Australian Journal of Basic and Applied Sciences, 10(15) October 2016, Pages: 194-208

AUSTRALIAN JOURNAL OF BASIC AND APPLIED SCIENCES

ISSN:1991-8178 EISSN: 2309-8414
Journal home page: www.ajbasweb.com

On Soft Generalized Star b**-Closed Sets and Soft Generalized Star b**-Continuous Functions In Soft Topological Spaces

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A R T I C L E I N F O

Article history:
Received 26 August 2016
Accepted 10 October 2016
Published 18 October 2016

Keywords:
*soft g*b**-closed set, soft g*b**-open* , soft $g*b** - \tilde{T}_{1/2}$ − space *set,* , soft $\tilde{T}_{g*b**}$ − space soft $\tilde{T}_b$ − space , soft g*b**-continuous function, soft g*b**-irresolute function.

A B S T R A C T

The purpose of this paper is to study a new concept in soft topological spaces, namely, soft generalized star b**-closed sets, as generalization of soft closed sets. This class of soft closed sets is placed properly between the classes of soft regular closed sets and soft closed sets and each of soft s*g-closed sets, soft semi generalized closed sets, soft generalized semi closed sets, soft generalized b-closed sets, soft generalized closed sets, soft regular generalized closed sets, soft regular w-closed sets, soft α-generalized closed sets and soft generalized α-closed sets respectively. As an application of soft generalized star b**-closed sets, we introduce and study new types of soft spaces, namely, soft $\tilde{T}_{g*b**}$ − space and soft $g*b** - \tilde{T}_{1/2}$ − space. Also, we use these soft closed sets to study new kinds of soft functions, namely, soft generalized star b**-continuous functions and soft generalized star b**-irresolute functions in soft topological spaces and we investigate the relationships between soft generalized star b**-continuous functions and each of soft continuous functions, soft regular continuous functions and other weaker forms of soft continuous functions.

INTRODUCTION

Molodtsov, D. (1999) introduced and study the concept of soft set theory to solve some problems in engineering, environment and economics. He successfully applied the soft set theory into several directions, such as game theory, operations research, Riemann integration, smoothness of functions and theory of probability and so on. Shabir, M. and M. Naz, (2011) introduced the notion of soft topological spaces. Yuksel, S. and *et al*. (2014), Akdag, M. and A. Ozkan, (2014), Chen, B. (2013) introduced and study soft regular open sets, soft b-open sets, soft α-open sets and soft semi-open sets respectively. Also, Al-Salem, S. (2014), Kannan, K. (2012), Arockiarani, I. and A. Arokia Lancy (2013), Kannan, K. and D. Rajalakshmi (2015), Seenivasan, V. and S. Kalaiselvi (2013) and Yuksel, S. and *et al*. (2014) introduced and studied soft generalized α-closed sets, soft generalized b-closed sets, soft generalized closed sets, soft α-generalized closed sets, soft generalized semi-closed sets, soft s*g-closed sets, soft semi generalized closed sets and soft regular generalized closed sets respectively. In the present paper, we introduce a new class of generalized soft closed sets in soft topological spaces, namely, soft generalized star b**-closed sets. The soft generalized star b**-closed sets is stronger than each of soft s*g-closed sets, soft semi generalized closed sets, soft generalized semi closed sets, soft generalized b-closed sets, soft generalized closed sets, soft regular generalized closed sets, soft regular w-closed sets, soft generalized α-closed sets and soft α-generalized closed sets and weaker than soft regular closed sets and soft closed sets. Also, we use these soft sets to define and study new kinds of soft functions, namely, soft generalized star b**-continuous functions and soft generalized star b**-irresolute functions in soft topological spaces and

To Cite This Article: Sabiha I.Mahmood, On Soft Generalized Star b**-Closed Sets and Soft Generalized Star b**-Continuous Functions In Soft Topological Spaces. ***Aust. J. Basic & Appl. Sci.,*** *10(15): 194-208, 2016*

we study the relation between these soft functions and each of soft regular continuous functions, soft continuous functions and other weaker forms of soft continuous functions.

1. Preliminaries:

Let $P(X)$ be the power set of X and E be the set of all parameters for X. Then:

Definition 1.1(Molodtsov, D., 1999):

A soft set over X is a pair (U,M), where U is a function given by $U:M \to P(X)$ and M is a non-empty subset of E.

Definition 1.2 (Nazmul, Sk. and S. K. Samanta, 2012):

If (U,M) is a soft set over X. Then $\tilde{m}=(e,\{m\})$ is said to be a soft point of (U,M) if $e \in M$ and $m \in U(e)$, and is denoted by $\tilde{m} \tilde{\in} (U,M)$.

Definition 1.3 (Shabir, M. and M. Naz, 2011):

If $\tilde{\tau}$ is a collection of soft sets over X. Then $\tilde{\tau}$ is said to be a soft topology on X if $\tilde{\tau}$ is satisfy the following:

i) $\tilde{\phi}$, $\tilde{X}$ belong to $\tilde{\tau}$.

ii) If $(U_1,E),(U_2,E) \tilde{\in} \tilde{\tau}$, then $(U_1,E) \tilde{\bigcap} (U_2,E) \tilde{\in} \tilde{\tau}$

iii) If $(U_\alpha,E) \tilde{\in} \tilde{\tau}$, $\forall \alpha \in \Lambda$, then $\bigcup_{\alpha\in\Lambda}(U_\alpha,E) \tilde{\in} \tilde{\tau}$.

The triplet $(X,\tilde{\tau},E)$ is said to be a soft topological space over X. The elements of $\tilde{\tau}$ are said to be soft open sets in $\tilde{X}$

Definition 1.4 (Cağman, N., 2011):

Let (A,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then:

i) $cl(A,E)=\tilde{\bigcap}\{\,(F,E):(F,E)$ is a soft closed set in $\tilde{X}$ and $(A,E)\tilde{\subseteq}(F,E)\,\}$ is called the soft closure of (A,E).

ii) $int(A,E)=\tilde{\bigcup}\{\,(O,E):(O,E)$ is a soft open set in $\tilde{X}$ and $(O,E)\tilde{\subseteq}(A,E)\,\}$ is called the soft interior of (A,E).

Definition 1.5 (Shabir, M. and M. Naz, 2011):

If $(X,\tilde{\tau},E)$ is a soft topological space and $(Y,E)\tilde{\subseteq}\tilde{X}$. Then $\tilde{\tau}_Y=\{(Y,E)\tilde{\bigcap}(O,E):(O,E)\tilde{\in}\tilde{\tau}\}$ is said to be a soft subspace topology on (Y,E), and $((Y,E),\tilde{\tau}_Y,E)$ is said to be a soft subspace of $(X,\tilde{\tau},E)$.

Proposition 1.6:

Let $((Y,E),\tilde{\tau}_Y,E)$ be a soft subspace of a soft topological space $(X,\tilde{\tau},E)$ and $(A,E)\tilde{\subseteq}(Y,E)$. Then:

i) $cl_Y(A,E)=(Y,E)\tilde{\bigcap}cl(A,E)$ **(Nazmul, Sk. and S. K. Samanta, 2012)**

ii) If (Y,E) is soft open in $(X,\tilde{\tau},E)$, then $int_Y(A,E)=(Y,E)\tilde{\bigcap}int(A,E)$.

Definition 1.7:

A soft subset (F,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be:

i) soft regular open (soft r-open) set **(Yuksel, S. A., 2014)** if $int(cl(F,E))=(F,E)$.

ii) soft α-open set **(Akdag, M. and A. Ozkan, 2014)** if $(F,E)\tilde{\subseteq}int(cl(int(F,E)))$.

iii) soft semi-open (soft s-open) set **(Chen, B., 2013)** if $(F,E)\tilde{\subseteq}cl(int(F,E))$.

iv) soft b-open set **(Akdag, M. and A. Ozkan, 2014)** if $(F,E)\tilde{\subseteq}int(cl(F,E))\tilde{\bigcup}cl(int(F,E))$.

v) soft regular semi-open set if there is a soft regular open set (U,E) such that $(U,E)\tilde{\subseteq}(F,E)\tilde{\subseteq}cl(U,E)$.

