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Numerical Computations of Coupling Constants in the relativistic Quark-Meson interaction

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ABSTRACT

During recent years, heavy meson physics has received a wide attention both from theory and experiment. The coupling constants of the relativistic quark-meson interaction is computed in a generalized constituent quark model constrained in the meson phenomenology and spectrum. A detailed analysis of all sectors from the light-pseudo scalar and vector mesons to bottomonium is performed paying special attention to the existence and nature of some non-well-established states. We examine the interaction of quarks and mesons numerically, and using the quark relativistic model we compute the values of the coupling constants. This was done by calculating the renormalisation constants and the mass difference between the heavy mesons H, S, T and the heavy quarks. These results should serve as a complementary tool in distinguishing conventional quark model mesons from glue balls, hybrids or multi quark states.

INTRODUCTION

An effective theory for heavy mesons, implementing the heavy quark symmetries, has been very successful at the phenomenological level(Kang et al., 2014). Predictions are easily obtained once the unknown effective couplings are fixed from experimental data. Moreover, such an effective approach can be combined with chiral symmetry for light mesons, thus giving a simple framework for implementing the known approximate symmetries of quantum chromodynamics (QCD) (Butenschoen and Kniehl, 2011). The disadvantage of such an approach is the number of free parameters, which grows very rapidly if one tries to improve the calculations beyond leading order. In order to go beyond the symmetry approach, one should be able to derive the free couplings at the meson Lagrangian level from a more fundamental theory, for example directly from QCD. This is clearly a difficult task. In the long run the definitive answer will come from systematic first principles calculations, such as in lattice QCD, but at present heavy meson physical quantities such as spectra and form factors are still subject to extrapolations, even if recent improvements are impressive(Colangelo et al., 2011). Moreover, an alternative and intuitive way for interpolating between QCD and an effective theory would be interesting in itself, allowing us to understand better the underlying physics. Obviously there is a price to pay for any simplification that may allow us to calculate the parameters of an effective heavy meson theory without solving the non-perturbative QCD problem. Our point of view here will be to consider a quark-meson Lagrangian where transition amplitudes are represented by diagrams with heavy mesons attached to loops containing heavy and light constituent quarks. It should be kept in mind that what we study here is only a model and not full QCD. However, one can hope to describe the essential part of the QCD behaviour in some energy range and extract useful information from it. Since the model used in the present paper is based on an effective

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constituent quark-meson Lagrangian containing both light and heavy degrees of freedom, it is constrained by the known symmetries of QCD in the limit$m_q \to \infty$. A similar model, for light quarks only, was pioneered, in the context of chirally symmetric effective theories, several years ago(Colangelo et al., 2012).A related approach is the one based on the extended Nambu–Jona-Lasinio whose generalization to include both heavy and light quarks has been studied. In this approach path integral bosonization is used, which replaces the effective 4-quark interactions by Yukawa-type couplings of heavy and light quarks with heavy meson fields. One may think of going even further and try to bosonize directly QCD, but a number of approximations are needed and one usually ends up with a nonlocal Lagrangian.

The standard model (in particle Physics) postulated that there are two classes of indivisible matter particles; the quarks and the leptons (Lincoln, 2012), the mixture of the two particles makes an atom. The standard model, however, left other unanswered questions like; why are there two types of fundamental particles rather than a single one that handles everything?

The Constituent Quark- Model (CQM) Lagrangian:

A constituent-quark-meson model (CQM) is relativistically an effective field theory which describes the quark-meson interactions. The model was first proposed by Polosa (2000) and has been well developed later based on the work of(Ebert et al., 1996). For reviews on this see (Segovia et al., 2013).The main feature of the model is the incorporation of the flavour-spin symmetry in the heavy quark sector and the chiral symmetry in the light quark sector. However, the effective lagrangian predicting the two sectors comes from the bosonisation of a primary Nambu-Jona-Lasinio (NJL) interaction involving light and heavy quark fields (Ebert et al., 1996). Employing the CQM model to study the phenomenology of heavy meson physics reasonable results have been achieved, (Zhao, 2008), in essence, the model describes hadron phenomenology and hadronic reactions and has been used to mesons which contain heavy quarks (Segovia et al., 2009).

