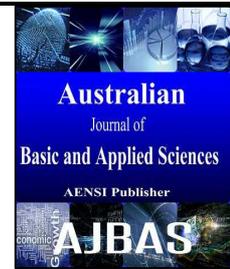




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On pre- α -Open Sets and Contra pre- α -continuous Functions and Contra pre- α -irresolute Functions in Topological Spaces

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ABSTRACT

In this paper, we introduce and study a new type of generalized open sets in topological spaces namely, pre- α -open sets and we show that the family of all pre- α -open sets in a topological space (X, τ) form a topology on X which is finer than τ . Also, we show that pre- α -open sets is stronger than each of semi-open sets, α -open sets, pre-open sets, b-open sets, β -open sets, generalized semi open sets, generalized α -open sets and α -generalized open sets and weaker than open sets. Moreover, we use these sets to define and study new classes of functions, namely, pre- α -continuous functions, contra pre- α -continuous functions, pre- α -irresolute functions and contra pre- α -irresolute functions in topological spaces and we discuss the relation between these types of functions and each of contra continuous functions and other weaker forms of contra continuous functions.

INTRODUCTION

Levine, N. (1963;1970) introduced and investigated generalized open sets and semi-open sets respectively. Abd El-Monsef, M. and et.al (1983), Andrijevic, D. (1996), Mashhour, A. and et.al. (1982) and Njastad, O. (1965) introduced β -open sets, b-open sets, pre-open sets and α -open sets respectively. Also, Maki, H. and et.al (1993;1994), Arya, S. and Nour, T. (1990) and Khan, M. and et.al (2008) introduced and studied α -generalized open sets, generalized α -open sets, generalized semi open sets and s*-g-open sets respectively. In this paper, we introduce and study a new class of open sets, namely, pre- α -open sets and we show that the family of all pre- α -open sets in a topological space (X, τ) form a topology on X which is finer than τ . This class of open sets is placed properly between the class of open sets and each of β -open sets, b-open sets, semi-open sets, pre-open sets, α -open sets, α -generalized open sets, generalized α -open sets and generalized semi open sets respectively. The characterizations and basic properties of pre- α -open sets and pre- α -closed sets have been studied. Moreover, we use these open sets to define and study new classes of functions, namely, pre- α -continuous functions, contra pre- α -continuous functions, pre- α -irresolute functions and contra pre- α -irresolute functions in topological spaces and we study the relation between these types of functions and each of contra continuous functions and other weaker forms of contra continuous functions.

1. Preliminaries:

First we recall the following definitions, theorems, propositions and lemmas.

Definition 1.1:

A subset A of a topological space (X, τ) is called:

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- i) A β -open set (Abd El-Monsef, M.E., 1983) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$.
- ii) A b-open set (Andrijevic, D., 1996) if $A \subseteq \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$.
- iii) A semi-open (briefly s-open) set (Levine, N., 1963) if $A \subseteq \text{cl}(\text{int}(A))$.
- iv) A pre-open set (Mashhour, A.S., 1982) if $A \subseteq \text{int}(\text{cl}(A))$.
- v) An α -open set (Njastad, O., 1965) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$.

The α -closure (resp. semi-closure) of a subset A of a topological space (X, τ) is the intersection of all α -closed (resp. semi-closed) sets which contains A and is denoted by $\alpha\text{cl}(A)$ (resp. $\text{scl}(A)$). Clearly $\text{scl}(A) \subseteq \alpha\text{cl}(A) \subseteq \text{cl}(A)$.

Definition 1.2:

A subset A of a topological space (X, τ) is said to be:

- i) A generalized closed (briefly g-closed) set (Levine, N., 1970) if $\text{cl}(A) \subseteq V$ whenever $A \subseteq V$ and V is open in X .
- ii) An α -generalized closed (briefly α g-closed) set (Maki, H., 1994) if $\alpha\text{cl}(A) \subseteq V$ whenever $A \subseteq V$ and V is open in X .
- iii) A generalized α -closed (briefly α g-closed) set (Maki, H., 1993) if $\alpha\text{cl}(A) \subseteq V$ whenever $A \subseteq V$ and V is α -open in X .
- iv) A generalized semi-closed (briefly gs-closed) set (Arya, S.P. and T.M. Nour, 1990) if $\text{scl}(A) \subseteq V$ whenever $A \subseteq V$ and V is open in X .
- v) An s^* g-closed set (Khan, M., 2008) if $\text{cl}(A) \subseteq V$ whenever $A \subseteq V$ and V is semi-open in X .

