL}}}\right)^-\right), \qquad d_p^{*\mathrm{LL}} = \left(\left(d_p^{PIS^{\mathrm{LL}}}\right)^*, \left(d_p^{NIS^{\mathrm{LL}}}\right)^*\right).$$

where

$\left(d_p^{PIS^{\mathrm{LL}}}\right)^* = \underset{X\in\mathbb{M}}{Minimize}\, d_p^{PIS^{\mathrm{LL}}}(X)$ *and the solution is* $X^{PIS^{LL}}$,
$\left(d_p^{NIS^{\mathrm{LL}}}\right)^* = \underset{X\in\mathbb{M}}{Maximize}\, d_p^{NIS^{\mathrm{LL}}}(X)$ *and the solution is* $X^{NIS^{LL}}$,
$\left(d_p^{PIS^{\mathrm{LL}}}\right)^- = d_p^{PIS^{\mathrm{LL}}}\left(X^{NIS^{LL}}\right)$ *and* $\left(d_p^{NIS^{\mathrm{LL}}}\right)^- = d_p^{NIS^{\mathrm{LL}}}\left(X^{PIS^{LL}}\right)$.

II- Construct PIS Payoff table for problem (12):
At $p = 1$, use the simplex method or the interior point method ,
At $p \geq 2$, use the generalized reduced gradient method ,
and obtain

$$d_p^{-\mathrm{HL}} = \left(\left(d_p^{PIS^{\mathrm{HL}}}\right)^-, \left(d_p^{NIS^{\mathrm{HL}}}\right)^-\right), \qquad d_p^{*\mathrm{HL}} = \left(\left(d_p^{PIS^{\mathrm{HL}}}\right)^*, \left(d_p^{NIS^{\mathrm{HL}}}\right)^*\right).$$

III- Construct PIS Payoff table for problem (13) :
At $p = 1$, use the simplex method or the interior point method ,
At $p \geq 2$, use the generalized reduced gradient method ,
and obtain

$$d_p^{-\mathrm{LH}} = \left(\left(d_p^{PIS^{\mathrm{LH}}}\right)^-, \left(d_p^{NIS^{\mathrm{LH}}}\right)^-\right), \qquad d_p^{*\mathrm{LH}} = \left(\left(d_p^{PIS^{\mathrm{LH}}}\right)^*, \left(d_p^{NIS^{\mathrm{LH}}}\right)^*\right).$$

IV- Construct PIS Payoff table for problem (14) :
At $p = 1$, use the simplex method or the interior point method ,
At $p \geq 2$, use the generalized reduced gradient method ,
and obtain

$$d_p^{-\mathrm{HH}} = \left(\left(d_p^{PIS^{\mathrm{HH}}}\right)^-, \left(d_p^{NIS^{\mathrm{HH}}}\right)^-\right), \qquad d_p^{*\mathrm{HH}} = \left(\left(d_p^{PIS^{\mathrm{HH}}}\right)^*, \left(d_p^{NIS^{\mathrm{HH}}}\right)^*\right).$$

Step 7:

I- Construct the following satisfactory level model (for finite value of p), (Bellman, R.E. and L.A. Zadeh, 1970; Dauer, P. and M.S.A. Osman, 1985; Dubois, J.D. and A. Prade, 1980; Zimmermann, H.J., 1996), for problem (11):

$Maximize\ \delta^{\mathrm{LL}}$
subject to (15)

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$\left(\frac{d_p^{PIS^{\text{LL}}}(X)-(d_p^{PIS^{\text{LL}}})^*}{\left(d_p^{PIS^{\text{LL}}}\right)^- -\left(d_p^{PIS^{\text{LL}}}\right)^*}\right) \geq \delta^{\text{LL}}$,

$\left(\frac{\left(d_p^{NIS^{\text{LL}}}\right)^* - d_p^{NIS^{\text{LL}}}(X)}{\left(d_p^{NIS^{\text{LL}}}\right)^* -\left(d_p^{NIS^{\text{LL}}}\right)^-}\right) \geq \delta^{\text{LL}}$,

$X \in \mathbb{M}, \quad \delta^{\text{LL}} \in [0,1]$

whereδ^{LL} is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

II- Construct the following satisfactory level model (for finite value of p) for problem (12):

$Maximize\ \delta^{\text{HL}}$

subject to (16)

$\left(\frac{d_p^{PIS^{\text{HL}}}(X)-(d_p^{PIS^{\text{HL}}})^*}{\left(d_p^{PIS^{\text{HL}}}\right)^- -\left(d_p^{PIS^{\text{HL}}}\right)^*}\right) \geq \delta^{\text{HL}}$,