CQM model plays the role of a fundamental model since it contains besides mesons fields, also the elementary heavy and light quark fields with which the hadron theory must match at higher energy. The mechanism describing the bare quark and giving the constituent quarks its mass value is an intrinsic feature of the model itself.

2.1 Effective Theories:

Effective theories are those models conceived to describe the Physics of a certain system at the energy scale of the experiment through which one studies it, i.e. at the level of accuracy chosen to experimentally examine the system, (Polosa, 2000). In this case, the atomic Physics of the hydrogen atom is an effective theory of the hydrogen.

Effective models succeed in giving reliable phenomenological predictions where fundamental theories have many more technical and sometimes principle problems. Quantum-Chromo-Dynamics (QCD) is the most important example of a fundamental theory, i.e., a theory which is derived from first principles, describing the intimate nature of strong interactions and the building fields of matter, which has deep troubles in dealing with the low energy hadron world. This is because of the partial theoretical comprehension of the confinement mechanism of quarks in the hadrons. Therefore, to deal with hadrons, it is always necessary to implement some low energy model, effective in the energy regions where the hadronic processes are expected to be studied.

2.2 Heavy Quark Effective Theory (HQET):

The Physics here is that of mesons containing one heavy quark, i.e. bottom b, or charm c. The HQET describes processes where a heavy quark interacts via soft gluons with the light degrees of freedom. The heavy scale, in this case, is clearly m_Q; the heavy quark mass and the other physical scale for the processes of interest is Λ_{QCD}.

Let the velocity of the hadron containing the heavy quark be V_μ, and its momentum P_Q can be written, by introducing a residual momentum k of the order Λ_{QCD}, as:

$$P_Q = m_Q V_\mu + k \qquad (1)$$

From this equation, we extract the dominant part $m_Q V_\mu$ of the heavy quark momentum defining a new field Q_v, given by;

$$Q_v(x) = e^{(im_q V_x)} Q(x) = h_v(x) + H_v(x) \qquad (2)$$

The field h_v is the large component field, and if the quark Q is exactly on shell, it is the only term present in equation (2) above. H_v is the small component field, and is of the order $1/m_Q$ which is integrated out when deriving the HEQT effective Lagrangian. HQET predicts the existence of two degenerate doublets: $(0^+, 1^+)$ and $(1^+, 2^+)$ for each heavy quark c or b.

Chiral Lagrangians:

It is natural to divide quarks into two classes by comparing their Lagrangian mass with Λ_{QCD}. The Up (U) and down (D) quarks belong definitely to the light quarks class, however, the situation for the strange quark is not so clear, but it is usually considered to belong to the light quark class, though non-negligible mass corrections are expected. Thus, for light quarks, we mean quark mass $m_Q \ll \Lambda_{QCD}$ or $m_Q \to 0$.

Therefore, considering the limit $m_Q \to 0$, the QCD lagrangian for these three quarks possesses an $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$symmetry which is spontaneously broken down to $SU(3)_V \otimes U(1)_V$,

The lightest Pseudo scalar particles of the octet $\pi, K, \bar{K}$ and η are identified with the Goldstone bosons corresponding to the broken generators. Of course, due to the explicit symmetry breaking given by the quark mass term, the mesons acquire a mass. However, the interactions among Goldstone bosons can be described by the chiral perturbation theory,

The interactions of the Goldstone fields with matter fields such as baryons, heavy mesons or light vector mesons can be described by using the theory of non-linear representations as discussed in the classical paper by Callen *et al.*

In constructing the chiral effective theory, we follow the instruction which describes the general effective theory and then write the lagrangian which contains Σ field that is consistent with relativistic invariance, PCT, and QCD chiral symmetry.

