Radial Basis Function Method For Modelling Leaf Surface from Real Leaf Data

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Abstract

The leaves participate in the plant development and are an integral component of any plant model. Therefore accurate leaf models are essentials in many applications, such as the development of a plant model, modelling droplet movement, spraying and spreading droplet on the leaf surface as well as in studying biological system such as photosynthesis and a canopy light nature. A smooth surface is important to assist the droplet motion that govern by a differential equation or partial differential equation. The leaf size, shape and position are significant in many ways. For instance the precipitation amount of pesticide or nutrients can be measured if a leaf model is manageable. This can be achieve from a surface fit to a data set measured by device such as a laser scanner or a sonic digitiser. The work presented here will structure the foundation for a theoretical revision of water droplets paths on leaves. The primary study will suppose that the droplet experiences external force includes gravity and an internal force includes viscosity and surface tension. It will be vital to create a continuous surface fit and this is certain by interpolation the data value. The problems stated in this research include: firstly, the choice of points from the set of the data. The selection of RBF and its parameter c. We selected the Hardy’s multiquadrics in combination with the Rippa’s algorithm to decide the width parameter. Secondly, the use of global and local multiquadric RBF interpolates. To validate our method we apply it to two sets of scattered data points. The first set is taken from Franke (Franke (1982)) while the second set is sampled by Loch (Loch (2004)) from an Anthurium and Frangipani leaf using a laser scanner. Numerical results confirm that the proposed technique is revealed to constructs an accurate and realistic surface of the leaves. The approach described in this paper is valid to scattered data and has the possibility for application to the numerical solution of partial differential equations.

Introduction

The leaves model are imperative in the plant development and are important in any plant model. Modelling of plant has been research over the last decades by Davydov and Zeilfelder (2004), while leaves model considered recently by Loch (Oqielat (2017); Lisa et al. (2016); Kemphorne et al. (2015); Oqielat et al. (2011); Oqielat et al. (2009); Oqielat et al. (2007); Dorr et al. (2014); Loch, 2004; Turner et al. (2008); Belward et al. (2008); Oqielat (2017)). Loch used laser scanner to collect leaf data points for ear Elephant’s, Anthurium, Flame and Frangipani leaves and then applied finite element method to construct leaf model. Our goal in this paper is to use scattered data interpolation method based on radial basis function (RBF) to reconstruct the leaf surface model. The scattered data points interpolation problem is given by:

Given M distinct scattered points $(x_i, y_i)^T$, and their function values $f_i, i = 1, 2, \ldots, P$, find a function $F : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ that interpolates these data satisfying
\[ F(x_i, y_i) = f_i, i = 1, \ldots, P. \] (1)

To evaluate the accuracy of the RBF method, we apply it to two sets of real data points sampled from a Frangipani and Anthurium leaf surface using laser scanner.

The research presented in this paper is comprised of four main sections. In § 2, an overview of the radial basis function (RBF) is presented. In § 3, the RBF technique is evaluated using six test functions and data points chosen from Franke (Franke (1982)). The quality of the approximation of the methods is measured numerically using the maximum error and the root mean square error (RMS). In § 4, Frangipani and Anthurium leaf surface is constructed using the RBF method. Finally, conclusion and future work is presented in § 5.

**Radial Basis Functions Method**

The approximation radial basis function \( F \) to the function \( f(x) \), is given by:

\[
F(x) = \sum_{i=1}^{N} a_i R(r_i), \quad x \in \mathbb{R}^2
\]

where \( r_i = \|x - x_i\| \) with \( \| \| \) denoting the Euclidean norm. The centres of the approximation RBF are \( x_i, i = 1, 2, \ldots, N \). If the coefficients of the RBF \( a_i, i = 1, \ldots, N \) satisfies the system

\[
Aa = F \quad \text{with} \quad A_{ij} = R(\|x_j - x_i\|) \quad i, j = 1, \ldots, N
\]

and

\[
F = (f_1, \ldots, f_N)^T.
\]

then \( F(x) \) is called interpolation function of \( f \) at \( x_1, \ldots, x_N \).