The complement of a g-closed (resp. α g-closed, α g-closed, gs-closed, s^* g-closed) set is called a g-open (resp. α g-open, gs-open, s^* g-open) set.

Definition 1.3:

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be:

- i) Contra continuous (Dontchev, J., 1996) if $f^{-1}(V)$ is closed set in X for every open set V in Y .
- ii) Contra semi-continuous (briefly contra s-continuous) (Dontchev, J. and T. Noiri, 1999) if $f^{-1}(V)$ is s-closed set in X for every open set V in Y .
- iii) Contra α -continuous (Jafari, S. and T. Noiri, 2001) if $f^{-1}(V)$ is α -closed set in X for every open set V in Y .
- iv) Contra pre-continuous (Jafari, S. and T. Noiri, 2002) if $f^{-1}(V)$ is pre-closed set in X for every open set V in Y .
- v) Contra b-continuous (Nasef, A.A., 2005) if $f^{-1}(V)$ is b-closed set in X for every open set V in Y .
- vi) Contra β -continuous (Caldas, M. and S. Jafari, 2001) if $f^{-1}(V)$ is β -closed set in X for every open set V in Y .
- vii) Contra generalized continuous (briefly Contra g-continuous) (Jafari, S., 2007) if $f^{-1}(V)$ is g-closed set in X for every open set V in Y .
- viii) Contra generalized semi continuous (briefly Contra gs-continuous) (Dontchev, J. and T. Noiri, 1999) if $f^{-1}(V)$ is gs-closed set in X for every open set V in Y .
- ix) Contra generalized α -continuous (briefly Contra α g-continuous) (Alli, K., 2013) if $f^{-1}(V)$ is α g-closed set in X for every open set V in Y .
- x) Contra α -generalized continuous (briefly Contra α g-continuous) (Jafari, S. and T. Noiri, 2001) if $f^{-1}(V)$ is α g-closed set in X for every open set V in Y .
- xi) Contra s^* g-continuous if $f^{-1}(V)$ is s^* g-closed set in X for every open set V in Y .

Definition 1.4 (Mrsevic, M., 1986):

Let A be a subset of a topological space (X, τ) . Then:

$\bigcap \{U \in \tau : A \subseteq U\}$ is called the kernel of A and is denoted by $\ker(A)$.

Lemma 1.5 (Jafari, S. and T. Noiri, 1999):

Let A and B be subsets of a topological space (X, τ) . Then:

- i) $x \in \ker(A)$ if and only if $A \cap F \neq \emptyset$ for any closed subset F of X containing x .
- ii) $A \subseteq \ker(A)$ and if A is open in X , then $A = \ker(A)$.
- iii) If $A \subseteq B$, then $\ker(A) \subseteq \ker(B)$.

Definition 1.6 (Mashhour, A., 1982):

Let (X, τ) be a topological space and $A \subseteq X$. Then:

- i) The pre-closure of A , denoted by $\text{pcl}(A)$ is the intersection of all pre-closed subsets of X which contains A .
- ii) The pre-interior of A , denoted by $\text{pint}(A)$ is the union of all pre-open subsets of X which are contained in A .

Proposition 1.7 (Jun, Y.B., 2008; Al-khazraji, R.B., 2004):

Let (X, τ) be a topological space and $A, B \subseteq X$. Then:

- i) $\text{int}(A) \subseteq \text{pint}(A) \subseteq A$.
- ii) $A \subseteq \text{pcl}(A) \subseteq \text{cl}(A)$.
- iii) A is pre-closed iff $\text{pcl}(A) = A$.
- iv) $\text{pcl}(\text{pcl}(A)) = \text{pcl}(A)$.
- v) $X - \text{pint}(A) = \text{pcl}(X - A)$.
- vi) If $A \subseteq B$, then $\text{pcl}(A) \subseteq \text{pcl}(B)$.
- vii) $\bigcup_{\alpha \in \Lambda} \text{pcl}(U_\alpha) \subseteq \text{pcl}(\bigcup_{\alpha \in \Lambda} U_\alpha)$.
- viii) $x \in \text{pcl}(A)$ iff for every pre-open set U containing x , $U \cap A \neq \emptyset$.
- ix) If V is an open set in X and A is a pre-open set in X , then $V \cap A$ is a pre-open set in X .