The chiral lagrangian with two derivatives is regarded as non-trivial term and is given as;

$$\mathcal{L}_2 = \frac{f_\pi^2}{4} T_r\left[\delta_\mu \Sigma \delta^\mu \Sigma^+\right] \tag{3}$$

f_π is the pion decay constant which is one of the energy scales characterizing the pion (the other one is its mass). The Σ field is described as the exponential of NG boson fields given by;

$$\Sigma(x) = e^{\frac{2\pi(x)i}{f\pi}} \tag{4}$$

where $\pi(x) = \pi^a(x)T^a$

2.4 The Effective Lagrangian:

As discussed previously in section one, the CQM lagrangian is made up of two terms combining the HQET and the chiral symmetry, as in the equation below;

$$\mathcal{L}_{CQM} = \mathcal{L}_{\ell\ell} + \mathcal{L}_{\hbar\ell} \tag{5}$$

In this equation, the first term $\mathcal{L}_{\ell\ell}$ involves only the light quark degrees of freedom, χ and the Pseudoscalar light mass octet, this constituent quark model was originally suggested by Manohar and Georgi (1984).

The lowest order of the $\mathcal{L}_{\ell\ell}$ in a derivative expansion can be written as:

$$\mathcal{L}_{\ell\ell} = \bar{\chi}[\gamma.(i\partial + v)]\chi + \bar{\chi}\gamma.\mathrm{A}\gamma_5\chi - m\chi\bar{\chi} + \frac{f_\pi^2}{8}\left(\partial_\mu\Sigma^+\partial^\mu\Sigma\right) \tag{6a}$$

In this equation (6a), all the terms are chiral invariant except the mass term. The terms involved in this equation are define as:

$$\left.\begin{array}{c} \chi = \xi q,\ \ \Sigma = \xi^2 \ \ and\ \xi = \ e^{i\pi/f_\pi} \\ The\ q\ are\ the\ light\ quark\ fields\ (u,d,s)\ and\ f_\pi = 130MeV\ is\ the\ pion\ decay\ constant \end{array}\right\} \tag{6b}$$

However, the two auxiliary fields v^μ and A^μ are given by:

$$\left.\begin{array}{l} v^\mu = \frac{1}{2}\{\xi^+\partial^\mu\xi + \xi\partial^\mu\xi^+\} \\ \mathrm{A}^\mu = \frac{i}{2}\{\xi^+\partial^\mu\xi - \xi\partial^\mu\xi^+\} \end{array}\right\} \tag{6c}$$

The first part of equation (6c) describes the interactions between light quarks and light mesons. Similarly, the coupling of the light quark to an odd number of pions is mediated in the second part of equation (6c). The mass m in equation (6a) is dynamically generated according to the CQM model mechanism.

We see that in equation (6a), there are no gluons and the light fields are defined differently. This absence of gluons is plausible since the lagrangian originates from the bosonisation of an underlying NJL interaction lagrangian where gluons are absent from the start, Polosa (2000).

Meanwhile, the effective Lagrangian containing heavy and light quarks together with meson fields is given by:

$$\mathcal{L}_{\hbar\ell} = \bar{Q}_v(iv.\partial)Q_v - \left[\bar{\chi}\left(\bar{H} + \bar{S} + i\bar{T}^\mu\frac{\partial_\mu}{\Lambda_x}\right)Q_v + h.c.\right] + \frac{1}{2G_3}T_r[(\bar{H}+\bar{S})(H-S)] + \frac{1}{2G_4}T_r\left[\bar{T}^\mu T_\mu\right] \tag{7a}$$

Where the first term in equation (7a) is the heavy quark kinetic terms of HEQT, and Q_v is the effective heavy quark field of the HQET given in equation (2) and Λ_x is the momentum scale characterising the convergence of the derivative expansion, usually taken as an estimate of the energy scale associated to spontaneous chiral symmetry breaking given by

$$\Lambda_x = 4\pi f_\pi \simeq 1GeV \tag{7b}$$

From a theoretical point of view, the G3 G3' couplings could have been determined by Δ_H and Δ_S, the experimental values of the mass differences between the mesons belonging to H and S multiplets respectively and their heavy quark constituent masses. The dynamical information $\frac{1}{2G_3} = -\frac{1}{2G_3'}$ is crucial for the CQM calculations of the coupling constants because there are not sufficient experimental data to determine two different constants, therefore, the dynamical information coming from NJL-model must be adopted as proposed by (Ebert et al., 1996) and so the modified version was used in equation (7a). The part containing the T field cannot be extracted from a bosonized NJL contact interaction and requires a new coupling G_4.

