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Mathematical Model on Influenza Disease with Re-Susceptibility

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ABSTRACT

Background: Influenza is a highly contagious respiratory disease, which has a property to re-susceptibility. Objective: A mathematical model used for the study of the stability on an epidemic disease. Methodology: In this model peoples are divided into the five compartments as susceptible, infective, treatment, recovered and re-suspected peoples. During the disease duration, if the number of the patients increases then the result of stability will be unstable due to losing immunity. Conclusion: The Routh Hurwitz Principle proves that the model SITRS is unstable in the human population.

INTRODUCTION

In the study of epidemics, susceptible-infected-recovered (SIR) model, which is developed by W. O. Kermack and A. G. Mckendrick *et al.*, (1927) play an important role in that field. SIR model explains the various stages of the epidemic diseases in the human population. According to Kermack. W. O and McKendrick. A. G., the susceptible-infected-recovered (SIR) model:

$$\frac{dS}{dt} = -\beta S\, I$$

$$\frac{dI}{dt} = \beta S\, I - \alpha\, I$$

$$\frac{dR}{dt} = \alpha\, I$$

Vinod Kumar Bais, Deepak Kumar and Pooja, *et al.*, (2016) provides that the study of basic reproduction number R_0 which explains the various stages of the epidemic disease and the concept of immunity to fight against epidemic disease. B. D. Greenbaum, S. Cocco, A. J. Levine and R. Monasson *et al.*, (2014) and G.G. Alcaraz and C.V.D. Leon *et al.*, (2012) explains an epidemic disease by mathematical method, where the basic reproduction number identified that the diseases will spread or not. If the value of $R_0 < 1$, then the infection dies out from the population and if the value of $R_0 > 1$, then the possibility of spreading an infection in human population is high. Furthermore investigate disease free equilibrium, steady state, the force of infection and the crude approximation etc. for the study of stability of the disease.

Influenza is very highly transmissible respiratory disease and its indicators are fever, sore throat, coughing, weakness, runny nose, chills, muscle pains, headache, nausea and vomiting. Influenza disease spreads through the air by coughs or sneezes, creating aerosols containing the virus. Influenza can also be spread by through contact with bird droppings or nasal secretions, or through contact with contaminated sides.

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Fig. 1: Seasonal Influenza

J. A. Belser *et al.*, (2012); A. Ramis *et al.*, (2014); J. A. Collera *et al.*, (2013) and M. Mohamadhasani and M. Haveshki *et al.*, (2011) describe the suitable temperature to spread the infection is in October and the infectious agent takes their high frequency in January, when the maximum number of cases exposed. In India, laboratory test for influenza and their respiratory infection are not widely available everywhere, except few metro cites in India.

A virus is an infectious agent that can replicate inside the host cell. A virus must replicate mRANs from their genomes to produces proteins and reproduces in numbers. It is notable that the patient becomes infected from the infection agent and after getting the disease the people with their weak immunity and then the number of patient increases with time.

Mathematical Model:

Due to the weak immunity from an epidemic disease, patient may become again suspect. Here, patient in the recovered compartment has temporary immunity to the infection, but may be returns to the susceptible compartment.

This work represents simultaneous differential equations which are formed for the susceptible to influenza (S), infected with influenza (I), Treatment for Influenza (T), completely recovered from influenza (R), Vinod *et al.*, (2016).

Susceptible to influenza (S), β is transmission rate of influenza and μ is losing immunity rate respectively,

$$\frac{dS}{dt} = -\beta SI + \mu R \tag{1}$$

Infected with influenza (I), γ rate of selection treatment, *d* death rate due to influenza.

$$\frac{dI}{dt} = \beta SI - \gamma I - dI \tag{2}$$

Treatment for Influenza (T), σ removal rate from influenza due to treatment,

$$\frac{dT}{dt} = \gamma I - \sigma T \tag{3}$$

Completely removal from influenza (R) and,

$$\frac{dR}{dt} = \sigma T - \mu R \tag{4}$$

After simplification the above mathematical model can be written as:

$$\left.\begin{aligned} \frac{dS}{dt} &= -\beta SI + \mu R \\ \frac{dI}{dt} &= \beta SI - \gamma I - dI \\ \frac{dT}{dt} &= \gamma I - \sigma T \\ \frac{dR}{dt} &= \sigma T - \mu R \end{aligned}\right\} \tag{A*}$$

The Jacobian of equation (A*) of the infectious disease is given by

$$J = \begin{pmatrix} -\beta\bar{I} & -\beta\bar{S} & 0 & \mu \\ \beta\bar{I} & \beta\bar{S} - \gamma - d & 0 & 0 \\ 0 & \gamma & -\sigma & 0 \\ 0 & 0 & \sigma & -\mu \end{pmatrix} \tag{5}$$