The soft α-closure **(Akdag, M. and A. Ozkan, 2014)** (resp. soft semi-closure **(Chen, B., 2013)** , soft b-closure **(Akdag, M. and A. Ozkan, 2014))** of a soft subset (F,E) of a soft topological space $(X,\tilde{\tau},E)$ is the intersection of all soft α-closed (resp. soft semi-closed, soft b-closed) sets that contains (F,E) and is denoted by $\alpha cl(F,E)$ (resp. $scl(F,E)$, $bcl(F,E)$). Clearly $bcl(F,E)\tilde{\subseteq} scl(F,E)\tilde{\subseteq}\alpha cl(F,E)\tilde{\subseteq} cl(F,E)$.

Definition 1.8:

A soft subset (F,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be:

i) soft generalized closed (briefly soft g-closed) set **(Kannan, K., 2012)** if $cl(F,E)\tilde{\subseteq}(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft open in $(X,\tilde{\tau},E)$.

ii) soft generalized semi-closed (briefly soft gs-closed) set **(Arockiarani, I. and A. Arokia Lancy, 2013)** if $scl(F,E)\tilde{\subseteq}(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft open in $(X,\tilde{\tau},E)$.

iii) soft semi-generalized closed (briefly soft sg-closed) set **(Seenivasan, V. and S. Kalaiselvi, 2013)** if $scl(F,E)\tilde{\subseteq}(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft semi-open in $(X,\tilde{\tau},E)$.

iv) soft α-generalized closed (briefly soft αg-closed) set **(Arockiarani, I. and A. Arokia Lancy, 2013)** if $\alpha cl(F,E)\subseteq(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft open in $(X,\tilde{\tau},E)$.

v) soft generalized α-closed (briefly soft gα-closed) set **(Al-Salem, S. M., 2014)** if $\alpha cl(F,E)\subseteq(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft α-open in $(X,\tilde{\tau},E)$.

vi) soft regular generalized closed (briefly soft rg-closed) set **(Yuksel, S. A., 2014)** if $cl(F,E)\tilde{\subseteq}(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft regular open in $(X,\tilde{\tau},E)$.

vii) soft regular w-closed (briefly soft rw-closed) set if $cl(F,E)\tilde{\subseteq}(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft regular semi-open in $(X,\tilde{\tau},E)$.

viii) soft s*g-closed set **(Kannan, K. and D. Rajalakshmi, 2015)** if $cl(F,E)\tilde{\subseteq}(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft semi-open in $(X,\tilde{\tau},E)$.

ix) soft generalized b-closed (briefly soft gb-closed) set **(Al-Salem, S. M., 2014)** if $bcl(F,E)\tilde{\subseteq}(V,E)$ whenever $(F,E)\tilde{\subseteq}(V,E)$ and (V,E) is soft open in $(X,\tilde{\tau},E)$.

The complement of a soft g-closed (resp. soft gs-closed, soft sg-closed, soft αg-closed, soft gα-closed, soft rg-closed, soft rw-closed, soft s*g-closed, soft gb-closed) set is called a soft g-open (resp. soft gs-open, soft sg-open, soft αg-open, soft gα-open, soft rg-open, soft rw-open, soft s*g-open, soft gb-open) set. The family of all soft g-closed (resp, soft gs-closed, soft s*g-closed) subsets of $(X,\tilde{\tau},E)$ is denoted by $SGC(\tilde{X})$ (resp. $SGSC(\tilde{X})$, $SS*GC(\tilde{X})$)

Definition 1.9:

A soft function $f:(X,\tilde{\tau},E)\to(Y,\tilde{\sigma},E')$ is said to be:

i) soft continuous **(Nazmul, Sk. and S. K. Samanta, 2012)** if $f^{-1}((F,E'))$ is soft closed in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$

ii) soft generalized continuous (briefly soft g-continuous) **(Mahmood, S. I. , 2016)** if $f^{-1}((F,E'))$ is soft g-closed in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

iii) soft generalized semi continuous (briefly soft gs-continuous) **(Mahmood, S. I. , 2016)** if $f^{-1}((F,E'))$ is soft gs-closed set in $(X,\tilde{\tau},E)$ fore very soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

iv) soft generalized α-continuous (briefly soft gα-continuous) **(Mahmood, S. I. , 2016)** if $f^{-1}((F,E'))$ is soft gα-closed in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

v) soft α-generalized continuous (briefly soft αg-continuous) **(Mahmood, S. I. , 2016)** if $f^{-1}((F,E'))$ is soft αg-closed in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

vi) soft s*g-continuous **(Mahmood, S. I. , 2016)** if $f^{-1}((F,E'))$ is soft s*g-closed in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

vii) soft semi-generalized continuous (briefly soft sg-continuous) if $f^{-1}((F,E'))$ is soft sg-closed set in

$(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

viii) soft regular generalized continuous (briefly soft rg-continuous) if $f^{-1}((F,E'))$ is soft rg-closed set in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

ix) soft generalized b-continuous (briefly soft gb-continuous) if $f^{-1}((F,E'))$ is soft gb-closed set in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

x) soft regular w-continuous (briefly soft rw-continuous) if $f^{-1}((F,E'))$ is soft rw-closed set in $(X,\tilde{\tau},E)$ for every soft closed set (F,E') in $(Y,\tilde{\sigma},E')$.

Definition 1.10 (Kannan, K., 2012):

A soft topological space $(X,\tilde{\tau},E)$ is called soft $\tilde{T}_{1/2}$-space if every soft g-closed set in $(X,\tilde{\tau},E)$ is soft closed.

Proposition 1.11:

Let $((Y,E),\tilde{\tau}_Y,E)$ be a soft open subspace of a soft topological space $(X,\tilde{\tau},E)$. Then

i) If (A,E) is a soft b-open set in $(X,\tilde{\tau},E)$, then $(A,E)\tilde{\bigcap}(Y,E)$ is a soft b-open set in $((Y,E),\tilde{\tau}_Y,E)$.

ii) If (A,E) is a soft b-open set in $((Y,E),\tilde{\tau}_Y,E)$, then (A,E) is a soft b-open set in $(X,\tilde{\tau},E)$.

2. Basic Properties Of Soft Generalized Star b**-Closed Sets and Soft Generalized Star b**-Open Sets:

We introduce the following definition.

Definition 2.1:

A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is said to be soft generalized star b**-closed set (briefly soft g*b**-closed set) if $cl(A,E)\tilde{\subseteq}(U,E)$ whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft b-open in $(X,\tilde{\tau},E)$. The complement of a soft generalized star b**-closed set is said to be a soft generalized star b**-open set (briefly soft g*b**-open set). The class of all soft g*b**-open (resp. soft g*b**-closed) subsets of $(X,\tilde{\tau},E)$ is denoted by $SG*B**O(\tilde{X})$ (resp. $SG*B**C(\tilde{X})$).

Theorem 2.2:

Every soft regular closed set is a soft g*b**-closed set.

Proof:

Let (A,E) be any soft r-closed set and (U,E) be any soft b-open set in $\tilde{X}$ such that $(A,E)\tilde{\subseteq}(U,E)$. Since every soft r-closed set is soft closed, then $cl(A,E)\tilde{\subseteq}(U,E)$. Thus (A,E) is a soft g*b**-closed set.

Remark 2.3:

The converse of theorem (2.2) may not be true in general we see that in the following example:

Example 2.4:

Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F,E)\}$ be a soft topology over X, where $(F,E)=\{(e_1,F(e_1)),(e_2,F(e_2))\}=\{(e_1,\{a\}),(e_2,\{a\})\}$ $\Rightarrow$ $(G,E)=\{(e_1,\{b,c\}),(e_2,\{b,c\})\}$ is a soft g*b**-closed set in $\tilde{X}$, but is not soft regular closed in $\tilde{X}$, since $cl(int(G,E))=\tilde{\phi}\neq(G,E)$.

Theorem 2.5:

Every soft g*b**-closed set is soft g-closed, soft rg-closed, soft gs-closed and soft gb-closed set.

Proof:

Let (A,E) be any soft g*b**-closed set and (U,E) be any soft open set in $\tilde{X}$ such that $(A,E)\tilde{\subseteq}(U,E)$.