Theorem 1.8 (Navalagi, G. and Debadatta Roy Chaudhuri, 2009):

Let $X \times Y$ be the product space of topological spaces (X, τ) and (Y, σ) . If $A_1 \subseteq X$ and $A_2 \subseteq Y$.

Then $\text{pcl}(A_1) \times \text{pcl}(A_2) = \text{pcl}(A_1 \times A_2)$.

2. Properties Of pre- α -open Sets:

In this section we introduce a new type of sets, namely, pre- α -open sets and we show that the family of all pre- α -open subsets of a topological space (X, τ) form a topology on X which is finer than τ . Also, we study the relation between this type of open sets and each of open sets and other weaker forms of open sets.

Definition 2.1:

A subset A of a topological space (X, τ) is called a pre- α -open set if $A \subseteq \text{int}(\text{pcl}(\text{int}(A)))$. The complement of a pre- α -open set is defined to be pre- α -closed. The family of all pre- α -open subsets of X is denoted by $\tau^{\text{pre-}\alpha}$.

Clearly, every open set is a pre- α -open set, but the converse is not true as shown by the following example.

Example 2.2:

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}\}$ be a topology on X . Then $\{a, b\}$ is a pre- α -open set in X , since $\{a, b\} \subseteq \text{int}(\text{pcl}(\text{int}(\{a, b\}))) = \text{int}(\text{pcl}(\{a\})) = \text{int}(X) = X$. But $\{a, b\}$ is not open in X .

Theorem 2.3:

Every pre- α -open set is α -open (resp. $g\alpha$ -open, αg -open, β -open, pre-open, b-open) set.

Proof:

Let A be any pre- α -open set in X , then $A \subseteq \text{int}(\text{pcl}(\text{int}(A)))$. Since $\text{int}(\text{pcl}(\text{int}(A))) \subseteq \text{int}(\text{cl}(\text{int}(A)))$, thus $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, therefore A is an α -open set in X . Since every α -open set is $g\alpha$ -open (resp. αg -open, β -open, pre-open, b-open) set. Thus every pre- α -open set is α -open (resp. $g\alpha$ -open, αg -open, β -open, pre-open, b-open) set.

Remark 2.4:

The converse of theorem (2.3) may not be true in general. Consider the following example.

Example 2.5:

Let $X = \{a, b\}$ & $\tau = \{\emptyset, X\}$ be a topology on X . Then $\{a\}$ is a pre-open (resp. $g\alpha$ -open, αg -open, β -open, b-open) set in X , but is not pre- α -open set in X , since $\{a\} \not\subseteq \text{int}(\text{pcl}(\text{int}(\{a\}))) = \text{int}(\text{pcl}(\emptyset)) = \emptyset$.

Remark 2.6:

pre- α -open sets and s^*g -open sets are in general independent as shown by the following examples:

Example 2.7:

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X\}$ be a topology on X . Then $\{b\}$ is an s^*g -open set in X , but is not pre- α -open, since $\{b\} \not\subseteq \text{int}(\text{pcl}(\text{int}(\{b\}))) = \text{int}(\text{pcl}(\emptyset)) = \emptyset$. Also, in example (2.2) $\{a, b\}$ is a pre- α -open set in X , but is not s^*g -open, since $\{a, b\}^c = \{c\}$ is not s^*g -closed set in X , since $\{a, c\}$ is a semi-open set in X and $\{c\} \subseteq \{a, c\}$, but $\text{cl}(\{c\}) = \{b, c\} \not\subseteq \{a, c\}$.

Theorem 2.8:

Every pre- α -open set is semi-open and gs -open set.

Proof:

Let A be any pre- α -open set in X , then $A \subseteq \text{int}(\text{pcl}(\text{int}(A)))$. Since $\text{int}(\text{pcl}(\text{int}(A))) \subseteq \text{pcl}(\text{int}(A)) \subseteq \text{cl}(\text{int}(A))$, thus $A \subseteq \text{cl}(\text{int}(A))$, therefore A is a semi-open set in X . Since every semi-open set is gs -open. Thus every pre- α -open set is semi-open and gs -open set.

Remark 2.9:

The converse of theorem (2.8) may not be true in general. Consider the following example.