The characteristics equation for the system of the governing equation is given by $|J - \lambda I| = 0$

$$\begin{vmatrix} -\beta\bar{I} - \lambda & -\beta\bar{S} & 0 & \mu \\ \beta\bar{I} & \beta\bar{S} - \gamma - d - \lambda & 0 & 0 \\ 0 & \gamma & -\sigma - \lambda & 0 \\ 0 & 0 & \sigma & -\mu - \lambda \end{vmatrix} = 0$$

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \quad (6)$$

Table 1: Values of the constant terms

S.No.	Constant terms	Numerical Values
1	A	1
2	$B = \gamma + d + \sigma + \mu - \beta S + \beta I$	-512.273
3	$C = \beta\gamma I + \beta d I + \sigma\mu + (\mu + \sigma)(\gamma + d - \beta S + \beta I)$	-23.45587
4	$D = \beta I(\sigma + \mu)(\gamma + d) + \sigma\mu(\gamma + d - \beta S + \beta I)$	2.953473
5	$E = \sigma\mu\beta I(\gamma + d) - \sigma\mu\beta\gamma S$	-1.3970016

Now the Eigen values of the equation (6) are with the help of MATLAB as
Case 1: $\lambda = -0.17174908130392513692779661257655$, then the system is stable.
Case 2: $\lambda = 512.31877249882027044712962146013$, then the system is not stable.
Case 3: $\lambda = 0.062988291241827344899087576225323 + 0.10912949774678246487907769323446*i$
Case 4: $\lambda = 0.062988291241827344899087576225323 - 0.10912949774678246487907769323446*i$

All the parameters have been collected and analyzed from Indira Gandhi Medical College, Shimla (India).

Table 2: Parameter Estimation

Parameters	Values	
β transmission rate of influenza	1.38 per year	Vinod et al., 2016
γ infective is selected for treatment	0.75 per year	Vinod et al., 2016
d death rate due to influenza	0.20 per year	Vinod et al., 2016
α removal rate from the treatment	0.10 per year	Vinod et al., 2016
μ losing immunity	0.037 per year	Vinod et al., 2016
Initial Value of S_0	1	
Initial Value of I_0	0.01	
Initial Value of T_0	0.50	
Initial Value of R_0	0.20	

The basic reproduction number:

The governing equation (2) can be written as

$$I' > 0;$$
$$\beta S I - \gamma I - d I > 0;$$
$$(\beta S - \gamma - d) I > 0;$$
$$(\beta S - \gamma - d) > 0;$$
$$\beta S > (\gamma + d);$$
$$S > \frac{(\gamma + d)}{\beta};$$
$$\frac{\beta}{\gamma} S > \left(1 + \frac{d}{\gamma}\right);$$
$$R_0 > \left(1 + \frac{d}{\gamma}\right);$$
$$R_0 = 1.2667$$

$R_0 = 1.2667 > 1$, then it is the condition for the existence of an epidemic disease.

The calculated value of basic reproduction number R_0 is approximated 1.6 for the epidemic disease as Swine flu and 2-3 for Influenza, so there is need to give more attention of national health agencies to fight against epidemic disease in human population.

Case Study:

Suppose that the I_0 is the number of infective conformed from susceptibilities at time t = 0.
From equation (2)

$$\frac{dI}{dt} = \beta S\, I - \gamma\, I - d\, I$$

If $(\beta S - \gamma - d) < 0$ then $I' < 0$ and $S(t) \le S_0 \; \forall\, t$.

In this case, the infection "dies out" i.e. no epidemic can occur.

The force of Infection:

The force of infection of an epidemic disease in the human population, defined as

$$F = \beta I$$

Here, due to a large numbers of population, it consider a force of infection, that does not depend on the infective class but rather on their fraction with respect to the total population, such that

$$F = \frac{\beta}{N}$$

A Crude Approximation:

A crude approximation of the disease is to assume that the rate of transmission varies, and it is defined by the formula

$$\beta(t) = \beta_0 (1 + \alpha \, \cos 2\pi t)$$

Where $\beta_0 = transmission\ rate = 1.38$

$t = measured\ time = 1\ to\ 12\ (Months)$

$\alpha = an\ amplitude\ of\ seasonal\ variation\ (0 \le \alpha \le 1)$

Table 3:

S. No.	Time	Transmission rate
1	0	1.3938
2	1	1.3938
3	2	1.3938
4	3	1.3938
5	4	1.3938
6	5	1.3938
7	6	1.3938
8	7	1.3938
9	8	1.3938
10	9	1.3938
11	10	1.3938
12	11	1.3938
13	12	1.3938

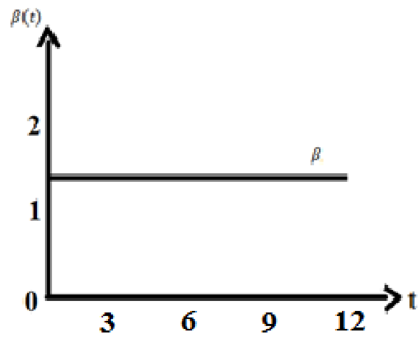

Fig. 2: Graph

The simple explanation of the transmission variation with respect to time is as a straight line that is the variation of transmission of infection is uniform.