$\left(\frac{\left(d_p^{NIS^{\text{HL}}}\right)^* - d_p^{NIS^{\text{HL}}}(X)}{\left(d_p^{NIS^{\text{HL}}}\right)^* -\left(d_p^{NIS^{\text{HL}}}\right)^-}\right) \geq \delta^{\text{HL}}$,

$X \in \mathbb{M}, \quad \delta^{\text{HL}} \in [0,1]$

whereδ^{HL} is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

III- Construct the following satisfactory level model (for finite value of p) for problem (13):

$Maximize\ \delta^{\text{LH}}$

subject to (17)

$\left(\frac{d_p^{PIS^{\text{L}H}}(X)-(d_p^{PIS^{\text{L}H}})^*}{\left(d_p^{PIS^{\text{LL}}}\right)^- -\left(d_p^{PIS^{\text{LL}}}\right)^*}\right) \geq \delta^{\text{LH}}$,

$\left(\frac{\left(d_p^{NIS^{\text{LH}}}\right)^* - d_p^{NIS^{\text{LH}}}(X)}{\left(d_p^{NIS^{\text{L}H}}\right)^* -\left(d_p^{NIS^{\text{LH}}}\right)^-}\right) \geq \delta^{\text{LH}}$,

$X \in \mathbb{M}, \quad \delta^{\text{LH}} \in [0,1]$

whereδ^{LH} is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

IV- Construct the following satisfactory level model (for finite value of p) for problem (14):

$Maximize\ \delta^{\text{HH}}$

subject to (18)

$\left(\frac{d_p^{PIS^{\text{HH}}}(X)-(d_p^{PIS^{\text{HH}}})^*}{\left(d_p^{PIS^{\text{HH}}}\right)^- -\left(d_p^{PIS^{\text{HH}}}\right)^*}\right) \geq \delta^{\text{HH}}$,

$\left(\frac{\left(d_p^{NIS^{\text{HH}}}\right)^* - d_p^{NIS^{\text{HH}}}(X)}{\left(d_p^{NIS^{\text{HH}}}\right)^* -\left(d_p^{NIS^{\text{HH}}}\right)^-}\right) \geq \delta^{\text{HH}}$,

$X \in \mathbb{M}, \quad \delta^{\text{HH}} \in [0,1]$

whereδ^{HH} is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

Step 8:

If the solution of problems (15)-(18) yield optimal solutions $\left(\delta^{*^{\text{LL}}}, X^{*^{\text{LL}}}\right)$, $\left(\delta^{*^{\text{HL}}}, X^{*^{\text{HL}}}\right)$,$\left(\delta^{*^{\text{LH}}}, X^{*^{\text{LH}}}\right)$ and $\left(\delta^{*^{\text{HH}}}, X^{*^{\text{HH}}}\right)$then $X^{*^{\text{LL}}}$, $X^{*^{\text{HL}}}$, $X^{*^{\text{LH}}}$ and $X^{*^{\text{HH}}}$ are a nondominated solutions of problems (3)-(6) and compromise solutions of problem (1), then go to step (10). Otherwise go to step (9).

Step 9:

Go to step (3).

Step 10:

Stop.

Illustrative Numerical Example for the Proposed Algorithm (I):

The proposed algorithm (I) is used to solve the following RLMODM problem:

$Maximize\ f_1(X) = 7([2,3],[1,5])x_1 + 4([2,5],[2,7])\ x_2$

$Minimize\ f_2(X) = 5([2,3],[1,4])x_1 + 11([3,4],[2,9])\ x_2$

subject to

$X \in \mathbb{M} = \{3x_1 + 5x_2 \leq 35, 2x_1 - x_2 \leq 20, 5x_2 \leq 16\ , x_1 \geq 1, x_1, x_2 \geq 0\}$

Solution:

P^{LL}:

$Maximize F_1^{LL} = 14x_1 + 8x_2$

$Minimize\ F_2^{LL} = 10x_1 + 33x_2$

subject to

$X \in \mathbb{M}$

P^{HL}:

$Maximize F_1^{HL} = 21x_1 + 20\ x_2$

$Minimize F_2^{HL} = 15x_1 + 44x_2$

Subject to

$X \in \mathbb{M}$

P^{LH}:

$Maximize F_1^{LH} = 7x_1 + 8\ x_2$

$Minimize F_2^{LH} = 5x_1 + 22x_2$

subject to

$X \in \mathbb{M}$

P^{HH}:

$Maximize F_1^{HH} = 35x_1 + 28\ x_2$

$Minimize F_2^{HH} = 20x_1 + 99x_2$

subject to

$X \in \mathbb{M}$

Obtain *PIS* and *NIS* payoff tables for problem ***P^{LL}:***:

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Table 1: PIS payoff table for problem $\underline{\boldsymbol{P^{LL}}}$:

	$F_1^{LL}(X)$	$F_2^{LL}(X)$	x_1	x_2
$Maximize F_1^{LL}(X)$	151.5385*	129.2308164	10.38462	0.7692308
$Minimize\ F_2^{LL}(X)$	14	10*	1	0

PIS: f^{*LL}= (151.5385, 10)

Table 2:NIS payoff table for problem $\underline{\boldsymbol{P^{LL}}}$:

	$F_1^{LL}(X)$	$F_2^{LL}(X)$	x_1	x_2
$Minimize\ F_1^{LL}(X)$	14^-	10	1	0
$Maximize\ F_2^{LL}(X)$	114.2662	168.9333^-	6.3333	3.2

NIS: f^{-LL}= (14, 168.9333)

- Next, construct equation and obtain the following equations:

$$d_P^{PIS^{LL}} = \left[w_1^p\left(\frac{151.5385 - F_1^{LL}(X)}{151.5385 - 14}\right)^p + w_2^p\left(\frac{F_2^{LL}(X) - (10)}{(168.9333) - (10)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{LL}} = \left[w_1^p\left(\frac{F_1^{LL}(X) - 14}{151.5385 - 14}\right)^p + w_2^p\left(\frac{(168.9333) - F_2^{LL}(X)}{(168.9333) - (10)}\right)^p\right]^{1/p}$$

- Thus, problem (11) is obtained. In order to get numerical solutions, assume that w_1^p=w_2^p=*0.5* and *p=2*,

Table 3: PIS payoff table of problem (11) when p=2.

	$d_2^{PIS^{LL}}$	$d_2^{NIS^{LL}}$	$F_1^{LL}(X)$	$F_2^{LL}(X)$	x_1	x_2
$Min.\ d_2^{PIS^{LL}}$	75.21841*	0.1249037754^-	151.5385264	129.2305616	10.38462	0.7692308
$Max.\, d_2^{NIS^{LL}}$	0.3750962246^-	0.5005752*	151.5385264	129.2305616	10.38462	0.7692308

d_2^{*LL}=(75.21841,0.5005752),d_2^{-LL}=(0.3750962246, 0.1249037754).

- Now, it is easy to compute step (10) :

$Maximize\ \delta^{LL}$

Subject to

$3x_1 + 5x_2 \leq 35, 2x_1 - x_2 \leq 20, 5x_2 \leq 16\ , x_1 \geq 1, \quad x_1, x_2, x_3 \geq 0$

$$\left(\frac{d_1^{PIS^{LL}}(X) - 75.21841}{0.3750962246 - 75.21841}\right) \geq \delta^{LL}, \left(\frac{0.5005752 - d_1^{NIS^{LL}}(X)}{0.5005752 - 0.1249037754}\right) \geq \delta^{LL}, \delta^{LL} \in [0,1]\ .$$

The maximum "satisfactory level" (δ^{LL}=*0.3960653*) is achieved for the solution x_1^{*LL}=*1*,x_2^{*LL}=*1.930737*.

- Obtain *PIS* and *NIS* payoff tables for problem $\underline{\boldsymbol{P^{HL}}}$: :

Table 4: PIS payoff table for problem $\underline{\boldsymbol{P^{HL}}}$:

	$F_1^{HL}(X)$	$F_2^{HL}(X)$	x_1	x_2
$Maximize F_1^{HL}(X)$	233.4615*	189.6154552	10.38462	0.7692308
$Minimize\ F_2^{HL}(X)$	21	15*	1	0

PIS: f^{*HL}= (233.4615, 15)

Table 5: NIS payoff table for problem $\underline{\boldsymbol{P^{HL}}}$:

	$F_1^{HL}(X)$	$F_2^{HL}(X)$	x_1	x_2
$Minimize\ F_1^{HL}(X)$	21^-	15	1	0
$Maximize\ F_2^{HL}(X)$	196.9993	235.8^-	6.3333	3.2

NIS: f^{-HL}= (21, 235.8)