Example 2.10:

Let $X = \{a, b, c\}$ & $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ be a topology on X . Then the set $\{a, c\}$ is semi-open and gs -open set in X , but is not pre- α -open set in X , since $\{a, c\} \not\subseteq \text{int}(\text{pcl}(\text{int}(\{a, c\}))) = \text{int}(\text{pcl}(\{a\})) = \text{int}(\{a, c\}) = \{a\}$.

Remark 2.11:

αg -open sets and pre-open sets are in general independent as shown by the following examples:

Example 2.12:

Let (\mathbb{R}, μ) be the usual topological space. Then the set of rational numbers Q is a pre-open set, but is not αg -open. Also, in example (2.2) $\{b\}$ is an αg -open set, since $\{b\}^c = \{a, c\}$ is αg -closed, but is not pre-open, since $\{b\} \not\subseteq \text{int}(\text{cl}(\{b\})) = \text{int}(\{c, b\}) = \emptyset$.

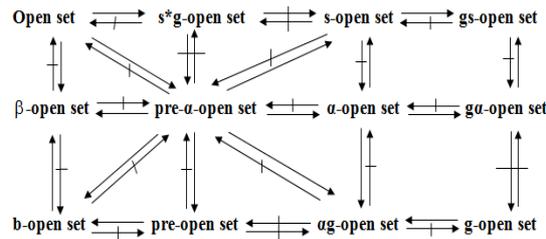
Remark 2.13:

$g\alpha$ -open sets and g -open sets are in general independent as shown by the following examples:

Example 2.14:

Let $X = \{a, b, c\}$ & $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$ be a topology on X . Then the set $\{a, b\}$ is a $g\alpha$ -open set in X , since $\{a, b\}^c = \{c\}$ is $g\alpha$ -closed, but is not g -open, since $\{a, b\}^c = \{c\}$ is not g -closed. Also, in example (2.2) $\{c\}$ is a g -open set in X , since, $\{c\}^c = \{a, b\}$ is g -closed, but is not $g\alpha$ -open, since $\{c\}^c = \{a, b\}$ is not $g\alpha$ -closed.

The following diagram shows the relationships between pre- α -open sets and each of open sets and other weaker forms of open sets:

**Theorem 2.15:**

A subset A of a topological space (X, τ) is a pre- α -open set if and only if there is an open set V of X such that $V \subseteq A \subseteq \text{int}(\text{pcl}(V))$.

Proof: \Rightarrow Assume that A is a pre- α -open set in X , then $A \subseteq \text{int}(\text{pcl}(\text{int}(A)))$. Since $\text{int}(A) \subseteq A$, then $\text{int}(A) \subseteq A \subseteq \text{int}(\text{pcl}(\text{int}(A)))$. Put $V = \text{int}(A)$, thus there is an open set V of X such that $V \subseteq A \subseteq \text{int}(\text{pcl}(V))$. Conversely, assume that there is an open set V of X such that $V \subseteq A \subseteq \text{int}(\text{pcl}(V))$. Since $V \subseteq A \Rightarrow V \subseteq \text{int}(A) \Rightarrow \text{pcl}(V) \subseteq \text{pcl}(\text{int}(A)) \Rightarrow \text{int}(\text{pcl}(V)) \subseteq \text{int}(\text{pcl}(\text{int}(A)))$. But $A \subseteq \text{int}(\text{pcl}(V))$, thus $A \subseteq \text{int}(\text{pcl}(\text{int}(A)))$. Therefore A is a pre- α -open set in X .

Theorem 2.16 (Jun, Y.B., 2008):

If V is an open set in (X, τ) , then $V \cap \text{pcl}(A) \subseteq \text{pcl}(V \cap A)$ for any subset A of X .

Theorem 2.17:

The family of all pre- α -open sets in a topological space (X, τ) form a topology on X .

Proof:

(i). Since $\emptyset \subseteq \text{int}(\text{pcl}(\text{int}(\emptyset)))$ and $X \subseteq \text{int}(\text{pcl}(\text{int}(X)))$, then $\emptyset, X \in \tau^{\text{pre-}\alpha}$.