Since every soft open set is soft b-open, then $cl(A,E)\,\tilde{\subseteq}\,(U,E)$. Therefore (A,E) is soft g-closed. Since every soft g-closed set is soft rg-closed (resp. soft gs-closed, soft gb-closed) set. Thus every soft g*b**-closed set is soft g-closed, soft rg-closed, soft gs-closed and soft gb-closed set.

Remark 2.6:

The converse of theorem (2.5) may not be true in general we see that in the following example:

Example 2.7:

Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$ be a soft topology over X, where (F_1,E) $=\{(e_1,\{a\}),(e_2,\{a\})\}$ and $(F_1,E)=\{(e_1,\{b,c\}),(e_2,\{b,c\})\}\Rightarrow(G,E)=\{(e_1,\{b\}),(e_2,\{b\})\}$ is soft g-closed (resp. soft rg-closed, soft gs-closed, soft gb-closed) in $\tilde{X}$, but is not soft g*b**-closed in $\tilde{X}$, since $(G,E)\,\tilde{\subseteq}\,\{(e_1,\{a,b\},(e_2,\{a,b\})\}$ and $\{(e_1,\{a,b\},(e_2,\{a,b\})\}$ is soft b-open in $\tilde{X}$, but $cl(G,E)=$ $\{(e_1,\{b,c\}),(e_2,\{b,c\})\}\,\tilde{\not\subset}\,\{(e_1,\{a,b\},(e_2,\{a,b\})\}$.

Theorem 2.8:

Every soft g*b**-closed set is soft s*g-closed, soft rw-closed, soft αg-closed, soft gα-closed and soft sg-closed set.

Proof:

Let (A,E) be any soft g*b**-closed set and (U,E) be any soft semi-open set in $\tilde{X}$ such that $(A,E)\,\tilde{\subseteq}$ (U,E). Since every soft semi-open set is soft b-open, then $cl(A,E)\,\tilde{\subseteq}\,(U,E)$. Therefore (A,E) is soft s*g-closed. Since every soft s*g-closed set is soft rw-closed (resp. soft αg-closed, soft gα-closed, soft sg-closed) set. Thus every soft g*b**-closed set is soft s*g-closed, soft rw-closed, soft αg-closed, soft gα-closed and soft sg-closed set.

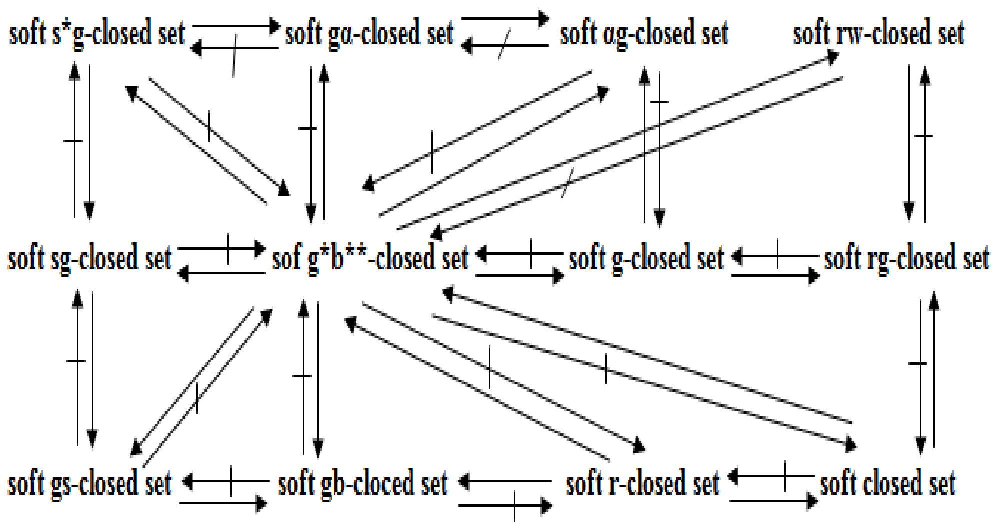
Remark 2.9:

The converse of theorem (2.8) may not be true in general we see that in the following example:

Example 2.10:

Let $X=\{a,b,c\}$, $E=\{e_1,e_2\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F,E)\}$ be a soft topology over X, where $(F,E)=\{(e_1,\{a,c\}),$ $(e_2,\{a,c\})\}\Rightarrow(G,E)=\{(e_1,\{b,c\}),(e_2,\{b,c\})\}$ is a soft s*g-closed (resp. soft rw-closed, soft gα-closed, soft αg-closed, soft sg-closed) in $\tilde{X}$, but is not soft g*b**-closed in $\tilde{X}$, since $(G,E)\,\tilde{\subseteq}\,(G,E)$ and (G,E) is soft b-open in $\tilde{X}$, but $cl(G,E)=\tilde{X}\,\tilde{\not\subset}\,(G,E)$.

The following diagram show the relationships between soft g*b**-closed sets and some other soft closed sets:



Definition 2.11:

Let (A,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then:

i) The soft g*b**-closure of (A,E), denoted by $g*b**cl(A,E)$ is the intersection of all soft g*b**-closed sets in $(X,\tilde{\tau},E)$ which contains (A,E).

ii) The soft g*b**-interior of (A,E), denoted by $g*b**int(A,E)$ is the union of all soft g*b**-open sets in $(X,\tilde{\tau},E)$ which are contained in (A,E).

Theorem 2.12:

Let (A,E) be a soft subset of a soft topological space $(X,\tilde{\tau},E)$. Then $\tilde{x}\tilde{\in} g*b**cl(A,E)$ if and only if for any soft g*b**-open set (U,E) containing $\tilde{x}$, $(A,E)\tilde{\bigcap}(U,E)\neq\tilde{\phi}$.

Proof:

$\Rightarrow$ Let $\tilde{x}\tilde{\in} g*b**cl(A,E)$ and suppose that, there is a soft g*b**-open set (U,E) in $\tilde{X}$ s.t $\tilde{x}\tilde{\in}(U,E)$ and $(A,E)\tilde{\bigcap}(U,E)=\tilde{\phi}\Rightarrow(A,E)\tilde{\subseteq}\tilde{X}-(U,E)$ which is soft g*b**-closed in $\tilde{X}\Rightarrow g*b**cl(A,E)\tilde{\subseteq}$ $g*b**cl(\tilde{X}-(U,E))=\tilde{X}-(U,E)$. But $\tilde{x}\tilde{\in}(U,E)\Rightarrow\tilde{x}\tilde{\notin}\tilde{X}-(U,E)\Rightarrow\tilde{x}\tilde{\notin} g*b**cl(A,E)$, this is a contradiction.

Conversely, Suppose that, for any soft g*b**-open set (U,E) containing $\tilde{x}$, $(A,E)\tilde{\bigcap}(U,E)\neq\tilde{\phi}$. To prove that $\tilde{x}\tilde{\in} g*b**cl(A,E)$. Suppose that $\tilde{x}\tilde{\notin} g*b**cl(A,E)$, then by definition (2.11) there is a soft g*b**-closed set (F,E) in $\tilde{X}$ such that $\tilde{x}\tilde{\notin}(F,E)$ and $(A,E)\tilde{\subseteq}(F,E)$. Since $\tilde{x}\tilde{\notin}(F,E)\Rightarrow\tilde{x}\tilde{\in}\tilde{X}-(F,E)$ which is soft g*b**-open in $\tilde{X}$. But $(A,E)\tilde{\subseteq}(F,E)\Rightarrow(A,E)\tilde{\bigcap}\tilde{X}-(F,E)=\tilde{\phi}$, this is a contradiction. Thus $\tilde{x}\tilde{\in} g*b**cl(A,E)$.

Theorem 2.13:

A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is a soft g*b**-closed set if and only if $cl(A,E)-(A,E)$ does not contain any non-null soft b-closed set in $\tilde{X}$.

