Australian Journal of Basic and Applied Sciences, 10(15) October 2016, Pages: 41-47

AUSTRALIAN JOURNAL OF BASIC AND APPLIED SCIENCES

ISSN:1991-8178 EISSN: 2309-8414
Journal home page: www.ajbasweb.com

Thickness effect of porous fins on laminar mixed convection in horizontal concentric annuli

Yacine Ould-Amer

Affiliation USTHB – Faculty of Mechanical and Process Engineering (FGMGP),Laboratory of Multiphase Transport and Porous Media (LTPMP), Département Energétique, B.P. 32 El-Alia Bab Ezzouar 16111 Algiers, Algeria

Address For Correspondence:
Yacine Ould-Amer, USTHB- Faculty of Mechanical and Process Engineering (FGMGP), Laboratory of Multiphase Transport and Porous Media (LTPMP), Département Energétique, B.P. 32 El-Alia Bab Ezzouar 16111 Algiers, Algeria.
E-mail: youldamer@usthb.dz

ARTICLE INFO

Article history:
Received 26 August 2016
Accepted 10 October 2016
Published 18 October 2016

Keywords:
Mixed convection, concentric annuli, porous, thickness

ABSTRACT

Laminar mixed convection in a horizontal annulus with the presence of porous fins has been numerically investigated. With attached four porous fins on the inner cylinder, steady 3D laminar mixed convection is presented for the fully developed region of horizontal concentric annuli. Results are presented for three values of porous fins thickness and a range of the values of the Grashoff number and Darcy number. Results are presented in the form of contours plots of the streamlines and for the temperature isotherms, and in terms of the overall heat transfer coefficients and friction factor. The average Nusselt number increases significantly with an increase of the thickness of porous fins. With the use of the four porous fins, the friction factor is consequently increased compared with the situation without porous fins. The decrease of the Darcy number leads to an increase of the friction factor. If the fully fluid case is taken as a reference, the use of porous fins is justified only when the ratio of the average Nusselt number to the friction factor is enhanced.

INTRODUCTION

Mixed convection in annular passages has many applications in the areas of refrigeration, chemical process, cooling of turbine rotors, thermal storage systems, food industries, electrical transport and nuclear energy production. Double pipe heat exchangers in chemical process, gas – cooled electrical cables, double pipe heat exchangers and the cooling of nuclear fuel rods represent industrial examples in which annular flow heat transfer is encountered. In these situations, the forced convection in the duct flow is often significantly influenced by body forces, which tend to produce secondary flows in the duct cross section. The main focus of these systems is to enhance the overall heat transfer coefficient. In the last decades, the use of porous material to improve the heat transfer is of topical interest. Field applications of heat transfer in porous media can be found in Nield and Bejan (2006). Vanover and Kulacki (1987) conducted experiments in a porous annulus with the inner cylinder heated by constant heat flux and the outer cylinder isothermally cooled. The medium consisted of 1- and 3-mm glass beads saturated with water. They found that when the Rayleigh number is large the values of the Nusselt number for mixed convection may be lower than the free convection values. They attributed this to restructuring of the flow as forced convection begins to play a dominant role. Laminar mixed convection in the thermal entry region of concentric horizontal annulus of radius 2 has been experimentally studied by Mohammed *et al*. (2010). The experimental setup consists of a stainless annulus with inner tube subjected to a constant wall heat flux boundary condition and an adiabatic outer annulus. Their experimental results show that the free convection tends to decrease the heat transfer at low Reynolds number while to increase the heat

To Cite This Article: Yacine Ould-Amer., Thickness effect of porous fins on laminar mixed convection in horizontal concentric annuli. ***Aust. J. Basic & Appl. Sci.,*** *10(15): 41-47, 2016*

transfer for high Reynolds number. Ould-Amer (2010) carried out numerical analysis to investigate the performance of an innovative thermal system to improve the heat transfer in horizontal annulus. With attached four porous blocks on the inner cylinder, steady laminar mixed convection is examined for the fully developed region of horizontal concentric annuli. Their results show the significant improve in heat transfer with highly conductive porous medium; however, the friction factor is increased compared with situations without porous blocks. Venugopal *et al*. (2010) studied experimentally the mixed convection in vertical duct filled with metallic porous structure. The main focus of their work is to examine the potential of porous insert for augmenting heat transfer from the heated wall of vertical duct under forced flow conditions. The objective of this paper is to evaluate the effect of porous fins thickness on mixed convection, on the flow and heat transfer in horizontal annulus. Four porous fins are fixed on the inner cylinder. The analysis is performed for laminar flow and for thermally and hydrodynamically developed conditions. This work is an extension of previous study of Ould-Amer (2010, 2013 and 2016).