The lagrangian in equation (7a) comprises three terms containing respectively H, S, and T. The H and S are degenerate in mass in the light-sector chirally symmetric phase. Similarly, the H, S and T are the effective super fields corresponding to the doublet, (0^-, 1^-), (0^+, 1^+) and (1^+, 2^+) respectively, and has the following explicit matrix representations;

$$\left.\begin{aligned} H &= \frac{1+\gamma.\upsilon}{2}\left(P_\mu^*\gamma^\mu - P\gamma_5\right) \\ S &= \frac{1+\gamma.\upsilon}{2}\left(P_{1\mu}^*\gamma^\mu\gamma_5 - P_0\right)\ and \\ T^\mu &= \frac{1+\gamma.\upsilon}{2}\left(P_2^{*\mu\nu}\gamma_\nu - \sqrt{\frac{3}{2}}P_{1\nu}^*\gamma_5\left(g^{\mu\nu} - \frac{1}{3}\gamma(\gamma^\mu - \nu^\mu)\right)\right) \end{aligned}\right\} \tag{7c}$$

where $P, P^{*\mu}, P_0$ and P_1 are the annihilation operators of the Pseudoscalar, vector, scalar and axial vector mesons respectively. These vertices for Hand S mesons have been derived from bosonization. The vertex involving T field is instead a phenomenological term, introduced according to the philosophy of effective theories ($\Lambda = 1GeV$).
Therefore, putting equations (6a) and (7a) into (5) yields;

$$\mathcal{L}_{CQM} = \bar{\chi}[\gamma.(i\partial + \upsilon)]\chi + \bar{\chi}\gamma.\mathrm{A}\gamma_5\chi - m\chi\bar{\chi} + \frac{f_\pi^2}{8}\left(\partial_\mu\Sigma^+\partial^\mu\Sigma\right) + \bar{Q}_\nu(i\upsilon.\partial)Q_\nu - \left[\bar{\chi}\left(\bar{H} + \bar{S} + i\bar{T}^\mu\frac{\partial_\mu}{\Lambda_x}\right)Q_\nu + h.c.\right] + \frac{1}{2G_3}T_r[(\bar{H} + \bar{S})(H - S)] + \frac{1}{2G_4}T_r\left[\bar{T}^\mu T_\mu\right] \tag{8}$$

An important feature of the effective lagrangian in equation (8) is that $\mathcal{L}_{CQM}$ describes the interaction vertex of the heavy-meson with heavy and light quarks, which makes the study of the phenomenology of heavy-meson physics at quark level feasible. In this equation it, is assumed that the effective quark meson lagrangian can be justified as a remnant of a four quark interaction of the NJL type by partial bosonization.

Employing this model to study the phenomenology of heavy meson physics have shown reasonable results, (Deandrea, 1999, Deandrea et al., 1998)

Regularization:

The CQM model neither incorporates quarks confinement nor includes gluons fields. As such, this appeared to be a strong limitation of the model, but, it is physically much more important working with non-confining models that possess chiral symmetry and its spontaneous breakdown than with confining models where the chiral symmetry and its breakdown are not properly incorporated.

We will show here how we face the problem that CQM is not a confining model which is achieved by introducing an infrared cut-off μ. An essential condition of the nonperturbative QCD behaviour is the suppression of large momentum flows through light-quark lines in the loops. The model describes the interactions in terms of effective vertices between a light quark, a heavy quark, and a heavy meson. The heavy quark and heavy mesons are consistently described with HEQT, and therefore, the heavy quark propagator in the loop contains the residual momentum k that arises from the interaction with the light degrees of freedom.