Proof:

$\Rightarrow$ Let (F,E) be any soft b-closed subset of $\tilde{X}$ such that $(F,E)\tilde{\subseteq} cl(A,E)-(A,E)\Rightarrow(A,E)\tilde{\subseteq}$ $\tilde{X}-(F,E)$. Since (A,E) is a soft g*b**-closed set in $\tilde{X}$ and $\tilde{X}-(F,E)$ is soft b-open $\Rightarrow cl(A,E)\tilde{\subseteq}\tilde{X}-(F,E)$ $\Rightarrow(F,E)\tilde{\subseteq}\tilde{X}-cl(A,E)$. So $(F,E)\tilde{\subseteq}\tilde{X}-cl(A,E)\bigcap cl(A,E)=\tilde{\phi}\Rightarrow(F,E)=\tilde{\phi}$.

Conversely, suppose that (A,E) is a soft subset of $\tilde{X}$ such that $cl(A,E)-(A,E)$ does not contain any non-null soft b-closed set. Let (U,E) be a soft b-open subset of $\tilde{X}$ such that $(A,E)\tilde{\subseteq}(U,E)$. If $cl(A,E)\tilde{\not\subset}(U,E)\Rightarrow$ $cl(A,E)\tilde{\bigcap}\tilde{X}-(U,E)$ is a non-null soft b-closed subset of $cl(A,E)-(A,E)$. This is a contradiction. Thus (A,E) is a soft g*b**-closed set.

Corollary 2.14:

Let (A,E) be a soft g*b**-closed set in $\tilde{X}$. Then (A,E) is soft closed in $\tilde{X}$ if and only if $cl(A,E)-(A,E)$ is soft b-closed in $\tilde{X}$.

Proof:

$\Rightarrow$ Let (A,E) be a soft g*b**-closed set in $\tilde{X}$. If (A,E) is soft closed in $\tilde{X}\Rightarrow cl(A,E)=(A,E)\Rightarrow$ $cl(A,E)-(A,E)=\tilde{\phi}$, which is soft b-closed in $\tilde{X}$. **Conversely,** let $cl(A,E)-(A,E)$ be a soft b-closed set in $\tilde{X}$. Then by theorem (2.13) $cl(A,E)-(A,E)$ does not contain any non-null soft b-closed set in $\tilde{X}$. Since $cl(A,E)-(A,E)\tilde{\subseteq} cl(A,E)-(A,E)\Rightarrow cl(A,E)-(A,E)=\tilde{\phi}$. Hence (A,E) is soft closed.

Theorem 2.15:

Let $(B,E)\tilde{\subseteq}(A,E)\tilde{\subseteq}\tilde{X}$ where (A,E) is a soft g*b**-closed and soft open set in $\tilde{X}$. Then (B,E) is soft g*b**-closed relative to (A,E) iff (B,E) is soft g*b**-closed in $\tilde{X}$.

Proof:

We first note that, since $(B,E)\tilde{\subseteq}(A,E)$ and (A,E) is both a soft g*b**-closed and soft open set in $\tilde{X}$, then $cl(A,E)\tilde{\subseteq}(A,E)$ and thus $cl(B,E)\tilde{\subseteq}cl(A,E)\tilde{\subseteq}(A,E)$. Since $cl_A(B,E)=(A,E)\cap cl(B,E) \Rightarrow cl_A(B,E)=cl(B,E)$. If (B,E) is soft g*b**-closed relative to (A,E) and (U,E) is soft b-open subset of $\tilde{X}$ such that $(B,E)\tilde{\subseteq}(U,E) \Rightarrow (B,E)=(B,E)\tilde{\cap}(A,E)\tilde{\subseteq}(U,E)\tilde{\cap}(A,E)$. By proposition (1.11) $(U,E)\tilde{\cap}(A,E)$ is soft b-open in (A,E). Since (B,E) is soft g*b**-closed relative to $(A,E) \Rightarrow cl(B,E)=cl_A(B,E)\tilde{\subseteq}(U,E)\tilde{\cap}(A,E)\tilde{\subseteq}(U,E)$. Therefore (B,E) is a soft g*b**-closed set in $\tilde{X}$.

Conversely, let (B,E) be a soft g*b**-closed set in $\tilde{X}$ and (U,E) be a soft b-open set in (A,E) such that $(B,E)\tilde{\subseteq}(U,E)$. Since (A,E) is soft open in $\tilde{X}$, then by proposition (1.11) (U,E) is a soft b-open set in $\tilde{X}$. Since (B,E) is soft g*b**-closed in $\tilde{X} \Rightarrow cl(B,E)\tilde{\subseteq}(U,E)$. Thus $cl_A(B,E)=cl(B,E)\tilde{\cap}(A,E)\tilde{\subseteq}(U,E)\tilde{\cap}(A,E)\tilde{\subseteq}(U,E)$. Therefore (B,E) is a soft g*b**-closed set relative to (A,E).

Theorem 2.16:

If (A,E) and (B,E) are soft g*b**-closed sets in $\tilde{X}$. Then $(A,E)\tilde{\cup}(B,E)$ is also a soft g*b**-closed set in $\tilde{X}$.

Proof:

Suppose that (A,E) and (B,E) are g*b**-closed sets in $\tilde{X}$. To prove that $(A,E)\tilde{\cup}(B,E)$ is soft g*b**-closed. Let (O,E) be a soft b-open set in $\tilde{X}$ such that $(A,E)\tilde{\cup}(B,E)\tilde{\subseteq}(O,E) \Rightarrow (A,E)\tilde{\subseteq}(O,E)$ & $(B,E)\tilde{\subseteq}(O,E)$. Since (A,E) and (B,E) are soft g*b**-closed sets $\Rightarrow cl(A,E)\tilde{\subseteq}(O,E)$ & $cl(B,E)\tilde{\subseteq}(O,E)$. Hence $cl((A,E)\tilde{\cup}(B,E))=cl(A,E)\tilde{\cup}cl(B,E)\tilde{\subseteq}(O,E)$. That is $cl((A,E)\tilde{\cup}(B,E))\tilde{\subseteq}(O,E)$. Therefore $(A,E)\tilde{\cup}(B,E)$ is a soft g*b**-closed set in $\tilde{X}$.

Theorem 2.17:

If (A,E) is a soft g*b**-closed set in $\tilde{X}$ and $(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}cl(A,E)$, then (B,E) is also a soft g*b**-closed set in $\tilde{X}$.

Proof:

Let (O,E) be a soft b-open set in $\tilde{X}$ such that $(B,E)\tilde{\subseteq}(O,E) \Rightarrow (A,E)\tilde{\subseteq}(O,E)$. Since (A,E) is soft g*b**-closed, then $cl(A,E)\tilde{\subseteq}(O,E)$. Since $(B,E)\tilde{\subseteq}cl(A,E) \Rightarrow cl(B,E)\tilde{\subseteq}cl(cl(A,E))=cl(A,E)\tilde{\subseteq}(O,E)$. Therefore (B,E) is also a soft g*b**-closed set in $\tilde{X}$.

Theorem 2.18:

A soft subset (A,E) of a soft topological space (X,τ,E) is soft g*b**-open iff $(F,E)\tilde{\subseteq}int(A,E)$ whenever (F,E) is a soft b-closed subset of $\tilde{X}$ and $(F,E)\tilde{\subseteq}(A,E)$.

Proof:

$\Rightarrow$ Suppose that (A,E) is soft g*b**-open and $(F,E)\tilde{\subseteq}(A,E)$, where (F,E) is soft b-cloced $\Rightarrow \tilde{X}-(A,E)\tilde{\subseteq}\tilde{X}-(F,E)$. Since $\tilde{X}-(F,E)$ is soft b-open and $\tilde{X}-(A,E)$ is soft g*b**-closed $\Rightarrow cl(\tilde{X}-(A,E))\tilde{\subseteq}\tilde{X}-(F,E)$. Since $cl(\tilde{X}-(A,E))=\tilde{X}-int(A,E) \Rightarrow \tilde{X}-int(A,E)\tilde{\subseteq}\tilde{X}-(F,E)$. Therefore $(F,E)\tilde{\subseteq}int(A,E)$. **Conversely,** suppose that $(F,E)\tilde{\subseteq}int(A,E)$ whenever (F,E) is soft b-closed and $(F,E)\tilde{\subseteq}(A,E)$.