Analysis:

As is shown in Fig. 1, four porous fins are fixed on outside of the inner cylinder. The porous fins are considered homogeneous, isotropic and saturated by an incompressible fluid. The outer cylinder is insulated while the inner cylinder has circumferentially uniform surface temperature and axially uniform heat transfer rate. In the fully developed region of the flow, the velocity components u, v and w in the φ, r and z coordinates become independent of the axial distance z. The density of fluid is considered constant except in the buoyancy term (Boussinesq approximation). A local thermal equilibrium takes place between the fluid and the porous medium. Because of the symmetry about the vertical center line shown in Fig. 1, the analysis is confined to a right half of the annulus.

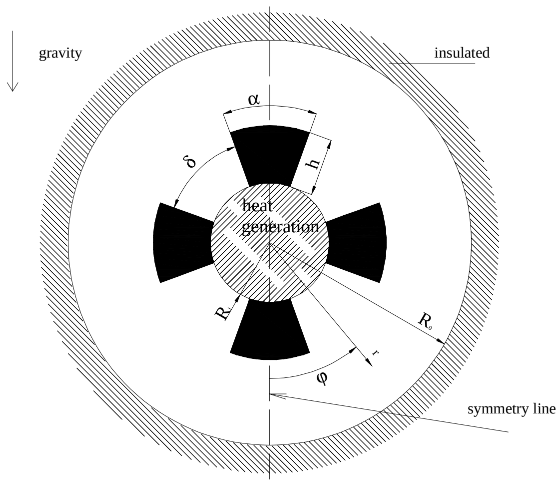


Fig. 1: Physical model

The continuity, momentum and energy equations that govern the physical situation can be written in dimensionless form as follows:

$$\frac{\partial U}{\partial \varphi}+\frac{\partial}{\partial \eta}(\eta V)=0 \tag{1}$$

$$\frac{1}{\varepsilon^2}\left[V\frac{\partial V}{\partial \eta}+\frac{U}{\eta}\frac{\partial V}{\partial \varphi}-\frac{U^2}{\eta}\right]=-\frac{\partial P}{\partial \eta}-\mathrm{Gr}\theta\cos\varphi+\mathrm{R_V}\left[\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta\frac{\partial V}{\partial \eta}\right)+\frac{1}{\eta^2}\frac{\partial^2 V}{\partial \varphi^2}-\frac{V}{\eta^2}-\frac{2}{\eta^2}\frac{\partial U}{\partial \varphi}\right]-\lambda\left[\frac{V}{\mathrm{Da}}+\frac{C_F}{\sqrt{\mathrm{Da}}}\left|\vec{V}\right|V\right] \tag{2}$$

$$\frac{1}{\varepsilon^2}\left[V\frac{\partial U}{\partial \eta}+\frac{U}{\eta}\frac{\partial U}{\partial \varphi}+\frac{UV}{\eta}\right]=-\frac{1}{\eta}\frac{\partial P}{\partial \varphi}+\mathrm{Gr}\theta\sin\varphi+\mathrm{R_V}\left[\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta\frac{\partial U}{\partial \eta}\right)+\frac{1}{\eta^2}\frac{\partial^2 U}{\partial \varphi^2}-\frac{U}{\eta^2}+\frac{2}{\eta^2}\frac{\partial V}{\partial \varphi}\right]-\lambda\left[\frac{U}{\mathrm{Da}}+\frac{C_F}{\sqrt{\mathrm{Da}}}\left|\vec{V}\right|U\right] \tag{3}$$