The advantage of using RBF method can be seen in proposing a smooth surface depiction to approximate the function values at points. This method has many applications in fields such as medical imaging (Beatson et al. (2003)), software to drive laser scanners (Carr et al. (2001)), and the solution of partial differential equations by Hardy (Hardy (1990)). Powell (Powell (1991)) review the theory of RBF.

The computational costs in assessing the RBF to large sets of data points can become time consuming because a large compact matrix system of size \( N \times N \) has to be solved in order to calculate the RBF coefficients \( a_i, i = 1, 2, \ldots, N \) in equation (4), this system can become ill-conditioned with very small in magnitude singular values (Franke (1982)). Franke compared around 30 interpolation schemes and found that the most accurate schemes were based on fitting RBFs and the application of global methods should be limited to sets of up to 100-200 data points. Beatson (Beatson et al. (2001)) used fast evaluation techniques to reduce the cost of computing the radial basis function.

Well known examples of radial basis function methods include thin plate splines, Gaussian RBF and Hardy’s multiquadric (Hardy (1990)) which is adopted in this paper. The multiquadric RBF is given by:

\[
R(\|x - x_i\|) = \sqrt{\|x - x_i\|^2 + c^2}. \quad (5)
\]

The accuracy of RBF interpolant depends strongly on the parameter \( c \) which this parameter is specified by the user, see for example (Niceno (2003)). For some values of \( c \) the problem may become ill-conditioned. Franke (Franke (1982)) and Foley (Foley (1987)) used \( c = 1.25 \frac{D}{\sqrt{n}} \) where \( D \) is the diameter of the minimal circle enclosing all data points. Hardy (Hardy (1990)) suggested a value of \( c = 0.815d \) where and \( d \) is the distance between a data point and its closest neighbour.

Franke (Franke (1982)) studied the accuracy of the inverse multiquadric and the multiquadric interpolant and found that the choice of the parameter \( c \) has great impact on the accuracy of the RBF. Carlson (Carlson (1991)) recurred the computation of the RMS error with different choices of \( c \) and stated the optimal value of \( c \) that minimizes the RMS. Rippa (Rippa (1999)) performed experiment on the influence that the parameter \( c \) has on the approximation quality achieved using Gaussian, inverse multiquadric and multiquadric interpolant. Rippa proved that the value of \( c \) has great impact on the approximation quality of these RBFs. Rippa considered two sets of data points and nine different test functions defined on the unit square. Nine test functions and two sets of data are considered by Rippa. He constructed the data vector \( F = (f_1, f_2, \ldots, f_N)^T \) by computing each test function over the set of data points so that
An algorithm is proposed by Rippa for the choice of a good value for the parameter $c$ that minimizes the RMS error between the RBF interpolant and the unknown function from which the data vector $F$ was sampled. The value of $c$ is selected by minimizing the cost function. The `mnbrak` and `brent` routines from Numerical Recipes (Press et al., 1992) were used to do the minimization. The `mnbrak` routine is given some tolerance and two initial values $c_1$ and $c_2$. It returns three numbers $b_1$, $b_2$, and $b_3$ that bracket the minimum. After bracketing the minimum, the three numbers and the tolerance parameter are passed into the function `brent` that uses Brent’s method to minimize the cost function. We refer to the minimum value of the cost function as the ”good value” of $c$.

The cost function is given by:

$$C(c) = \|H(c)\|_1,$$

(12)

and

$$c_{opt} = \arg \min_{c \in \mathbb{R}} \|H(c)\|_1.$$

(13)

**The solution of the linear system $Aa = F$**

The solution of equation (4) is a unique if and only if the matrix $A$ in (6) is invertible. However, the approximation solution of the linear system is computed by applying the truncated singular value decomposition (TSVD) of $A$ (Tony, 1990).

$$A = U \Sigma V^T = \sum_{i=1}^{N} u_i \sigma_i v_i^T,$$

(14)

where the left and right singular vectors $u_i$ and $v_i$ are the columns of the matrices $U$ and $V$, respectively, and $\sigma_i$ are the singular values of $A$.