To prove that (A,E) is soft g*b**-open. Let $\tilde{X}-(A,E)\tilde{\subseteq}(U,E)$, where (U,E) is soft b-open in $\tilde{X}$ $\Rightarrow$ $\tilde{X}-(U,E)\tilde{\subseteq}(A,E)$. Since $\tilde{X}-(U,E)$ is soft b-closed $\Rightarrow$ $\tilde{X}-(U,E)\tilde{\subseteq}\text{int}(A,E)$. Hence $\tilde{X}-\text{int}(A,E)\tilde{\subseteq}$ (U,E). Therefore $\text{cl}(\tilde{X}-(A,E))\tilde{\subseteq}(U,E)$. Thus $\tilde{X}-(A,E)$ is a soft g*b**-closed set .i.e. (A,E) is a soft g*b**-open set in $\tilde{X}$.

Theorem 2.19:

If (A,E) and (B,E) are soft g*b**-open sets in $\tilde{X}$. Then $(A,E)\tilde{\bigcap}(B,E)$ is also a soft g*b**-open set.

Proof:

The proof follows immediately from theorem (2.16) by showing that $\tilde{X}-((A,E)\tilde{\bigcap}(B,E))$ is soft g*b**- closed.

Theorem 2.20:

If (A,E) and (B,E) are soft separated soft g*b**-open sets in $\tilde{X}$. Then $(A,E)\tilde{\bigcup}(B,E)$ is also a soft g*b**-open set in $\tilde{X}$.

Proof:

$\Rightarrow$ Let (F,E) be any soft b-closed subset of $(A,E)\tilde{\bigcup}(B,E)$. Then $(F,E)\tilde{\bigcap}\text{cl}(A,F)\tilde{\subseteq}(A,E)$. By [1] $(F,E)\tilde{\bigcap}\text{cl}(A,F)$ is soft b-closed and hence by theorem (2.18) $(F,E)\tilde{\bigcap}\text{cl}(A,E)\tilde{\subseteq}\text{int}(A,E)$. Similarly, $(F,E)\tilde{\bigcap}\text{cl}(B,E)\tilde{\subseteq}\text{int}(B,E)$ .Now, $(F,E)=(F,E)\tilde{\bigcap}((A,E)\tilde{\bigcup}(B,E))\tilde{\subseteq}((F,E)\bigcap\text{cl}(A,E))\tilde{\bigcup}((F,E)\tilde{\bigcap}$ $\text{cl}(B,E))\tilde{\subseteq}\text{int}(A,E)\tilde{\bigcup}\text{int}(B,E)\tilde{\subseteq}\text{int}((A,E)\tilde{\bigcup}(B,E))$. Hence $(F,E)\tilde{\subseteq}\text{int}((A,E)\tilde{\bigcup}(B,E))$ and by theorem (2.18) $(A,E)\tilde{\bigcup}(B,E)$ is soft g*b**-open.

Corollary 2.21:

Let (A,E) and (B,E) be soft g*b**-closed sets and suppose that $\tilde{X}-(A,E)$ and $\tilde{X}-(B,E)$ are soft separated. Then $(A,E)\tilde{\bigcap}(B,E)$ is soft g*b**-closed.

Proof:

The proof follows immediately from theorem (2.20) by showing that $\tilde{X}-((A,E)\tilde{\bigcap}(B,E))$ is soft g*b**-open.

Theorem 2.22:

If (A,E) is a soft g*b**-open set in $\tilde{X}$ and $\text{int}(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(A,E)$, then (B,E) is also a soft g*b**-open set in $\tilde{X}$.

Proof:

Since $\text{int}(A,E)\tilde{\subseteq}(B,E)\tilde{\subseteq}(A,E)$ $\Rightarrow$ $\tilde{X}-(A,E)\tilde{\subseteq}\tilde{X}-(B,E)\tilde{\subseteq}\tilde{X}-\text{int}(A,E)=\text{cl}(\tilde{X}-(A,E))$ and $\tilde{X}-(A,E)$ is soft g*b**-closed, then by theorem (2.17) $\tilde{X}-(B,E)$ is soft g*b**-closed. Thus (B,E) is soft g*b**-open .

Theorem 2.23:

A soft subset (A,E) of a soft topological space $(X,\tilde{\tau},E)$ is soft g*b**-closed if and only if $\text{cl}(A,E)-(A,E)$ is soft g*b**-open .

Proof:

$\Rightarrow$ suppose that (A,E) is soft g*b**-closed in $\tilde{X}$. To prove that $\text{cl}(A,E)-(A,E)$ is soft g*b**-open. Let $(F,E)\tilde{\subseteq}\text{cl}(A,E)-(A,E)$ where (F,E) is soft b-closed in $\tilde{X}$, then by theorem (2.13) $(F,E)=\tilde{\phi}$ and hence $(F,E)\tilde{\subseteq}$ $\text{int}(\text{cl}(A,E)-(A,E))$. Therefore by theorem (2.18) $\text{cl}(A,E)-(A,E)$ is soft g*b**-open.

Conversely, suppose that $cl(A,E)-(A,E)$ is soft g*b**-open and $(A,E)\tilde{\subseteq}(O,E)$, where (O,E) is a soft b-open set in $\tilde{X}$. Now, $cl(A,E)\tilde{\bigcap}(\tilde{X}-(O,E))\tilde{\subseteq}cl(A,E)\tilde{\bigcap}(\tilde{X}-(A,E))=cl(A,E)-(A,E)$. Since $cl(A,E)\tilde{\bigcap}(\tilde{X}-(O,E))$ is soft b-closed and $cl(A,E)-(A,E)$ is soft g*b**-open, then $cl(A,E)\tilde{\bigcap}(\tilde{X}-(O,E))\tilde{\subseteq}$ $int(cl(A,E)-(A,E))=\tilde{\phi}$. Therefore $cl(A,E)\tilde{\bigcap}(\tilde{X}-(O,E))=\tilde{\phi}\Rightarrow cl(A,E)\tilde{\subseteq}(O,E)$. Thus (A,E) is soft g*b**-closed.

*3. Applications Of Soft Generalized Star b**-Closed Sets:*

In this section, we introduce soft $g*b**-\tilde{T}_{1/2}$-spaces and soft $\tilde{T}_{g*b**}$-spaces as an application of soft g*b**-closed sets in soft topological spaces and study some of their properties.

Definition 3.1:
A soft topological space $(X,\tilde{\tau},E)$ is said to be a soft $g*b**-\tilde{T}_{1/2}$-space if every soft g*b**-closed set in $\tilde{X}$ is soft closed.

Theorem 3.2:
A soft topological space $(X,\tilde{\tau},E)$ is soft $g*b**-\tilde{T}_{1/2}$-space iff every soft singleton soft subset of $\tilde{X}$ is either soft open or soft b-closed.

Proof:
$\Rightarrow$ Suppose that $(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$-space .To prove that every soft singleton soft subset of $\tilde{X}$ is either soft open or soft b-closed. Let $\tilde{x}\tilde{\in}\tilde{X}$ and suppose that $\{\tilde{x}\}$ is not soft b-closed in $\tilde{X}\Rightarrow\tilde{X}-\{\tilde{x}\}$ is not soft b-open in $\tilde{X}$. So $\tilde{X}$ is the only soft b-open set of $\tilde{X}$ containing $\tilde{X}-\{\tilde{x}\}$, therefore $cl(\tilde{X}-\{\tilde{x}\})\tilde{\subseteq}\tilde{X}\Rightarrow\tilde{X}-\{\tilde{x}\}$ is soft g*b**-closed. By assumption $\tilde{X}-\{\tilde{x}\}$ is a soft closed set in $\tilde{X}\Rightarrow\{\tilde{x}\}$ is a soft open set in $\tilde{X}$.

Conversely, let (A,E) be a soft g*b**-closed set of $\tilde{X}$. To prove that $(A,E)=cl(A,E)$. Clearly $(A,E)\tilde{\subseteq}cl(A,E)$. Now, let $\tilde{x}\tilde{\in}cl(A,E)$, by assumption $\{\tilde{x}\}$ is either soft open or soft b-closed.

Case(i): If $\{\tilde{x}\}$ is soft open $\Rightarrow\{\tilde{x}\}\tilde{\bigcap}(A,E)\neq\tilde{\phi}\Rightarrow\tilde{x}\tilde{\in}(A,E)$.