$$\frac{1}{\varepsilon^2}\left[V\frac{\partial W}{\partial \eta}+\frac{U}{\eta}\frac{\partial W}{\partial \varphi}\right]=1+R_V\left[\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta\frac{\partial W}{\partial \eta}\right)+\frac{1}{\eta^2}\frac{\partial^2 W}{\partial \varphi^2}\right]-\lambda\frac{W}{Da} \quad (4)$$

$$\left[V\frac{\partial \theta}{\partial \eta}+\frac{U}{\eta}\frac{\partial \theta}{\partial \varphi}+\frac{4\frac{W}{\overline{\overline{W}}}}{Pr(1+RR)}\right]=\frac{1}{Pr}\left[\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(R_C\eta\frac{\partial \theta}{\partial \eta}\right)\theta+\frac{1}{\eta^2}\frac{\partial}{\partial \varphi}\left(R_C\frac{\partial \theta}{\partial \eta}\right)\right] \quad (5)$$

With dimensionless boundary conditions:

$$\eta=\eta_i \quad W=U=V=0 \quad \theta=0 \quad (6)$$

$$\eta=\eta_o \quad W=U=V=0 \quad \frac{\partial\theta}{\partial\eta}=0 \quad (7)$$

$$\varphi=0,\pi \quad U=0 \quad \frac{\partial V}{\partial \varphi}=\frac{\partial W}{\partial \varphi}=\frac{\partial \theta}{\partial \varphi}=0 \text{ (symmetry line)} \quad (8)$$

For convenience, the pressure is redefined as explained in Ould-Amer (2010). In the energy equation, the relation between axial temperature gradient and the average heat flux at the surface of the inner cylinder is given in Ould-Amer (2010). At the interfaces which separate the fully fluid zone and the porous fins, the continuity of flow, pressure, temperature, stress and also energy are adopted. The porosity is set equal to one in the fully fluid zone and the λ parameter and the conductivity ratio R_C are defined as follows:

$$\lambda=\begin{cases}0 & \text{in the fully fluid zone}\\ 1 & \text{in the porous bloks}\end{cases} \qquad R_C=\begin{cases}1 & \text{in the fully fluid zone}\\ k_P/k_f & \text{in the porous bloks}\end{cases}$$

Numerical Procedure:

The set of differential equations governing mixed convection in the horizontal concentric annuli is transformed into a system of algebraic equations with the use of the control volume approach. The SIMPLER algorithm is used for the calculation of the flow field and temperature. The system of algebraic equations is solved iteratively by means of the Thomas algorithm. Under–relaxation factors are introduced to avoid the divergence of the strongly non–linear system. A non uniform staggered grid of 74×64 nodes, over the entire domain, was selected on the basis of a grid sensitivity study. Convergence is controlled in terms of the relative error for the variables *U*, *V*, W, *P*, θ and the mass residual in each control volume. The convergence criterion is:

$$\left|\frac{\Phi_{ij}^{n+1}-\Phi_{ij}^{n}}{\Phi_{ij}^{n}}\right|<10^{-5} \text{ and } \left|\text{residual mass}\right|_{\text{in each control volume}}<10^{-5} \quad (9)$$

with Φ corresponding to U, V, W, P or θ, and n and $n+1$ indicating two consecutive iterations. The discontinuity between the porous fins and fully fluid zone is handled with the use of the harmonic mean formulation suggested by Patankar (1980). The porosity discontinuity at the interface between purely fluid zone and porous fins is evaluated by linear interpolation. The present code was validated by comparing the results obtained with our code for the fully fluid case, with those of Nieckele and Patankar (1985). Good agreement between the results is obtained (table 1). The situation of fully fluid case, i.e. without porous fins, is obtained by setting the Darcy number equal to a large value (typically infinite) and the porosity, R_V and R_C to one.