The kinematic condition for a heavy meson having mass M to decay into its free constituent quarks is:

$$M > m_q + m$$

While the meson momentum $P_q = m_q v + k$ Therefore, the kinematic condition above is equivalent to the condition$v.k > m$, and if the meson is at rest this means, $k_o > m$. Hence, the condition holds only when the residual momentum k is larger than m, as it should be in a non-confining model (because for any lower value of k, one is in the energy region where confinement must be considered).

However, the constituent light mass is determined by:

$$\langle\Sigma\rangle = m = \widetilde{m} + 8mI_1(m^2) \tag{9}$$

Where the chiral ray Σ defines a direction of the condensate in the flavour space, and $\Sigma\Sigma^+ = 1$ and $N_c = 3$

The I_1 (given in the appendix) is calculated with a UV and an IR cut-off introduced according to the Shwinger's regularization method. As the infrared cut-off varies, the m value varies accordingly, and therefore we choose an infrared cut-off $\mu = m$ which means the running momenta in the CQM loops is of the size of the heavy quark residual momenta, even though Deandrea et al. (1998) observed that the numerical outcome of the subsequent calculations is not strongly dependant on the value of Λ.

Ebert et al. (1996) Showed the plot of m against μ obtained from (9) for a fixed value of Λ. This plot has a typical shape of a second order phase transition order parameter with a critical μ at $\mu_c \approx 550MeV$. For $\mu > \mu_c$, m is zero, i.e., the chiral symmetry is unbroken. Meanwhile, for $\mu = m = 300MeV$, one is in the broken (physical) phase at the edge of a plateau. Hence, we say for larger values of μ, m decreases and at μ_c it vanishes.

Therefore, the boundary energy values of the effective theory chosen to be $\mu = 300MeV$, $\Lambda \approx \Lambda_\chi \approx 1.25GeV$(The choice of this ultraviolet cutoff is given by the scale of the chiral symmetry breaking $\Lambda_x = 4\pi f_\pi$) and the light constituent mass is dynamically generated by a NJL gap equation: m=300MeV.

Since the model is not confining, the range of the infrared behaviour cannot extend below the energies of the order of Λ_{QCD} .Therefore, the introduction of the infrared is practically to drop the unknown confinement part of the quark interaction. We then make a choice for the prescription to implementing the cut-offs in the calculations, and for a non renormalizable model, this step is one of the definition of the model itself, Polosa (2000) confirmed that the proper time Shwinger regularization has shown to be the most adequate for these purposes (perhaps a different choice is followed in Bardeen and Hill (1994)).

The following is produced after continuation of the light propagator in the Euclidean domain;

$$\int d^4k_E \frac{1}{k_E^2+m^2} \rightarrow \int d^4k_E \int_{1/\mu^2}^{1/\Lambda^2} ds e^{-s(k_E^2+m^2)} \quad (10)$$

In this equation μ and Λ are infrared and ultraviolet cutoffs respectively and m is the constituent light quark mass, whose nonvanishing value implies the mass splitting between the H and S multiplets. The values of μ, Λ and m are in principle adjustable parameters.

Renormalization constants and masses:

The CQM self-energy diagram for two contracting vertices Q-Meson-q (figure below) can be applied for the computation of the constants G_3 and G_4 appearing in the quark-meson effective lagrangian.

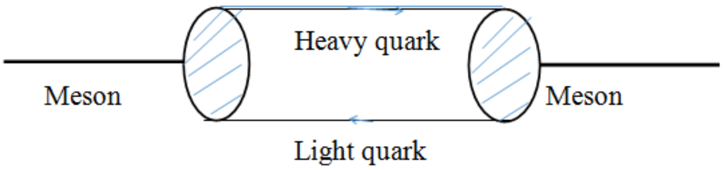

Fig. 1: CQM self-energy diagram for heavy meson field

The loop integral (for in and out H, S, and T fields) can be given by applying the standard loop integrals as follows:

$$Tr[-\bar{H}\prod_H(v.k)H] = iN_c \int \frac{d^4l}{(2\pi)^4} \frac{Tr[H.(l-k+m)\bar{H}]}{[(l-k)^2-m^2+i\epsilon][v.l+i\epsilon]} \quad (11)$$