- Next, construct equation and obtain the following equations:

$$d_P^{PIS^{HL}} = \left[w_1^p\left(\frac{233.4615 - F_1^{HL}(X)}{233.4615 - 21}\right)^p + w_2^p\left(\frac{F_2^{HL}(X) - (15)}{(235.8) - (15)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{HL}} = \left[w_1^p\left(\frac{F_1^{HL}(X) - 21}{233.4615 - 21}\right)^p + w_2^p\left(\frac{(235.8) - F_2^{HL}(X)}{(235.8) - (15)}\right)^p\right]^{1/p}$$

- Thus, problem (12) is obtained. In order to get numerical solutions, assume that w_1^p=w_2^p=*0.5* and *p=2*,

Table 6: PIS payoff table of problem (12) when p=2.

	$d_2^{PIS^{HL}}$	$d_2^{NIS^{HL}}$	$F_1^{HL}(X)$	$F_2^{HL}(X)$	x_1	x_2
$Min.\ d_2^{PIS^{HL}}$	116.1814*	0.5108208411^-	233.461636	189.6154552	10.38462	0.7692308
$Max.\, d_2^{NIS^{HL}}$	0.395415433^-	0.5002369*	233.461636	189.6154552	10.38462	0.7692308

d_2^{*HL}=(116.1814, 0.5002369),d_2^{-HL}=(0.395415433, 0.5108208411).

- Now, it is easy to compute step (10) :

$Maximize\ \delta^{\mathrm{HL}}$

Subject to

$3x_1 + 5x_2 \le 35, 2x_1 - x_2 \le 20, 5x_2 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$$\left(\frac{d_1^{PIS^{HL}}(X) - 116.1814}{0.395415433 - 116.1814}\right) \ge \delta^{HL}, \left(\frac{0.5002369 - d_1^{NIS^{\mathrm{HL}}}(X)}{0.5002369 - 0.5108208411}\right) \ge \delta^{HL}, \qquad \delta^{HL} \in [0,1]\ .$$

The maximum "satisfactory level" ($\delta^{\mathrm{HL}}=1$) is achieved for the solution $x_1^{*\mathrm{HL}}=1.234568, x_2^{*\mathrm{HL}}=1.234568$.

- Obtain *PIS* and *NIS* payoff tables for problem $\boldsymbol{P^{LH}}$:

Table 7: PIS payoff table for problem $\boldsymbol{P^{LH}}$:

	$F_1^{LH}(X)$	$F_2^{LH}(X)$	x_1	x_2
$Maximize F_1^{LH}(X)$	78.84615*	86.8460776	10.38462	0.7692308
$Minimize\ F_2^{LH}(X)$	7	5*	1	0

PIS: $f^{*^{LH}}$= (78.84615, 5)

Table 8: NIS payoff table for problem $\boldsymbol{P^{LH}}$:

	$F_1^{LH}(X)$	$F_2^{LH}(X)$	x_1	x_2
$Minimize\ F_1^{LH}(X)$	7^-	5	1	0
$Maximize\ F_2^{LH}(X)$	96.9331	102.0667^-	6.3333	3.2

NIS: $f^{-^{LH}}$= (7, 102.0667)

- Next, construct equation and obtain the following equations:

$$d_P^{PIS^{\mathrm{LH}}} = \left[w_1^p \left(\frac{78.84615 - F_1^{LH}(X)}{78.84615 - 7}\right)^p + w_2^p \left(\frac{F_2^{LH}(X) - (5)}{(102.0667) - (5)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{\mathrm{LH}}} = \left[w_1^p \left(\frac{F_1^{LH}(X) - 7}{78.84615 - 7}\right)^p + w_2^p \left(\frac{(102.0667) - F_2^{LH}(X)}{(102.0667) - (5)}\right)^p\right]^{1/p}$$

- Thus, problem (13) is obtained. In order to get numerical solutions, assume that $w_1^p=w_2^p=0.5$ and $p=2$,

Table 9: PIS payoff table of problem (13) when p=2.

	$d_2^{PIS^{\mathrm{LH}}}$	$d_2^{NIS^{\mathrm{LH}}}$	$F_1^{LH}(X)$	$F_2^{LH}(X)$	x_1	x_2
$Min.\ d_2^{PIS^{\mathrm{LH}}}$	38.87445*	0.5284722909^-	78.84615	68.8460776	10.38462	0.7692308
$Max.\ d_2^{NIS^{\mathrm{LH}}}$	0.3288773472^-	0.5008807*	78.84615	68.8460776	10.38462	0.7692308

$d_2^{*\mathrm{LH}}$=(38.87445, 0.5008807), $d_2^{-\mathrm{LH}}$=(0.3288773472, 0.5284722909).