Table 1: Comparison of present results with those of Nieckele and Patankar (1985)

Gr	Present code		[9]	
	$\frac{f\,Re}{(f\,Re)_0}$	$\frac{\overline{\overline{Nu_b}}}{Nu_0}$	$\frac{f\,Re}{(f\,Re)_0}$	$\frac{\overline{\overline{Nu_b}}}{Nu_0}$
10^3	1.0000	1.0000	1.0000	1.0000
10^4	1.0002	1.0027	1.0000	1.0000
10^5	1.0115	1.1156	1.0083	1.1250
10^6	1.0793	1.4650	1.0833	1.4500
10^7	1.2692	1.7859	1.2667	1.8000

RESULTS AND DISCUSSION

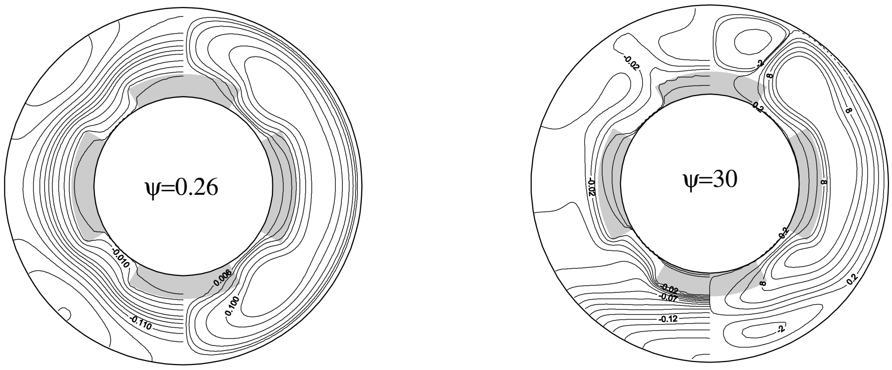
The numerical solutions are obtained for $\varepsilon=0.9$, $C_F=0.011$, $Pr=0.7$, $R_V=1$, $RR=2$, $\alpha=60°$, $\delta=30°$ and $R_C=100$. Two values of Darcy number are considered 10^{-3} and 10^{-4}. The Grashoff number is varied from 10 to 10^7. Three values for the porous fins thickness are considered $(H=0.125, H=0.25, H=0.375)$.

Australian Journal of Basic and Applied Sciences, 10(15) October 2016, Pages: 41-47

In this section, the flow structure and the heat transfer will be analyzed and discussed. The results include some local details of the flow and temperature fields and the overall heat transfer coefficient and friction factor.

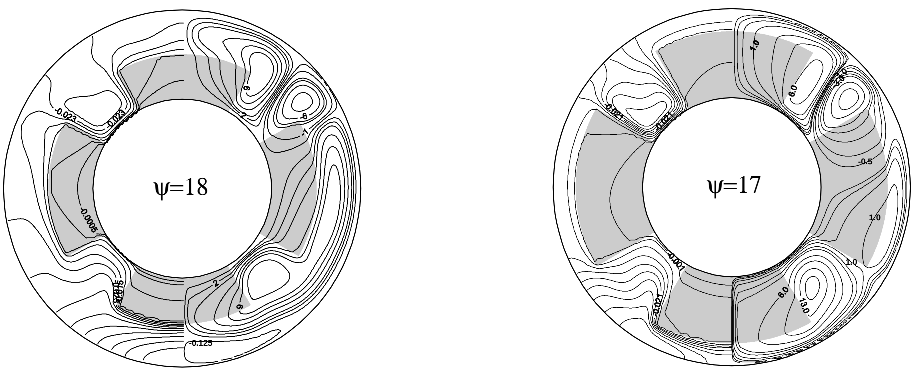
A. Streamlines and isotherms:

The secondary flow pattern and isotherms are displayed in Figures 2 and 3. Because of the symmetry about the vertical diameter, the isotherms are plotted in the left half and the streamlines in the right half. The value of ψ listed for each figure is a measure of the strength of the secondary flow; ψ is the maximum value of the dimensionless stream function. At $H = 0.125$ or $H = 0.25$ and for all values of Da when $Gr \leq 10^5$, the secondary flow forms a symmetrical eddy rotating in the clockwise direction, for instance Figure 2a. The isotherms are nearly circular with a deformation between two consecutive fins; this deformation is much pronounced when $H > 0.125$. As the Grashoff number increases, the secondary motion becomes stronger as evidenced by the higher values of ψ, in this situation, three or four eddies are formed. At higher values of Gr, the isotherm pattern shows the thermal boundary layer near the porous fins and between them close to heated inner duct. The presence of the porous fins causes the distortion of the isotherms between two consecutive porous fins, in this region the heat transfer decreases. The isotherms plots show that the thermal boundary layer along the surface of the two lower porous fins get thinner as H goes from 0.125 to 0.375, indicating improved heat transfer. However, in the upper region of the cross section the non-uniformities of temperatures occur, this behavior lead to the decrease of heat transfer in this zone. The presence of the porous fins causes a supplementary resistance to the fluid flow as evidenced by the deformation of streamlines near the porous fins, when H goes from 0.125 to 0.375. That is why the intensity of the secondary flow decreases when the thickness of the porous fins is larger. Indeed, the secondary flow is decelerated causing a significant increase in the flow resistance, the contribution of the secondary flow in heat transfer is thereby reduced.



a) $Gr = 10^4$ b) $Gr = 10^7$

Fig. 2: Isotherms and streamlines for $Da = 10^{-4}$ and H=0.125



a) $Gr = 10^7$ and H=0.25 b) $Gr = 10^7$ and H=0.375

Fig. 3: Isotherms and streamlines for $Da = 10^{-4}$

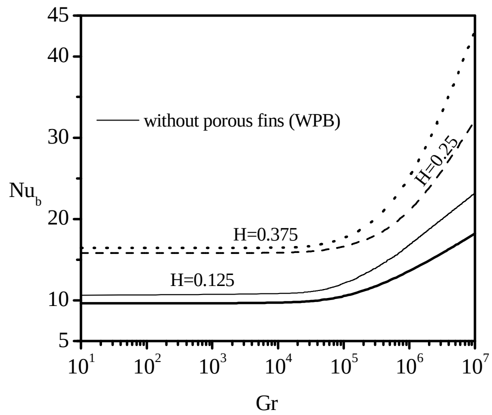
B. Overall heat transfer coefficient:

For the design purposes, the overall heat transfer in the device is of primary interest. The overall heat transfer results are presented in the form of the average Nusselt number Nu_b. The average Nusselt number is evaluated as:

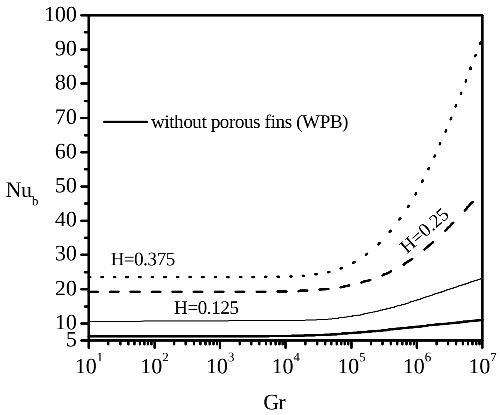
$$Nu_b = -\frac{1}{\theta_b} \quad (10)$$

Where θ_b is the dimensionless bulk temperature of the fluid.

The variation of Nu_b with Gr is shown in Figures 4a and 4b for various thickness of porous fins. The influence of buoyancy is to increase Nu_b; but this becomes noticeable only after a certain threshold value of Gr is exceeded (approximately $Gr > 10^5$). Figures 4a and 4b show that the use of the porous fins improve the heat transfer, more particularly with higher values of porous fins thickness. Indeed, for $Gr = 10^7$ and $Da = 10^{-4}$, Nu_b goes approximately from 20 for H=0.125 to 40 for H=0.375, i.e. an enhancement of 50%. The better improvement is obtained with $Da = 10^{-3}$. It is interesting to note that more permeable are the porous fins better is the heat transfer. These enhancements are attributable to the combined effect of highly conductive porous fins, buoyancy and the presence of much larger pore volume. Indeed, the porous fins acting as extended surfaces for heat transfer.