We applied TSVD (Moroney, 2006) to cast off the small singular values regarding to the benchmark where the singular values that are equal to, or less than, the product of the largest singular value with a chosen target $\varepsilon$ (machine epsilon) are ignored. Thus, if $\sigma_i \leq \sigma_i \varepsilon$ we ignore $\sigma_i$, $i = 2, \ldots, N$. A new matrix $A_t$ is then formed with rank $t$ defined by:

$$B_t = \sum_{i=1}^{t} u_i \sigma_i v_i^T,$$

(15)

and the solution to (6) is then approximated by:

$$a = A_t^+ F = \sum_{i=1}^{t} \frac{u_i^T F}{\sigma_i} v_i,$$

(16)

where the matrix $A_t^+$ is the pseudoinverse of the matrix $A_t$. 

$$S(x_i) = f_i, i = 1, 2, \ldots, N.$$ 

(6)
**The Local and Global RBF Approximations:**

In this paper, we have applied interpolated methods based on multi-quadratic radial basis functions for two types of scattered data. In particular, two variants of methods are investigated, which are referred to as the local multi-quadratic RBF interpolation $F_m(x)$ and the global multi-quadratic RBF interpolator $F_N(x)$. The global RBF method uses all points $(N)$ on the surface to construct the RBF interpolant, while the local RBF method uses only a subset of points $(m)$ on the surface for this purpose. We now elaborate on each of these variants.

**Global Method:**

Global multi-quadratic RBF interpolant.

Given $N$ data points $x_1, x_2, ..., x_N$ (our case $N = 100$) and data vector $f = (f_1, f_2, ..., f_N)^T$, determine the interpolating RBF $F_N(x)$ that interpolates the data vector $f$.

**Local Method:**

Local multi-quadratic RBF interpolant.

Given $N$ data points $x_1, x_2, ..., x_N$ and data vector $f = (f_1, f_2, ..., f_N)^T$, choose only a subset of $m$ data values $f = (f_1, f_2, ..., f_m)^T$ and data points $x_1, x_2, ..., x_m$, (typically $m = 20$) and $m = 40$ such that these $m$ points represent the closest $tm$ points to each point of interest on the surface around which the RBF interpolant is constructed. We now outline this procedure in the following Algorithm.

**Algorithm 1: Construction Surfaces using the RBF Method:**

1. **INPUT:** $N$ data points $\{(x_i, f_i), i = 1, ..., N\}$ and corresponding data vector.
2. **Step 1:** Select a subset of $n \subset N$ data points to build the surface triangulation.
3. **Step 2:** Compute the RBF linear system (4) using either the local method OR the global method.
4. **Step 3:** Estimate the parameter $c$ either locally OR globally.
5. **Step 4:** Solve the linear system using the TSV method.
6. **Step 5:** Apply either the global method OR the local method to construct the surface value.

**Numerical Experimentation for the Franke Data Set:**

The numerical experiments for the surface fitting technique showed in section § 2 is presented in this section. A data set taken from Franke (Franke (1982)) are used to evaluate the accuracy of our method either globally or locally. Franke data sets consists of two subsets (100 points and 33 points) and six test functions (see ogielat et al. (2007); ogielat et al. (2009)). The global RBF interpolant is constructed using all $N = 100$ data points and the used to estimate the surface values for the 33 points. While, the local RBF interpolant is constructed by choosing the closest $20$ or $40$ points to each point of interest. The parameter $c$ in the two cases was estimated either globally using the $n = 100$ data points, or locally using a selection of $m = 20$ or $m = 40$ neighboring data points for each surface value. To assess the quality of the method, we computed the root mean square error (RMS) given by:

$$\text{RMS} = \sqrt{\frac{1}{q} \sum_{i=1}^{q} [F(a, b) - f(a, b)]^2}, \quad (19)$$

where $f(a, b)$ is the exact value of the function and $F(a, b)$ is the estimate value at the equivalent points.

**Table 1:** The RMS error comparison using the global multi-quadratic RBF ($N = 100$ points) and local multi-quadratic RBF interpolants ($m = 20$ or $m = 40$ points) for the six test functions. The parameter $c$ was computed globally by Rippa method using the $n = 100$ points.