$$Tr[+\bar{S}\prod_S(v.k)S] = iN_c \int \frac{d^4l}{(2\pi)^4} \frac{Tr[S.(l-k-m)\bar{S}]}{[(l-k)^2-m^2+i\epsilon][v.l+i\epsilon]} \quad (11)$$

and for T states, we get

$$Tr\left[+\bar{T}_\mu\prod_T(v.k)T^\mu\right] = \frac{iN_c}{\Lambda_\chi{}^2} \int \frac{d^4l}{(2\pi)^4} \frac{Tr[T^\nu.(l_\mu-k_\mu)(l-k+m)(l_\nu-k_\nu)\bar{T}^\mu]}{[(l-k)^2-m^2+i\epsilon][v.l+i\epsilon]} \quad (13)$$

Expanding these three equations around Δ_H, Δ_S *and* Δ_T respectively, we get;

$$\Pi(v.k) \cong \Pi(\Delta) + \Pi'(\Delta)(v.k - \Delta) \quad (14)$$

The residual momentum k fluctuates smoothly around$(M - mQ)v$, i.e.

$$k^\mu = \Delta v^\mu - q^\mu \quad (15)$$

Where q is a parameterized of the fluctuation and Δ is given as Δ= $M - mQ$

This expansion can be substituted in the self-energy expression for the H, S and T fields respectively and subtracting from $\mathcal{L}_{\hbar\ell}$ the counter-terms $[-\bar{H}\prod_H(v.k)H]$, $Tr[+\bar{S}\prod_S(v.k)S]$ and $Tr\left[+\bar{T}_\mu\prod_T(v.k)T^\mu\right]$ we obtain a modified kinetic part of the effective lagrangian $\mathcal{L}_{\hbar\ell}$ given by;

$$\mathcal{L}_{ren}^{hl} = -Tr[\bar{H}_{ren}(iv.\partial - \Delta_H)H_{ren}] + Tr[\bar{S}_{ren}(iv.\partial - \Delta_S)S_{ren}] + Tr\left[\bar{T}_{ren}^\mu(iv.\partial - \Delta_T)T_{ren\mu}\right] \quad (16)$$

Provided the constants G_3 and G_4 satisfy the conditions that:

$$\frac{1}{2G_3} = \Pi_H(\Delta_H) = \Pi_S(\Delta_S) \quad (17)$$

$$\frac{1}{2G_4} = \Pi_T(\Delta_T) \quad (18)$$

And the renormalize fields are:

$$H_{ren} = \frac{H}{\sqrt{Z_H}} \quad (19)$$

$$S_{ren} = \frac{S}{\sqrt{Z_S}} \tag{20}$$
$$T_{ren} = \frac{T}{\sqrt{Z_T}} \tag{21}$$
The renormalization constants are defined as:
$$Z_j^{-1} = \left(\frac{d}{dx}\Pi(x)\right)_{x\to\Delta_j} \tag{22}$$
Here, the $j = H, S, T.$ and Δ_H,Δ_S, Δ_T are the mass differences between the heavy mesons H, S, T and the heavy quark.

Meanwhile, in the chiral Lagrangian approach for heavy meson states, the fundamental fields of the Lagrangian are the meson fields. And therefore, this model becomes more fundamental approach because together with meson fields, it also includes the quark fields.

Δ_H is the free CQM variable and it cannot be deduce from the model, but can only be fix by reasonable numerical values. Whereas, the Δ_S, and Δ_T can be known by fixing Δ_H.