Analysis of Stability for the model:

We have the equation from for a fourth-order polynomial from the equation (6)

$$A\,\lambda^4 + B\,\lambda^3 + C\,\lambda^2 + D\,\lambda + E = 0; \tag{7}$$

The Routh-Hurwitz principle for the equation (7) is given by the matrix

$$\begin{pmatrix} B & A & 0 & 0 \\ D & C & B & A \\ 0 & E & D & C \\ 0 & 0 & 0 & E \end{pmatrix}$$

The inequalities for the stabilities of the matrix are $A > 0$; $\Delta_1 = B > 0$; $\Delta_2 = BC - AD > 0$; $\Delta_3 = BCD - AD^2 - B^2E > 0$ and $\Delta_4 = E \neq 0$ (8)

Where $A = 1, B = \gamma + d + \sigma + \mu - \beta S + \beta I, C = \beta\gamma I + \beta dI + \sigma\mu + (\mu + \sigma)(\gamma + d - \beta S + \beta I)$
$D = \beta I(\sigma + \mu)(\gamma + d) + \sigma\mu(\gamma + d - \beta S + \beta I), E = \sigma\mu\beta I(\gamma + d) - \sigma\mu\beta\gamma S$

By calculating numerically, the results do not satisfy the inequalities (8). So, the given system is not a locally asymptotically stable.

Graphical Analysis for the disease:

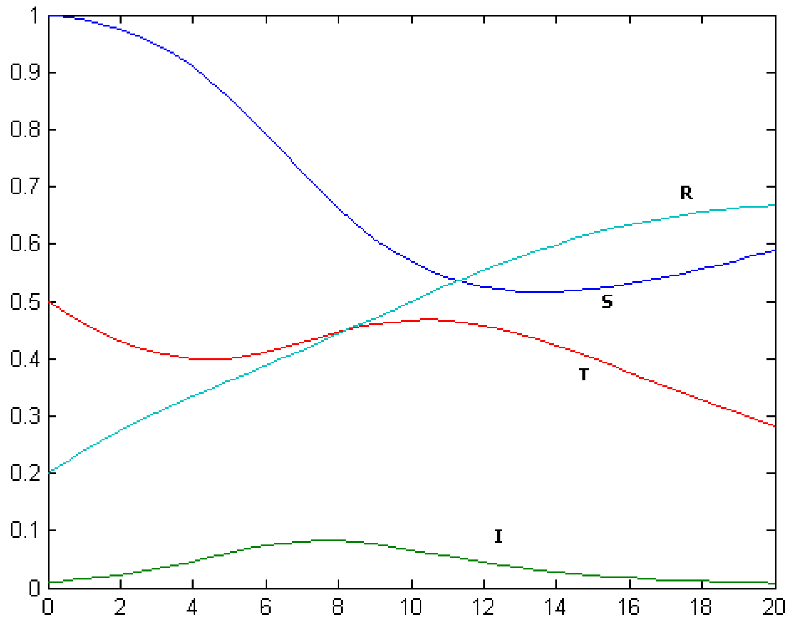

Fig. 3: Graph to represent the behavior of susceptible to influenza (S), infected with influenza (I), Treatment for Influenza (T), and completely recovered from influenza (R).

Graph represents that the susceptible rate (S) decrease with respect to time and removal (R) from the disease due to treatment. Graphically solutions prove that the variables are not an asymptotically stable. Simulation results are showing the trajectories and behavior of SITRS model.

RESULT AND DISCUSSION

1. The numbers of the new infection increases in the population because the basic reproduction number is positive and greater than one.
2. The force of infection is also increases in the population and given by $\propto \frac{\eta}{R_0}$, the number of suspected class will diseases with respect to time.
3. The numerical data is verified and analyzed by MATLAB Graph analysis.

Conclusion:

This work changes the traditional point of view of stability of a system with respect to epidemic disease modeling. This work touched mathematical challenges concerned in developing differential equations model. This mathematical model represents that the epidemic disease may be re-suspect from H1N1 Influenza virus, only in the case of temporary immunity. The Routh Hurwitz Principle proves that the given mathematical model SITRS is unstable in the human population.

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