(ii). Let $A, B \in \tau^{\text{pre-}\alpha}$. To prove that $A \cap B \in \tau^{\text{pre-}\alpha}$. By theorem (2.15), there are $U, V \in \tau$ s.t $U \subseteq A \subseteq \text{int}(\text{pcl}(U))$ and $V \subseteq B \subseteq \text{int}(\text{pcl}(V))$. Notice that $U \cap V \in \tau$ and $U \cap V \subseteq A \cap B$. Hence,

$$\begin{aligned} A \cap B &\subseteq \text{int}(\text{pcl}(U)) \cap \text{int}(\text{pcl}(V)) = \text{int}(\text{int}(\text{pcl}(U)) \cap \text{pcl}(V)) \\ &\subseteq \text{int}(\text{pcl}(\text{int}(\text{pcl}(U)) \cap V)) \quad (\text{by theorem (2.16)}) \\ &\subseteq \text{int}(\text{pcl}(\text{pcl}(U) \cap V)) \\ &\subseteq \text{int}(\text{pcl}(\text{pcl}(U \cap V))) \quad (\text{by theorem (2.16)}) \\ &= \text{int}(\text{pcl}(U \cap V)) \quad (\text{by proposition (1.7),iv}) \end{aligned}$$

Therefore $U \cap V \subseteq A \cap B \subseteq \text{int}(\text{pcl}(U \cap V))$. Thus by theorem (2.15), $A \cap B \in \tau^{\text{pre-}\alpha}$.

(iii). Let $\{V_\alpha : \alpha \in \Lambda\}$ be any family of pre- α -open sets in X , then $V_\alpha \subseteq \text{int}(\text{pcl}(\text{int}(V_\alpha)))$ for each $\alpha \in \Lambda$. Therefore by proposition ((1.7) vii), we get:

$$\begin{aligned} \bigcup_{\alpha \in \Lambda} V_\alpha &\subseteq \bigcup_{\alpha \in \Lambda} \text{int}(\text{pcl}(\text{int}(V_\alpha))) \subseteq \text{int}(\bigcup_{\alpha \in \Lambda} \text{pcl}(\text{int}(V_\alpha))) \subseteq \text{int}(\text{pcl}(\bigcup_{\alpha \in \Lambda} \text{int}(V_\alpha))) \\ &\subseteq \text{int}(\text{pcl}(\text{int}(\bigcup_{\alpha \in \Lambda} V_\alpha))). \text{ Then } \bigcup_{\alpha \in \Lambda} V_\alpha \in \tau^{\text{pre-}\alpha}. \end{aligned}$$

Therefore $\tau^{\text{pre-}\alpha}$ is a topology on X .

Theorem 2.18:

Let B be a subset of a topological space (X, τ) . Then the following statements are equivalent:

- i) B is pre- α -closed.
- ii) $\text{cl}(\text{pint}(\text{cl}(B))) \subseteq B$.
- iii) There is a closed subset F of X such that $\text{cl}(\text{pint}(F)) \subseteq B \subseteq F$.

Proof:

(i) \Rightarrow (ii). Since B is a pre- α -closed set in $X \Rightarrow X - B$ is pre- α -open $\Rightarrow X - B \subseteq \text{int}(\text{pcl}(\text{int}(X - B))) \Rightarrow X - B \subseteq \text{int}(\text{pcl}(X - \text{cl}(B)))$. By proposition ((1.7),v), we get $X - \text{pint}(\text{cl}(B)) = \text{pcl}(X - \text{cl}(B))$. Hence $X - B \subseteq \text{int}(X - \text{pint}(\text{cl}(B))) \Rightarrow X - B \subseteq X - \text{cl}(\text{pint}(\text{cl}(B))) \Rightarrow \text{cl}(\text{pint}(\text{cl}(B))) \subseteq B$.

(ii) \Rightarrow (iii). Since $\text{cl}(\text{pint}(\text{cl}(B))) \subseteq B$ and $B \subseteq \text{cl}(B)$, then $\text{cl}(\text{pint}(\text{cl}(B))) \subseteq B \subseteq \text{cl}(B)$. Put $F = \text{cl}(B)$, thus there is a closed subset F of X such that $\text{cl}(\text{pint}(F)) \subseteq B \subseteq F$.

(iii) \Rightarrow (i). Assume that there is a closed subset F of X such that $\text{cl}(\text{pint}(F)) \subseteq B \subseteq F$. Hence $X - F \subseteq X - B \subseteq X - \text{cl}(\text{pint}(F)) = \text{int}(X - \text{pint}(F))$. Since $X - \text{pint}(F) = \text{pcl}(X - F)$, then $X - F \subseteq X - B \subseteq \text{int}(\text{pcl}(X - F))$. Hence $X - B$ is a pre- α -open set in X . Thus B is a pre- α -closed set in X .