<table>
<thead>
<tr>
<th>Function</th>
<th>$c$</th>
<th>Global Method</th>
<th>Local Method $m=40$</th>
<th>Local Method $m=20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>0.2506</td>
<td>2.3e-003</td>
<td>2.6e-003</td>
<td>2.6e-003</td>
</tr>
<tr>
<td>$F_2$</td>
<td>0.1560</td>
<td>5.3e-003</td>
<td>5.3e-003</td>
<td>5.3e-003</td>
</tr>
<tr>
<td>$F_3$</td>
<td>0.5907</td>
<td>7.4e-005</td>
<td>1.1e-004</td>
<td>2.9e-004</td>
</tr>
</tbody>
</table>
Table 2: The RMS error comparison using the local multiquadric RBF interpolant \( m = 20 \) or \( m = 40 \) points for the six test functions. The parameter \( c \) was computed locally by Rippa method using the same \( (m = 20 \) or \( m = 40 \) ) points.

<table>
<thead>
<tr>
<th>Function</th>
<th>Local Method (( m=40 ))</th>
<th>Local Method (( m=20 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 )</td>
<td>([c_{min} \ c_{max}])</td>
<td>([c_{min} \ c_{max}])</td>
</tr>
<tr>
<td>( F_1 )</td>
<td>[0.1565 3.4001]</td>
<td>2.8e-003</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>[0.1009 2.1613]</td>
<td>5.2e-003</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>[0.4645 3.8262]</td>
<td>4.2e-004</td>
</tr>
<tr>
<td>( F_4 )</td>
<td>[1.1355 2.6431]</td>
<td>5.0e-006</td>
</tr>
<tr>
<td>( F_5 )</td>
<td>[0.3086 0.5959]</td>
<td>3.7e-004</td>
</tr>
<tr>
<td>( F_6 )</td>
<td>[1.4729 4.3317]</td>
<td>4.9e-004</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

Tables 1 and 2 show the RMS errors for the six test functions using the global and local multiquadric RBF method where the parameter \( c \) computed in both cases using Rippa method. We observe that the RMS errors obtained using the global RBF method is more accurate than the RMS produced using the local RBF method. Moreover, the RMS error produced using the local RBF method constructed with \( m = 40 \) points was always found to be more accurate than the RMS produced for the surface representation constructed from \( m = 20 \) points (for both cases whether \( c \) is approximated globally or locally). Furthermore, estimating the \( c \) value locally (Table 2) helps to produce a more accurate RMS for some of the functions than using \( c \) computed globally (Table 1). However, the parameter \( c \) computed using global methods is less computationally costly than using \( c \) locally because a dense matrix system (6) of size either 20×20 or 40×40 needs to be solved at each time the local RBF is estimated. Table 3 shows as expected the global values of \( c \) were always contained in the local ranges of \( c \) given for each of the functions.

In summary, using the global RBF is more accurate and less costly than using the local RBF. Now, we investigate the suitability of the global RBF method for a real leaf data set in the following section.

Application of the multiquadric RBF technique to a real leaf data set:

A set of data points sampled from real leaf surface using laser scanner are requires to reconstruct the surface of a leaf. In this paper we used a real data points of Frangipani and Anthurium leaves sampled using a laser scanner by Loch (Loch (2004)). The multiquadric RBF are then applied to estimate the surface values for the Anthurium and Frangipani leaves. The Anthurium leaf data set consists of 4,688 leaf surface points and 79 boundary points, see figure 1. While Frangipani leaf data set comprises of 3,388 leaf surface points and 17 boundary points, see figure 3.

Fig. 1: The data points for the Anthurium Leaf. There are 79 boundary points (represented by the red dots) and 4,555 surface points (represented by the green dots)
Fig. 2: The model of the Anthurium leaf surface generated from the points (shown in figure 1) using the global RBF method.