Case(ii): If $\{\tilde{x}\}$ is soft b-closed and $\tilde{x}\tilde{\notin}(A,E)\Rightarrow cl(A,E)-(A,E)$ contains a non-null soft b-closed set $\{\tilde{x}\}$. By theorem (2.13) this is a contradiction since (A,E) is soft g*b**-closed. Hence $\tilde{x}\tilde{\in}(A,E)$ and (A,E) is soft closed. Therefore $(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$-space.

Theorem 3.3:
Every soft $\tilde{T}_{1/2}$-space is a soft $g*b**-\tilde{T}_{1/2}$-space.

Proof:
Let $(X,\tilde{\tau},E)$ be a soft $\tilde{T}_{1/2}$-space and (F,E) be any soft g*b**-closed set in $\tilde{X}$, then by theorem (2.5) (F,E) is a soft g-closed set in $\tilde{X}$. Since $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_{1/2}$-space, then by definition (1.10), (F,E) is soft closed. Thus $(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$-space.

Remark 3.4:
The converse of theorem (3.3) may not be true in general we see that in the following example:

Example 3.5:
Let $X=\{a,b\}$, $E=\{e\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi}\}$ be a soft topology over X. Since $SG*B**C(\tilde{X})=\{\tilde{X},\tilde{\phi}\}=$ soft closed sets in $\tilde{X}\Rightarrow$ $(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$-space. But $(X,\tilde{\tau},E)$ is not a soft $\tilde{T}_{1/2}$-space, since $SGC(\tilde{X})=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}\neq$ closed sets in $\tilde{X}$, where $(F_1,E)=\{(e,\{a\})\}$ and $(F_2,E)=\{(e,\{b\})\}$.

Definition 3.6:

A soft topological space $(X,\tilde{\tau},E)$ is said to be soft $\tilde{T}_b$ -space if every soft gs-closed set in $(X,\tilde{\tau},E)$ is soft closed.

Theorem 3.7:

Every soft $\tilde{T}_b$ -space is a soft $g*b**-\tilde{T}_{1/2}$ -space.

Proof:

Let $(X,\tilde{\tau},E)$ be a soft $\tilde{T}_b$ -space and (F,E) be any soft g*b**-closed set in $\tilde{X}$, then by theorem (2.5) (F,E) is a soft gs-closed set in $\tilde{X}$. Since $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_b$ -space, then (F,E) is soft closed. Thus $(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$ -space.

Remark 3.8:

The converse of theorem (3.7) may not be true in general we see that in the following example.

Example 3.9:

Let $X=\{a,b,c\}$, $E=\{e\}$ and $\tilde{\tau}=\{\tilde{X},\tilde{\phi},(F,E)\}$ be a soft topology over X, where $(F,E)=\{(e,\{a\})\}$. Since $SG*B**C(\tilde{X})=\{\tilde{X},\tilde{\phi},(G,E)\}=$ soft closed sets in $\tilde{X}$, where $(G,E)=\{(e,\{b,c\})\}\Rightarrow(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$ -space. But $(X,\tilde{\tau},E)$ is not a soft $\tilde{T}_b$ -space, since $SGSC(\tilde{X})=\{\tilde{X},\tilde{\phi},(G,E),(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}\neq$ soft closed sets in $\tilde{X}$, where $(F_1,E)=\{(e,\{a,b\})\}$, $(F_2,E)=\{(e,\{a,c\})\}$, $(F_3,E)=\{(e,\{b\})\}$ and $(F_4,E)=\{(e,\{c\})\}$

Definition 3.10:

A soft topological space $(X,\tilde{\tau},E)$ is said to be a soft $\tilde{T}_{g*b**}$ -space if every soft gs-closed set in $\tilde{X}$ is soft g*b**-closed.

Now we obtain a new characterization of soft $\tilde{T}_b$ -space.

Theorem 3.11:

A soft topological space $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_b$ -space iff it is soft $\tilde{T}_{g*b**}$ -space and soft $g*b**-\tilde{T}_{1/2}$ -space.

Proof:

$\Rightarrow$ If $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_b$ -space, then by theorem (3.7), $(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$ -space. To prove that $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_{g*b**}$ -space. Let (F,E) be any soft gs-closed set in $\tilde{X}$. Since $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_b$ -space, then (F,E) is soft closed. Hence (F,E) is soft g*b**-closed. Thus $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_{g*b**}$ - space.

Conversely, suppose that $(X,\tilde{\tau},E)$ is both soft $\tilde{T}_{g*b**}$ -space and soft $g*b**-\tilde{T}_{1/2}$ -space. To prove that $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_b$ -space. Let (F,E) be any soft gs-closed set in $\tilde{X}$. Since $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_{g*b**}$ -space, then (F,E) is a soft g*b**-closed set in $\tilde{X}$. Since $(X,\tilde{\tau},E)$ is a soft $g*b**-\tilde{T}_{1/2}$ -space, then (F,E) is soft closed. Thus $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_b$ -space.

Theorem 3.12:

If a soft topological space $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_{g*b**}$ -space, then every soft singleton soft subset of $\tilde{X}$ is either soft g*b**-open or soft semi-closed.

Proof:

$\Rightarrow$ Suppose that $(X, \tilde{\tau}, E)$ is a soft $\tilde{T}_{g*b**}$-space. To prove that every soft singleton soft subset of $\tilde{X}$ is either soft g*b**-open or soft semi-closed. Let $\tilde{x} \tilde{\in} \tilde{X}$ and suppose that $\{\tilde{x}\}$ is not a soft semi-closed set in $\tilde{X} \Rightarrow \tilde{X} - \{\tilde{x}\}$ is not a soft semi-open set in $\tilde{X}$. So $\tilde{X}$ is the only soft semi-open set of $\tilde{X}$ containg $\tilde{X} - \{\tilde{x}\}$, therefore $scl(\tilde{X} - \{\tilde{x}\}) \tilde{\subseteq} \tilde{X} \Rightarrow \tilde{X} - \{\tilde{x}\}$ is soft sg-closed. Hence $\tilde{X} - \{\tilde{x}\}$ is a soft gs-closed set in $\tilde{X}$. By assumption $\tilde{X} - \{\tilde{x}\}$ is a soft g*b**-closed set in $\tilde{X} \Rightarrow \{\tilde{x}\}$ is a soft g*b**-open set in $\tilde{X}$.

Remark 3.13:
The converse of theorem (3.12) may not be true in general. The space in example (3.9) is not a soft $\tilde{T}_{g*b**}$-space, but every soft singleton soft subset of $\tilde{X}$ is either soft g*b**-open or soft semi-closed.

*4. Soft Generalized Star b**-Continuous Functions and Soft Generalized Star b**-Irresolute Functions:*

In this section, we introduce soft g*b**-continuous functions and soft g*b**-irresolute functions in soft topological spaces and study some of their properties.

Definition 4.1:
A soft function $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E')$ from a soft topological space $(X, \tilde{\tau}, E)$ into a soft topological space $(Y, \tilde{\sigma}, E')$ is said to be soft generalized star b**-continuous (briefly soft g*b**-continuous) if $f^{-1}((V, E'))$ is a soft g*b**-closed set in $\tilde{X}$ for every soft closed set (V, E') in $\tilde{Y}$.

Theorem 4.2:
A soft function $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E')$ from a soft topological space $(X, \tilde{\tau}, E)$ into a soft topological space $(Y, \tilde{\sigma}, E')$ is soft g*b**-continuous iff $f^{-1}((V, E'))$ is a soft g*b**-open set in $\tilde{X}$ for every soft open set (V, E') in $\tilde{Y}$.

Proof:
It is Obvious.

Theorem 4.3:
Every soft continuous function is soft g*b**-continuous.

Proof:
Follows from the definition and the fact that every soft closed set is soft g*b**-closed.

Theorem 4.4:
Every soft g*b**-continuous function is soft g-continuous, soft rg-continuous, soft gs-continuous and soft gb-continuous.

Proof:
Follows from the theorem (2.5).