a) $Da = 10^{-4}$



b) $Da = 10^{-3}$

Fig. 4: Average Nusselt number

C. Friction factor:

The increased heat transfer in mixed convection and with presence of porous media does not come without a penalty. The pressure gradient required for a given mass flow rate through the duct also increases in magnitude. The axial pressure drop for the flow can be expressed in terms of the friction factor f, where f is given by the standard definition:

$$f = \frac{-\left(\frac{\partial p}{\partial z}\right) d}{\frac{1}{2}\rho \overline{w}^2} \quad (11)$$

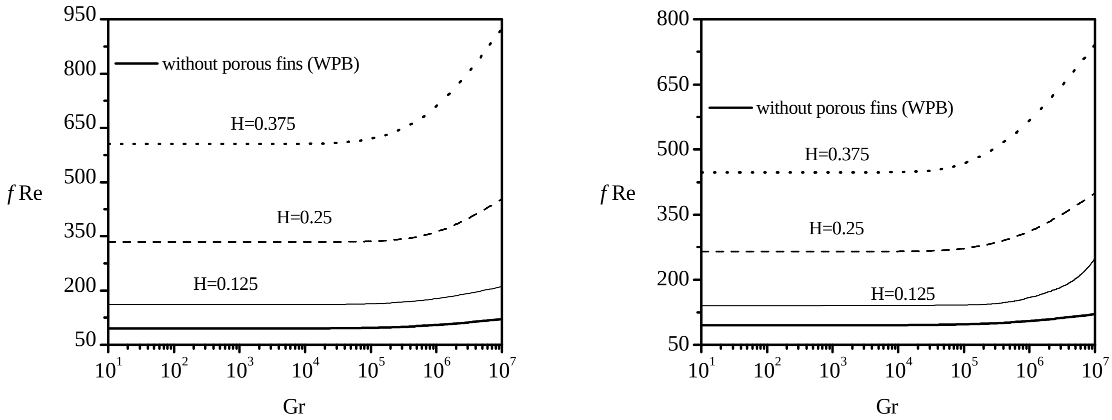
If previous equation is rewritten in terms of the dimensionless variables, there results:

$$f\,Re = \frac{2}{\overline{W}} \quad (12)$$

With

$$Re = \frac{\rho \overline{w} d}{\mu} \quad (13)$$

In the absence of the secondary flow, the product $f\,Re$ is constant, which depends only on the geometry and the permeability of the porous fins. Figures 5a and 5b show the product $f\,Re$ plotted as a function of Gr for different values of H. The influence of buoyancy becomes noticeable only after a certain threshold value of Gr is exceeded, since Stronger secondary flows are associated with increasing values of Gr. The increase in friction factor due to porous fins is seen to be significantly large with lower values of Darcy number and with higher values of H. Indeed, the flow resistance increases with the Darcy number decrease and with the increase in the thickness of porous fins.