Now to apply the multiquadric RBF method to the Frangipani and Anthurium leaf data a new reference plane for the data are required, see oqielat (oqielat et al. (2009)). The reference plane of the laser scanner leaf data points may not essentially correspond with the $xy$-plane in the data point coordinate system. To solve this issue a least squares fit to the leaf data points is then used as reference plane and then rotate the coordinate system to obtain the $xy$-plane as the new reference plane. This rotations can be succeed by rotating the normal vector of the reference plane about the $y$-axis into the $yz$-plane and then rotating about the $x$-axis into the $xz$-plane, for more details (see oqielat et al. (2007); oqielat et al. (2009)).

Finally, After we generated the new reference plane of the leaves data sets, The global multiquadric RBF method was applied to reconstruct the surface of those two leaves (shown in figure 2 and figure 4). For the Anthurium leaf, The global RBF approach based on using 212 points to generate one global RBF and then use it to estimate the surface values for all leaf data points. While, For the Frangipani leaf, The global RBF approach based on using 141 points to generate one global RBF and then use it to estimate the surface values for all leaf data points. The selection process for the 212 points and 141 points are explained in (see oqielat et al. (2007); oqielat et al. (2009)). In both cases we used Rippa method (Rippa (1999)) to estimate the parameter $c$ globally using the same points that we used to built the global RBF. The results obtained for this surface fitting method are shown in Table 3 and Table 4.

Fig. 3: The data points for the Frangipani Leaf. There are 17 boundary points (represented by the Red dots) and 3,388 surface points (represented by the green dots).

Fig. 4: The model of the Frangipani leaf surface generated from the points (shown in figure 3) using the global RBF method.

Numerical Experiments for the Leaf Surface:

In this section we show the outcome of applying the multiquadric RBF technique to the Frangipani and Anthurium leaf data points. As stated before a subset of the leaf surface data are used to construct one global RBF,
the remaining data points of the leaf data (say \( m \)) were used to evaluate the method quality using two error metrics. The first error metric is the relative root mean square error RMS (see equation 19), which is given by:

\[
Relative\ RMS = \frac{RMS}{\max(f_i) - \min(f_i)}, \quad i = 1, 2, \ldots, m
\]

The second error metric is the maximum error associated with the surface fit in relation to the maximum variation in z as

\[
Maximum\ Error = \frac{\max \left| F(a_i, b_i) - f_i \right|}{\max(f_i) - \min(f_i)},
\]

where \( F(a_i, b_i), i = 1, 2, \ldots, m \) are the multiquadric RBF estimated values at the data points \( (m) \) and \( f_i \) are the given function values at the same data points.

### Table 3: RMS and maximum error computed using the global multiquadric RBF for Anthurium leaf.

<table>
<thead>
<tr>
<th></th>
<th>Global Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum error</td>
<td>0.042</td>
</tr>
<tr>
<td>Relative RMS</td>
<td>0.006</td>
</tr>
<tr>
<td>Number of point tested</td>
<td>4555</td>
</tr>
</tbody>
</table>

### Table 4: RMS and maximum error computed using the global multiquadric RBF for Frangipani leaf.

<table>
<thead>
<tr>
<th></th>
<th>Global Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum error</td>
<td>0.097</td>
</tr>
<tr>
<td>Relative RMS</td>
<td>0.014</td>
</tr>
<tr>
<td>Number of point tested</td>
<td>3264</td>
</tr>
</tbody>
</table>

### RESULTS AND DISCUSSION

Table 3 and 4 shows the maximum errors and the relative RMS using the global RBF method for the Anthurium and Frangipani leaves shown in Figures 2 and 4. For the Anthurium and Frangipani leaf there were a total of 4555 and 3264 data points respectively used to evaluate the accuracy of the surface. In conclusion, from the results given in the tables 3 and 4 it appears that the global multiquadric RBF method produces an accurate leaf surface representation and this is confirm our finding for Franke data set given in section 3.

### Conclusions and Future Research:

In this research we present a surface fitting method, based on Radial basis function, for modelling leaf surface from three-dimensional scanned data points. We showed that the method produces an accurate leaf surface representation. The surface can be used to determine the path of pesticide or water droplet on a leaf surface. This model will help in the assessment of different pesticide formulations and then in the effectiveness of treatment.

### REFERENCES