- Now, it is easy to compute step (10) :

$Maximize\ \delta^{\mathrm{LH}}$

Subject to

$3x_1 + 5x_2 \le 35, 2x_1 - x_2 \le 20, 5x_2 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$$\left(\frac{d_2^{PIS^{LH}}(X) - 38.87445}{0.3288773472 - 38.87445}\right) \ge \delta^{LH}, \left(\frac{0.5008807 - d_2^{NIS^{\mathrm{LH}}}(X)}{0.5008807 - 0.5284722909}\right) \ge \delta^{LH}, \qquad \delta^{LH} \in [0,1]\ .$$

The maximum "satisfactory level" ($\delta^{\mathrm{LH}}=1$) is achieved for the solution $x_1^{*\mathrm{LH}}=1.234568, x_2^{*\mathrm{LH}}=1.234568$.

- Obtain *PIS* and *NIS* payoff tables for problem $\boldsymbol{P^{HH}}$:

Table 10: PIS payoff table for problem $\boldsymbol{P^{HH}}$:

	$F_1^{HH}(X)$	$F_2^{HH}(X)$	x_1	x_2
$Maximize F_1^{HH}(X)$	385*	283.8462492	10.38462	0.7692308
$Minimize\ F_2^{HH}(X)$	35	20*	1	0

PIS: $f^{*^{HH}}$= (385, 20)

Table 11: NIS payoff table for problem $\boldsymbol{P^{HH}}$:

	$F_1^{HH}(X)$	$F_2^{HH}(X)$	x_1	x_2
$Minimize\ F_1^{HH}(X)$	35^-	20	1	0
$Maximize\ F_2^{HH}(X)$	311.2655	443.4667^-	6.3333	3.2

NIS: $f^{-^{HH}}$= (35, 443.4667)

- Next, construct equation and obtain the following equations:

$$d_P^{PIS^{\mathrm{HH}}} = \left[w_1^p \left(\frac{385 - F_1^{HH}(X)}{385 - 35}\right)^p + w_2^p \left(\frac{F_2^{HH}(X) - (20)}{(443.4667) - (20)}\right)^p\right]^{1/p}$$

Citation: Shereen Fathy El-Feky and Tarek H.M. Abou-El-Enien, Compromise Solutions for Rough Multiple Objective Decision Making Problems. Australian Journal of Basic and Applied Sciences, 12(7): 112-119.

$$d_P^{NIS^{\text{HH}}} = \left[w_1^p \left(\frac{F_1^{HH}(X) - 35}{385 - 35} \right)^p + w_2^p \left(\frac{(443.4667) - F_2^{HH}(X)}{(443.4667) - (20)} \right)^p \right]^{1/p}$$

- Thus, problem (14) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and $p=2$,

Table 12: PIS payoff table of problem (14) when p=2.

	$d_2^{PIS^{\text{HH}}}$	$d_2^{NIS^{\text{HH}}}$	$F_1^{HH}(X)$	$F_2^{HH}(X)$	x_1	x_2
$Min.\ d_2^{PIS^{\text{HH}}}$	191.95*	0.534341143^-	385	283.8462492	10.38462	0.7692308
$Max.\ d_2^{NIS^{\text{HH}}}$	0.3115312836^-	0.5002225*	385	283.8462492	10.38462	0.7692308

$d_2^{*^{\text{HH}}}$=(191.95, 0.5002225),$d_2^{-^{\text{HH}}}$=(0.3115312836, 0.534341143).

- Now, it is easy to compute step (10) :

$Maximize\ \delta^{\text{HH}}$

Subject to

$3x_1 + 5x_2 \le 35, 2x_1 - x_2 \le 20, 5x_2 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$$\left(\frac{d_1^{PIS^{HH}}(X) - 191.95}{0.3115312836 - 191.95} \right) \ge \delta^{HH}, \left(\frac{0.5002225 - d_1^{NIS^{\text{HH}}}(X)}{0.5002225 - 0.534341143} \right) \ge \delta^{HH}, \qquad \delta^{HH} \in [0,1] .$$

The maximum "satisfactory level" ($\delta^{\text{HH}}=1$) is achieved for the solution $x_1^{*^{\text{HH}}}=1.234568$, $x_2^{*^{\text{HH}}}=1.234568$.