Proposition 2.19:

If A is a pre- α -open set in (X, τ) and $A \subseteq B \subseteq \text{int}(\text{pcl}(A))$, then B is a pre- α -open set in X .

Proof:

Since A is a pre- α -open set in X , then by theorem (2.15), there is an open set V of X such that $V \subseteq A \subseteq \text{int}(\text{pcl}(V))$. Since $A \subseteq B \Rightarrow V \subseteq B$. But $\text{int}(\text{pcl}(A)) \subseteq \text{int}(\text{pcl}(V)) \Rightarrow V \subseteq B \subseteq \text{int}(\text{pcl}(V))$. Thus B is a pre- α -open set in X .

Corollary 2.20:

If A is a pre- α -closed set in (X, τ) and $\text{cl}(\text{pint}(A)) \subseteq B \subseteq A$, then B is a pre- α -closed set in X .

Proof:

Since $X - A \subseteq X - B \subseteq X - \text{cl}(\text{pint}(A)) = \text{int}(X - \text{pint}(A)) = \text{int}(\text{pcl}(X - A))$, then by proposition (2.19), $X - B$ is a pre- α -open set in X . Thus B is a pre- α -closed set in X .

Theorem 2.21:

A subset A of a topological space (X, τ) is pre- α -clopen (pre- α -open and pre- α -closed) if and only if A is clopen (open and closed).

Proof: \Rightarrow Assume that A is a pre- α -clopen set in X , then A is pre- α -open and pre- α -closed in X . Thus $A \subseteq \text{int}(\text{pcl}(\text{int}(A)))$ and $\text{cl}(\text{pint}(\text{cl}(A))) \subseteq A$. But by proposition ((1.7), i, ii) we get, $\text{pcl}(A) \subseteq \text{cl}(A)$ and $\text{int}(A) \subseteq \text{pint}(A)$, then $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

Since $\text{int}(A) \subseteq A \Rightarrow \text{cl}(\text{int}(A)) \subseteq \text{cl}(A)$ (1)

Since $\text{int}(\text{cl}(\text{int}(A))) \subseteq \text{cl}(\text{int}(A))$, then

$$A \subseteq \text{int}(\text{cl}(\text{int}(A))) \subseteq \text{cl}(\text{int}(A)) \Rightarrow \text{cl}(A) \subseteq \text{cl}(\text{int}(A)) \quad (2)$$

Therefore from (1) and (2), we get $\text{cl}(\text{int}(A)) = \text{cl}(A)$ (3)

Similarly, since $A \subseteq \text{cl}(A) \Rightarrow \text{int}(A) \subseteq \text{int}(\text{cl}(A))$ (4)

Also, $\text{int}(\text{cl}(A)) \subseteq \text{cl}(\text{int}(\text{cl}(A))) \subseteq A$, then

$$\text{int}(\text{cl}(A)) \subseteq \text{int}(A) \quad (5)$$

Therefore from (4) and (5), we get $\text{int}(\text{cl}(A)) = \text{int}(A)$ (6)

Since $\text{int}(\text{cl}(A)) = \text{int}(A) \Rightarrow$

$$\text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(\text{int}(A)) = \text{cl}(A) \text{ (by (3))}.$$

But $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$, then $\text{cl}(A) \subseteq A$, since $A \subseteq \text{cl}(A)$, therefore $A = \text{cl}(A)$, hence A is closed in X . Similarly, since $\text{cl}(\text{int}(A)) = \text{cl}(A)$

$\Rightarrow \text{int}(\text{cl}(\text{int}(A))) = \text{int}(\text{cl}(A)) = \text{int}(A)$ (by (6)). But $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, then $A \subseteq \text{int}(A)$, since $\text{int}(A) \subseteq A$, therefore $A = \text{int}(A)$, hence A is open in X . Thus A is a clopen set in X .

\Leftarrow It is obvious.

Definition 2.22:

A subset N of a topological space (X, τ) is called a pre- α -neighborhood of a point x in X if there is a pre- α -open set O in X such that $x \in O \subseteq N$.

Remark 2.23:

Since every open set in X is pre- α -open, then every neighborhood of x is a pre- α -neighborhood of x , but the converse is not true in general. In example (2.2), $\{a, c\}$ is a pre- α -neighborhood of a point c , since $c \in \{a, c\} \subseteq \{a, c\}$. But $\{a, c\}$ is not a neighborhood of a point c .