Remark 4.5:
The converse of theorem (4.4) may not be true in general we see that in the following example:

Example 4.6:
Let $X = Y = \{a, b, c\}$ and $E = \{e\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ is a soft topology over Y, where $(F_1, E) = \{(e, \{a\})\}$ and $(F_2, E) = \{(e, \{a, c\})\}$. Hence $SG*B**C(\tilde{X}) = \{\tilde{X}, \tilde{\phi}\}$ and $SGC(\tilde{X}) = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$, where $(F_3, E) = \{(e, \{b\})\}$, $(F_4, E) = \{(e, \{c\})\}$, $(F_5, E) = \{(e, \{a, b\})\}$, $(F_6, E) = \{(e, \{b, c\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f(\tilde{a}) = \tilde{a}$, $f(\tilde{b}) = \tilde{c}$ & $f(\tilde{c}) = \tilde{b} \Rightarrow$ f is not soft g*b**-continuous, Since (F_6, E) is soft closed in $(Y, \tilde{\sigma}, E)$, but $f^{-1}(\{(F_6, E))$

$=(F_6, E)$ is not soft g*b**-closed in $(X, \tilde{\tau}, E)$. But f is soft g-continuous (resp. soft rg-continuous, soft gs-continuous and soft gb-continuous).

Theorem 4.7:

Every soft g*b**-continuous function is soft s*g-continuous, soft rw-continuous, soft gα-continuous, soft αg-continuous and soft sg-continuous.

Proof:

Follows from the theorem (2.8).

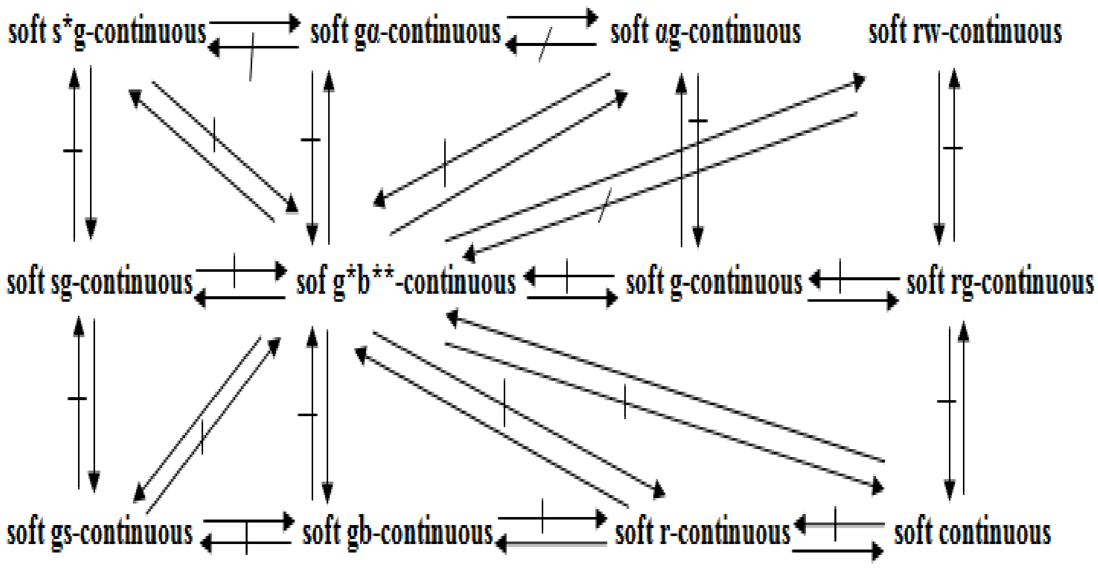
Remark 4.8:

The converse of theorem (4.8) may not be true in general we see that in the following example:

Example 4.9:

Let $X = Y = \{a, b, c\}$ and $E = \{e\}$. Then $\tilde{\tau} = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ is a soft topology over X and $\tilde{\sigma} = \{\tilde{Y}, \tilde{\phi}, (F_1, E), (G_1, E), (G_2, E)\}$ is a soft topology over Y, where $(F_1, E) = \{(e, \{a\})\}$, $(F_2, E) = \{(e, \{b, c\})\}$, $(G_1, E) = \{(e, \{b\})\}$ and $(G_2, E) = \{(e, \{a, b\})\}$. Hence $SG*B**(\tilde{X}) = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ and $SS*GC(\tilde{X}) = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (G_1, E), (G_2, E), (G_3, E), (G_4, E)\}$, where $(G_3, E) = \{(e, \{c\})\}$ and $(G_4, E) = \{(e, \{a, c\})\}$. Define $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E)$ by: $f(\tilde{a}) = \tilde{a}$, $f(\tilde{b}) = \tilde{b}$ & $f(\tilde{c}) = \tilde{c}$ $\Rightarrow$ f is not soft g*b**-continuous, Since (G_3, E) is soft closed in $(Y, \tilde{\sigma}, E)$, but $f^{-1}(\{(G_3, E)) = (G_3, E)$ is not soft g*b**-closed in $(X, \tilde{\tau}, E)$. But f is soft s*g-continuous (resp. soft rw-continuous, soft gα-continuous, soft αg-continuous and soft sg-continuous).

The following diagram show the relationships between soft g*b**-continuous functions and some other soft continuous functions:



Theorem 4.10:

Let $f : (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, E')$ be a soft function such that $(X, \tilde{\tau}, E)$ is a soft $\tilde{T}_{g*b**}$-space. Then:

i) Every soft gs-continuous function is soft g*b**-continuous.
ii) Every soft sg-continuous function is soft g*b**-continuous.
iii) Every soft g-continuous function is soft g*b**-continuous.
iv) Every soft αg-continuous function is soft g*b**-continuous.
v) Every soft s*g-continuous function is soft g*b**-continuous.

Proof:

i) Let $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ be a soft gs-continuous function. To prove that $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ is soft g*b**-continuous. Let (F,E') be any soft closed subset of $\tilde{Y}$. Since f is soft gs-continuous, then $f^{-1}((F,E'))$ is soft gs-closed in $\tilde{X}$. Since $(X,\tilde{\tau},E)$ is a soft $\tilde{T}_{g*b**}$-space, then $f^{-1}((F,E'))$ is a soft g*b**-closed set in $\tilde{X}$. Hence f is soft g*b**-continuous.

(ii),(iii),(iv) & (v) follows from (i) and the fact that every soft sg-continuous (resp. soft g-continuous, soft αg-continuous, soft s*g-continuous) function is soft gs-continuous.

Proposition 4.11:

If $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ is a soft g*b**-continuous function, then $f(g*b**cl(A,E))\,\tilde{\subseteq}\,cl(f((A,E)))$ for every soft subset (A,E) of $\tilde{X}$.

Proof:

Since $f((A,E))\,\tilde{\subseteq}\,cl(f((A,E)))\Rightarrow(A,E)\,\tilde{\subseteq}\,f^{-1}(cl(f((A,E))))$. Since $cl(f((A,E)))$ is a soft closed set in $\tilde{Y}$ and f is soft g*b**-continuous, then by definition (4.1) $f^{-1}(cl(f((A,E))))$ is a soft g*b**-closed set in $\tilde{X}$ containing (A,E). Hence $g*b**cl(A,E)\,\tilde{\subseteq}\,f^{-1}(cl(f((A,E))))$. Therefore $f(g*b**cl(A,E))\,\tilde{\subseteq}$ $cl(f((A,E)))$ for every soft subset (A,E) of $\tilde{X}$.

Proposition 4.12:

If $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ is a soft g*b**-continuous function, then $f^{-1}(int(B,E'))\,\tilde{\subseteq}$ g*b**-int($f^{-1}((B,E'))$) for every soft subset (B,E') of $(Y,\tilde{\sigma},E')$.

Proof:

$\Rightarrow$ Since $int(B,E')\,\tilde{\subseteq}\,(B,E')\Longrightarrow f^{-1}(int(B,E'))\,\tilde{\subseteq}\,f^{-1}((B,E'))$. Since $int(B,E')$ is a soft open set in $(Y,\tilde{\sigma},E')$ and f is soft g*b**-continuous, then by theorem (4.2), $f^{-1}(int(B,E'))$ is a soft g*b**-open set in $(X,\tilde{\tau},E)$ such that $f^{-1}(int(B,E'))\,\tilde{\subseteq}\,f^{-1}((B,E'))$. Therefore $f^{-1}(int(B,E'))\,\tilde{\subseteq}$ g*b** int($f^{-1}((B,E'))$) for every soft subset (B,E') of $(Y,\tilde{\sigma},E')$.