The CQM expressions for $\Pi_H(\Delta_H)$, $\Pi_S(\Delta_S)$and $\Pi_T(\Delta_T)$ for the calculations of the renormalization constants $Z_{H,S,T}$:
$$\Pi_H(\Delta_H) = I_1 + (\Delta_H + m)I_3(\Delta_H) \tag{23}$$
$$\Pi_S(\Delta_S) = I_1 + (\Delta_S - m)I_3(\Delta_S) \tag{24}$$
$$\Pi_T(\Delta_T) = \frac{1}{\Lambda_\chi^2}\left[-\frac{I_1'}{4} + \frac{m+\Delta_T}{3}\left[I_0(\Delta_T) + \Delta_T I_1 + \left(\Delta_T^2 - m^2\right)I_3(\Delta_T)\right]\right] \tag{25}$$
And the field renormalization constants are:
$$\frac{1}{Z_H} = (\Delta_H + m)\frac{\partial I_3(\Delta_H)}{\partial \Delta_H} + I_3(H_H) \tag{26}$$
$$\frac{1}{Z_S} = (\Delta_S - m)\frac{\partial I_3(\Delta_S)}{\partial \Delta_S} + I_3(\Delta_S) \tag{27}$$
$$\frac{1}{Z_T} = \frac{1}{3\Lambda_\chi^2}\left[(\Delta_T^2 - m^2)\left((m + \Delta_T)\frac{\partial I_3(\Delta_T)}{\partial \Delta_T} + I_3(\Delta_T)\right) + (m + \Delta_T)\left(\frac{\partial I_0(\Delta_T)}{\partial \Delta_T} + I_1 + 2\Delta_T I_3(\Delta_T)\right) + I_0 + \Delta_T I_1\right] \tag{28}$$
The integrals I_0, I_1, I_1' and I_3 are given in appendix.

Conclusion:

To compute numerically, the couplings G_3, G_4, and the renormalisation constants; we fix the free CQM parameterΔ_H. The values will take the range of $\Delta_H = 0.3, 0.4, 0.5 GeV$. The interaction of quarks and mesons was examined numerically, and using the quark relativistic model we compute the values of the coupling constants. This was done by calculating the renormalisation constants and the mass difference between the heavy mesons H, S, T and the heavy quarks. These results should serve as a complementary tool in distinguishing conventional quark model mesons from glue balls, hybrids or multi quark states.

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Appendix:

$$I_0(\Delta) = \frac{iN_c}{16\pi^4}\int \frac{d^4k}{(v.k + \Delta + i\epsilon)}$$

$$= \frac{N_c}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} \frac{ds}{s^{3/2}} e^{-s(m^2-\Delta^2)} \left(\frac{3}{2s} + m^2 + \Delta^2\right)\left\{1 + erf\left(\Delta\sqrt{s}\right)\right\} - \frac{\Delta N_c m^2}{16\pi^2}\Gamma\left(-1, \frac{m^2}{\Lambda^2}, \frac{m^2}{\mu^2}\right)$$

$$I_1 = \frac{iN_c}{16\pi^4}\int d^4k \frac{k^2}{(k^2 - m^2)} = \frac{N_c m^2}{16\pi^2}\Gamma\left(-1, \frac{m^2}{\Lambda^2}, \frac{m^2}{\mu^2}\right)$$

$$I_1' = \frac{iN_c}{16\pi^4}\int d^4k \frac{k^2}{(k^2 - m^2)} = \frac{N_c m^4}{8\pi^2}\Gamma\left(-2, \frac{m^2}{\Lambda^2}, \frac{m^2}{\mu^2}\right)$$

$$I_2 = \frac{-iN_c}{16\pi^4}\int d^4k \frac{k^2}{(k^2 - m^2)} = \frac{N_c}{16\pi^2}\Gamma\left(0, \frac{m^2}{\Lambda^2}, \frac{m^2}{\mu^2}\right)$$

$$I_3(\Delta) = -\frac{iN_c}{16\pi^4}\int \frac{d^4k}{(k^2 - m^2)(v.k + \Delta + i\epsilon)} = \frac{N_c}{16\pi^4}\int_{1/\Lambda^2}^{1/\mu^2} \frac{ds}{s^{3/2}} e^{-s(m^2-\Delta^2)}\left\{1 + erf\left(\Delta\sqrt{s}\right)\right\}$$

In these equations,

$$\Gamma(\alpha, x_0, x_1) = \int_{x_0}^{x_1} t^{\alpha-1}e^{-t}dt$$

is the incomplete gamma function, while erf is the error function defined by;

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}dx$$