Propositions 2.24:

A subset A of a topological space (X, τ) is pre- α -open if and only if it is a pre- α -neighborhood of each of its points.

Proof:

\Rightarrow If A is a pre- α -open set in X , then $x \in A \subseteq A$ for each $x \in A$. Hence A is a pre- α -neighborhood of each of its points.

Conversely, assume that A is a pre- α -neighborhood of each of its points. Then for each $x \in A$, there is a pre- α -open set V_x in X such that $x \in V_x \subseteq A$. Hence $\bigcup_{x \in A} V_x \subseteq A$. Since $A \subseteq \bigcup_{x \in A} V_x$, thus

$$A = \bigcup_{x \in A} V_x. \text{ Therefore } A \text{ is a pre-}\alpha\text{-open set in } X, \text{ since it is a union of pre-}\alpha\text{-open sets.}$$

Definition 2.25:

Let A be a subset of a topological space (X, τ) . Then:

i) The pre- α -closure of A , denoted by $p\text{-}\alpha\text{-cl}(A)$ is the intersection of all pre- α -closed subsets of X which contains A .

ii) The pre- α -interior of A , denoted by $p\text{-}\alpha\text{-int}(A)$ is the union of all pre- α -open sets in X which are contained in A .

Theorem 2.26:

Let A and B be subsets of a topological space (X, τ) . Then:

i) $\text{int}(A) \subseteq p\text{-}\alpha\text{-int}(A) \subseteq A$ and $A \subseteq p\text{-}\alpha\text{-cl}(A) \subseteq \text{cl}(A)$.

ii) $p\text{-}\alpha\text{-int}(A)$ is a pre- α -open set in X and $p\text{-}\alpha\text{-cl}(A)$ is a pre- α -closed set in X .

- iii) If $A \subseteq B$, then $p\text{-}\alpha\text{-int}(A) \subseteq p\text{-}\alpha\text{-int}(B)$ and $p\text{-}\alpha\text{-cl}(A) \subseteq p\text{-}\alpha\text{-cl}(B)$.
- iv) A is pre- α -open iff $p\text{-}\alpha\text{-int}(A) = A$ and A is pre- α -closed iff $p\text{-}\alpha\text{-pcl}(A) = A$.
- v) $p\text{-}\alpha\text{-int}(A \cap B) = p\text{-}\alpha\text{-int}(A) \cap p\text{-}\alpha\text{-int}(B)$ and $p\text{-}\alpha\text{-pcl}(A \cup B) = p\text{-}\alpha\text{-pcl}(A) \cup p\text{-}\alpha\text{-pcl}(B)$.
- vi) $p\text{-}\alpha\text{-int}(p\text{-}\alpha\text{-int}(A)) = p\text{-}\alpha\text{-int}(A)$ and $p\text{-}\alpha\text{-cl}(p\text{-}\alpha\text{-cl}(A)) = p\text{-}\alpha\text{-pcl}(A)$.
- vii) $x \in p\text{-}\alpha\text{-int}(A)$ iff there is a pre- α -open set U in X s.t $x \in U \subseteq A$.
- viii) $x \in p\text{-}\alpha\text{-pcl}(A)$ iff for every pre- α -open set U containing x , $U \cap A \neq \emptyset$.

Proof:

It is obvious.

Proposition 2.27:

Let (X, τ) and (Y, σ) be topological spaces. If $A_1 \subseteq X$ and $A_2 \subseteq Y$. Then $A_1 \times A_2$ is a pre- α -open set in $X \times Y$ if and only if A_1 and A_2 are pre- α -open sets in X and Y respectively.

Proof:

\Leftarrow Since A_1 and A_2 are pre- α -open sets in X and Y respectively, then by definition (2.1), we get $A_1 \subseteq \text{int}(\text{pcl}(\text{int}(A_1)))$ and $A_2 \subseteq \text{int}(\text{pcl}(\text{int}(A_2)))$. Hence $A_1 \times A_2 \subseteq \text{int}(\text{pcl}(\text{int}(A_1))) \times \text{int}(\text{pcl}(\text{int}(A_2))) = \text{int}(\text{pcl}(\text{int}(A_1) \times \text{pcl}(\text{int}(A_2))))$. By theorem (1.8) $\text{pcl}(A_1) \times \text{pcl}(A_2) = \text{pcl}(A_1 \times A_2)$. Therefore $A_1 \times A_2 \subseteq \text{int}(\text{pcl}(\text{int}(A_1 \times A_2)))$. Thus $A_1 \times A_2$ is a pre- α -open set in $X \times Y$. By the same way, we can prove that A_1 and A_2 are pre- α -open sets in X and Y respectively if $A_1 \times A_2$ is a pre- α -open set in $X \times Y$.