Theorem 4.13:

Let $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ be a soft function from a soft topological space $(X,\tilde{\tau},E)$ into a soft topological space $(Y,\tilde{\sigma},E')$. Then the following statements are equivalent:

i) For each soft point $\tilde{x}\,\tilde{\in}\,\tilde{X}$ and each soft open set (V,E') in $\tilde{Y}$ with $f(\tilde{x})\,\tilde{\in}\,(V,E')$, there is a soft g*b**- open set (U,E) in $\tilde{X}$ such that $\tilde{x}\,\tilde{\in}\,(U,E)$ and $f((U,E))\,\tilde{\subseteq}\,(V,E')$.

ii) For each soft subset (A,E) of $\tilde{X}$, $f(g*b**cl(A,E))\,\tilde{\subseteq}\,cl(f((A,E)))$.

iii) For each soft subset (B,E') of $\tilde{Y}$, $g*b**cl(f^{-1}((B,E')))\,\tilde{\subseteq}\,f^{-1}(cl(B,E'))$.

Proof:

$(i)\rightarrow(ii)$.

Suppose that (i) holds and let $\tilde{y}\,\tilde{\in}\,f(g*b**cl((A,E)))$ and let (V,E') be any soft open set of $\tilde{y}$ in $\tilde{Y}$. Since $\tilde{y}\,\tilde{\in}\,f(g*b**cl((A,E)))\Rightarrow\exists\;\tilde{x}\,\tilde{\in}\,g*b**cl(A,E)$ s.t $f(\tilde{x})=\tilde{y}$. Since $f(\tilde{x})\,\tilde{\in}\,(V,E')$, then by (i) $\exists$ a soft g*b**-open set (U,E) in $\tilde{X}$ s.t $\tilde{x}\,\tilde{\in}\,(U,E)$ and $f((U,E))\,\tilde{\subseteq}\,(V,E')$. Since $\tilde{x}\,\tilde{\in}\,g*b**cl(A,E)$, then by theorem (2.12), $(U,E)\,\tilde{\bigcap}\,(A,E)\neq\tilde{\phi}$ and hence $f((A,E))\,\tilde{\bigcap}\,(V,E')\neq\tilde{\phi}$. Therefore we have $\tilde{y}=f(\tilde{x})\,\tilde{\in}$ $cl(f((A,E)))$. Hence $f(g*b**cl(A,E))\,\tilde{\subseteq}\,cl(f((A,E)))$ for each soft subset (A,E) of $\tilde{X}$.

$(ii)\rightarrow(i)$

Suppose that (ii) holds and let $\tilde{x} \tilde{\in} \tilde{X}$ and (V, E') be any soft open set in $\tilde{Y}$ containing $f(\tilde{x})$. Let $(A, E) = f^{-1}(\tilde{Y} - (V, E')) \Rightarrow \tilde{x} \tilde{\notin} (A, E)$. Since $f(g*b**cl((A, E))) \tilde{\subseteq} cl(f((A, E))) \tilde{\subseteq} \tilde{Y} - (V, E') \Rightarrow g*b**cl(A, E) \tilde{\subseteq} f^{-1}(\tilde{Y} - (V, E')) = (A, E)$. Since $\tilde{x} \tilde{\notin} (A, E) \Rightarrow \tilde{x} \tilde{\notin} g*b**cl(A, E)$ and by theorem (2.12) there exists a soft g*b**-open set (U, E) containing $\tilde{x}$ such that $(U, E) \tilde{\bigcap} (A, E) = \tilde{\phi}$ and hence $f((U, E)) \tilde{\subseteq} f(\tilde{X} - (A, E))) \tilde{\subseteq} (V, E')$.

$(ii) \rightarrow (iii)$.

Suppose that (ii) holds and let (B, E') be any soft subset of $\tilde{Y}$. Replacing (A, E) by $f^{-1}((B, E'))$ we get from (ii) $f(g*b**cl(f^{-1}((B, E')))) \tilde{\subseteq} cl(f(f^{-1}((B, E')))) \tilde{\subseteq} cl(B, E')$. Hence $g*b**cl(f^{-1}(B, E')) \tilde{\subseteq} f^{-1}(cl(B, E'))$ for every soft subset (B, E') of $\tilde{Y}$.

$(iii) \rightarrow (ii)$.

Suppose that (iii) holds and let $(B, E') = f((A, E))$ where (A, E) is a soft subset of $\tilde{X}$. Then we get from (iii), $g*b**cl(A, E) \tilde{\subseteq} g*b**cl(f^{-1}(f((A, E)))) \tilde{\subseteq} f^{-1}(cl(f((A, E))))$. Therefore $f(g*b**cl(A, E)) \tilde{\subseteq} cl(f((A, E)))$ for every soft subset (A, E) of $\tilde{X}$.

Definition 4.14:

A soft function $f : (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E')$ from a soft topological space $(X, \tilde{\tau}, E)$ into a soft topological space $(Y, \tilde{\sigma}, E')$ is said to be soft generalized star b**- irresolute (briefly soft g*b**-irresolute) if $f^{-1}((V, E'))$ is soft g*b**-closed set in $\tilde{X}$ for every soft g*b**-closed set (V, E') in $\tilde{Y}$.

Theorem 4.15:

A soft function $f : (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E')$ from a soft topological space $(X, \tilde{\tau}, E)$ into a soft topological space $(Y, \tilde{\sigma}, E')$ is soft g*b**-irresolute iff $f^{-1}((V, E'))$ is soft g*b**-open set in $\tilde{X}$ for every soft g*b**-open set (V, E') in $\tilde{Y}$.

Proof:

It is Obvious.

Theorem 4.16:

Every soft g*b**-irresolute function is soft g*b**-continuous.

Proof:

Let $f : (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E')$ be a soft g*b**-irresolute function. To prove that $f : (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E')$ is soft g*b**-continuous. Let (F, E') be any soft closed subset of $\tilde{Y} \Rightarrow (F, E')$ is a soft g*b**-closed subset of $\tilde{Y}$. Since f is soft g*b**-irresolute $\Rightarrow f^{-1}((F, E'))$ is a soft g*b**-closed subset of $\tilde{X}$. Thus f is a soft g*b**-continuous function.

Theorem 4.17:

Let $f : (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E')$ be a soft g*b**-continuous function such that $(Y, \tilde{\sigma}, E')$ is a soft $g*b** - \tilde{T}_{1/2}$-space, then f is soft g*b**-irresolute.

Proof:

Let (F, E') be a soft g*b**-closed set in $\tilde{Y}$. Since $(Y, \tilde{\sigma}, E')$ is a soft $g*b** - \tilde{T}_{1/2}$-space, then (F, E') is soft closed in $\tilde{Y}$. Since f is soft g*b**-continuous, then $f^{-1}((F, E'))$ is soft g*b**-closed in $\tilde{X}$. Hence f is a soft g*b**-irresolute function.

Theorem 4.18:

If $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ and $g:(Y,\tilde{\sigma},E')\rightarrow(Z,\tilde{\eta},E'')$ are soft functions. Then:

i) If $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ and $g:(Y,\tilde{\sigma},E')\rightarrow(Z,\tilde{\eta},E'')$ are both soft g*b**-irresolute functions, then $g\circ f:(X,\tilde{\tau},E)\rightarrow(Z,\tilde{\eta},E'')$ is soft g*b**-irresolute.

ii) If $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ is soft g*b**-irresolute and $g:(Y,\tilde{\sigma},E')\rightarrow(Z,\tilde{\eta},E'')$ is soft g*b**-continuous, then $g\circ f:(X,\tilde{\tau},E)\rightarrow(Z,\tilde{\eta},E'')$ is soft g*b**-continuous.

iii) If $f:(X,\tilde{\tau},E)\rightarrow(Y,\tilde{\sigma},E')$ is soft g*b**-continuous and $g:(Y,\tilde{\sigma},E')\rightarrow(Z,\tilde{\eta},E'')$ is soft continuous, then $g\circ f:(X,\tilde{\tau},E)\rightarrow(Z,\tilde{\eta},E'')$ is soft g*b**-continuous.

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