a) $Da = 10^{-4}$ b) $Da = 10^{-3}$

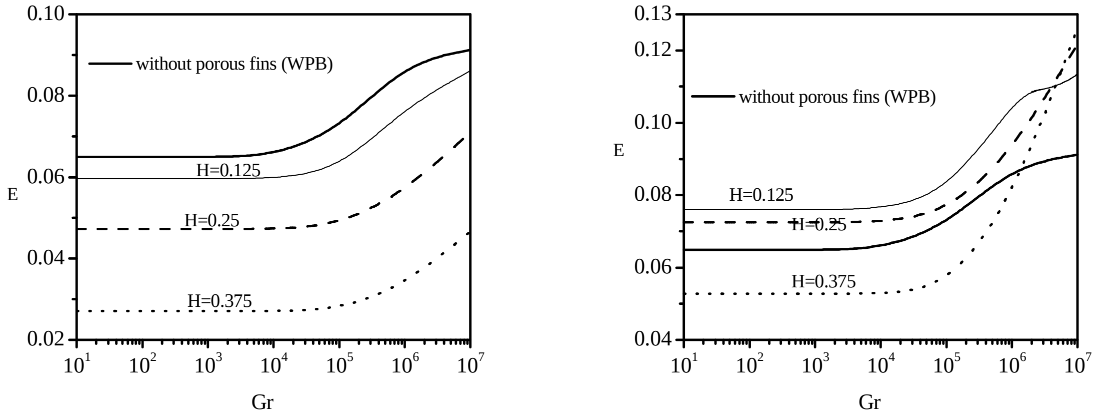
Fig. 5: friction factor

D. Ratio of mean Nusselt number to friction factor:

From previous discussion of the overall heat transfer coefficient and the friction factor, it appears that Nu_b increases with higher values of H; and on the other hand $f\,Re$ increases with the presence of the porous matrix. For this reason a new parameter must be taken into consideration in design calculations. It corresponds to ratio of the mean Nusselt number to the quantity $f\,Re$ defined as: $E = (Nu_b / f\,Re)$. Figures 6a and 6b show the ratio E plotted as a function of Gr for different values of H. The plot for the situation without porous fins is shown by solid line and is considered as reference. It can be seen that the ratio E is significantly influenced by the Darcy number and the thickness of porous fins. Thus The combined effect of highly conductive porous fins, the thickness H and the buoyancy is noticeable only for lower values of H (H<0.25) and for H=0.375 this occurs after a certain threshold value of Gr is exceed (approximately $Gr \geq 10^6$).

Conclusion:

Laminar mixed convection in horizontal concentric annuli in the presence of porous media has been numerically studied for various thicknesses of the porous bloks, Grashoff number, and Darcy number. It is found that the porous fins significantly affect heat transfer and the fluid flow characteristics. In each half of the annulus divided by the vertical center line, the secondary flow generated by buoyancy, in the presence of the porous structure, has single, two, three or four eddy pattern depending on the thickness of porous fins, Darcy number and Grashoff number. The use of highly conductive porous fins leads to a significant enhancement in heat transfer, particularly at H=0.125; however the friction factor increases with. The ratio of the average Nusselt number to friction factor is introduced to take account the challenge between increased heat transfer and decreased friction factor. The use of the porous structure, in the configuration proposed, is justified only when this parameter is greater than the values obtained for the fully fluid situation. This enhancement occurred for $Da = 10^{-3}$ and for two values of thickness (H=0.125 and H=0.25). For H=0.375, this enhancement occurs only when a certain threshold value of Gr is exceed.



a) $Da = 10^{-4}$ b) $Da = 10^{-3}$

Fig. 6: ratio of mean Nusselt number to friction factor

REFERENCES

Mohammed, H.A., A. Compo and R. Saidur, 2010. Experimental study of forced and free convective heat transfer in the thermal entry region of horizontal concentric annuli, Int. Com. Heat and Mass Transfer, 37: 739-747.

Nieckele, A.O. and S.V. Patankar, 1985. Laminar Mixed Convection in a Concentric Annulus with Horizontal Axis, ASME J. Heat Transfer, 107: 902-909.

Nield, D., and A. Bejan, 2006. Convection in Porous Media," 3rd edition, Springer, New York.

Ould-Amer, Y., 2016. 3D Fully Developed Laminar Mixed Convection in Horizontal Concentric Annuli with the Presence of Porous Blocks, IACSIT International Journal of Engineering and Technology, 8(2): 76-82.

Ould-Amer, Y., 2013. Optimal heat transfer in horizontal concentric annuli with the presence of porous structure, International conference on mechanical engineering and technology, Amsterdam 15-16 May 2013, Nederland, 77: 486-494. PISSN 2010-376X. EISSN 2010-3778.

Ould-Amer, Y., 2010. Laminar mixed convection in horizontal concentric annuli with four porous blocks attached on the outside of the inner cylinder, ASME 10th Biennial Conference on Engineering Systems Design and Analysis (ESDA2010), July 12–14, 2010, Istanbul, Turkey, Paper no. ESDA2010-24077, pp: 433-442.

Patankar, S.V., 1980. Numerical Heat Transfer and Fluid Flow, McGraw-Hill, New York.

Vanover, D.E. and F.A. Kulacki, 1987. Experimental Study of Mixed Convection in a Horizontal Porous Annulus, ASME HTD, 84: 61-66.

Venugopal, G., C. Balaji and S.P. Venkateshan, 2010. Experimental study of mixed convection heat transfer in a vertical duct filled with metallic porous structures, I. J. of Thermal Sciences, 49: 340-348.