The compromise solutions by using the proposed TOPSIS algorithm is compared with the vector of ideal solutions in Table (15). Thus, the proposed TOPSIS algorithm gives good compromise solutions.

Table 13:

Objective		Proposed TOPSIS Algorithm method (p=2)		Ideal Objective Vector	
		w_1^p=0.5	$w_2^p = 0.5$	PIS	NIS
$\boldsymbol{P^{LL}}$	F_1^{LL}	29.445896		151.5385	14
	F_2^{LL}	73.714321		10	168.9333
$\boldsymbol{P^{HL}}$	F_1^{HL}	50.617288		233.4615	21
	F_2^{HL}	72.839512		15	235.8
$\boldsymbol{P^{LH}}$	F_1^{LH}	18.51852		78.84615	7
	F_2^{LH}	33.33336		5	102.0667
$\boldsymbol{P^{HH}}$	F_1^{HH}	77.777784		385	35
	F_2^{HH}	146.913592		20	443.4667

Conclusions:

This paper extended TOPSIS approach to find compromise solutions for the RLMODM of mixed ($Maximize/Minimize$)-type. A new algorithm is presented for the proposed TOPSIS approach for solving these types of mathematical programming problems. Also, an illustrative numerical example is solved and compared the compromise solutions of the proposed algorithm with the vector of ideal solutions. Thus, the proposed TOPSIS algorithm gives good compromise solutions.

Abbreviations:

DM: Decision Maker,
CP: Compromise Programming,
PIS: Positive Ideal Solution,
NIS: Negative Ideal Solution,
MODM: Multiple Objective Decision Making,
TOPSIS: Technique for Order Preference by Similarity Ideal Solution,
RMODM: Multiple Objective Decision Making Problems with Rough parameters in the objective functions.
RLMODM : Linear Multiple Objective Decision Making Problem with rough parameters in the objective functions.

REFERENCES

Abou-El-Enien, T.H.M., 2013. TOPSIS Algorithms for Multiple Objectives Decision Making: Large Scale Programming Approach, *LAP LAMBERT Academic Publishing*, Germany.

Bellman, R.E. and L.A. Zadeh, 1970. Decision-making in fuzzy environment, *Management Science*, B 17: 141-164.

Dauer, P. and M.S.A. Osman, 1985. Decomposition of the Parametric Space in Multiobjective Convex Programs using the Generalized Tchebycheff Norm, *Journal of Mathematical Analysis and Applications*, 107(1): 156-166.

Dubois, J.D. and A. Prade, 1980. Fuzzy Sets and Systems: Theory and applications, *Academic Press*, New York.

Hwang, C.L., and A.S.M. Masud, 1979. Multiple Objective Decision Making Methods and Applications, Springer-Verlag, New York.

Hamzehee, A., M.A. Yaghoobi, M. Mashinchi, 2014. Linear programming with rough interval coefficients, Intelligent and Fuzzy Systems, 26: 1179-1189.

Lai, Y.J., T.Y. Liu, C.L. Hwang, 1994. TOPSIS for MODM, *European Journal of Operational Research*, 76: 486-500.

Osman, M.S., E.F. Lashein, E.A. Youness and T.E.M. Attya, 2011. "Mathematical Programming in Rough Environment", Optimization: A Journal of Mathematical Programming and Operations Research, 60(5): 603-611.

Pawlak, Z., 1991. "Rough Sets", International Journal of Computer and Information Sciences, 11(1982) 341-356.

Pawlak, Z., "Rough Sets", Springer,

Youness, E., 2006. "Characterizing Solutions of rough Programming Problems", European Journal of Operational Research, 1681019-1029.

Yu, P.L., 1973. "A Class of Solutions for Group Decision Problems", Management Science, 19: 936-946.

Yu, P.L., 1985. Multiple-Criteria Decision Making, Plenum Press, New York.

Zadeh, L.A., 1974. Fuzzy sets, Information and Control, 8: 338-353.

Zeleny, M., 1973. Compromise Programming.In Cochrane, J.L., Zeleny M. (Eds.) Multiple Criteria Decision Making. University of South Carolina Press. Columbia, pp: 262-301.

Zeleny, M., 1974. A Concept of Compromise Solutions and the Method of the Displaced Ideal. Computers and Operations Research, 1: 479-496.

Zeleny, M., 1982. Multiple Criteria Decision Making, McGraw-Hill, New York.

Zimmermann, H.J., 1996. Fuzzy Set Theory and Its Applications, Kluwer Academic Publishers, Boston, Third edition.