Contra Pre- α -Continuous Functions and Contra pre- α -Irresolute Functions:

In this section, we introduce new types of functions, namely, pre- α -continuous functions, contra pre- α -continuous functions, pre- α -irresolute functions and contra pre- α -irresolute functions in topological spaces and we study the relation between these types of functions and each of contra continuous functions and other weaker forms of contra continuous functions.

Definition 3.1:

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pre- α -continuous if $f^{-1}(U)$ is a pre- α -open set in X for every open set U in Y .

Definition 3.2:

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called contra pre- α -continuous if $f^{-1}(U)$ is a pre- α -closed set in X for every open set U in Y .

Proposition 3.3:

Every contra continuous function is contra pre- α -continuous.

Proof:

Follows from the definition (3.2) and the fact that every closed set is pre- α -closed.

Remark 3.4:

The converse of proposition (3.3) may not be true in general as shown by the following example:

Example 3.5:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ & $\sigma = \{\emptyset, Y, \{a\}, \{a, c\}\} \Rightarrow \tau^{\text{pre-}\alpha} = \{\emptyset, X, \{a\}, \{a, c\}, \{a, b\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = b$, $f(b) = c$ & $f(c) = a \Rightarrow f$ is not contra continuous, but f is contra pre- α -continuous, since $f^{-1}(Y) = X$, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(\{a, c\}) = \{b, c\}$, and $f^{-1}(\{a\}) = \{c\}$ are pre- α -closed sets in X .

Remark 3.6:

pre- α -continuous functions and contra pre- α -continuous functions are in general independent as shown by the following examples:

Example 3.7:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ & $\sigma = \{\emptyset, Y, \{a\}\} \Rightarrow \tau^{\text{pre-}\alpha} = \tau$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = a$, $f(b) = a$ & $f(c) = c \Rightarrow f$ is pre- α -continuous, but f is not contra pre- α -continuous, since $\{a\}$ is open set in Y , but $f^{-1}(\{a\}) = \{a, b\}$ is not pre- α -closed in X . Also, in example (3.5) f is contra pre- α -continuous, but is not pre- α -continuous, since $\{a\}$ is open set in Y , but $f^{-1}(\{a\}) = \{c\}$ is not pre- α -open in X .

Remark 3.8:

Contra pre- α -continuous functions and contra s^*g -continuous functions are in general independent as shown by the following examples:

Example 3.9:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X\}$ & $\sigma = \{\emptyset, Y, \{a\}\} \Rightarrow \tau^{\text{pre-}\alpha} = \tau$ and $\tau^{s^*g} = \{\emptyset, X, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = a$, $f(b) = b$ & $f(c) = c \Rightarrow f$ is contra s^*g -continuous, but f is not contra pre- α -continuous, since $\{a\}$ is open set in Y , but $f^{-1}(\{a\}) = \{a\}$ is not pre- α -closed in X . Also, in example (3.5) f is contra pre- α -continuous, but is not contra s^*g -continuous, since $\{a\}$ is open set in Y , but $f^{-1}(\{a\}) = \{c\}$ is not s^*g -closed in X .

Theorem 3.10:

Every contra pre- α -continuous function is contra α -continuous (resp. contra $g\alpha$ -continuous, contra αg -continuous, contra β -continuous, contra pre-continuous, contra b -continuous) function.

Proof:

Follows from the theorem (2.3).

Remark 3.11:

The converse of theorem (3.10) may not be true in general. Notice that in example (3.9) f is contra pre-continuous (resp. contra β -continuous, contra b -continuous, contra αg -continuous, contra $g\alpha$ -continuous) function, but f is not contra pre- α -continuous.

Theorem 3.12:

Every contra pre- α -continuous function is contra semi-continuous function and contra gs -continuous function.

Proof:

Follows from the theorem (2.8).

Remark 3.13:

The converse of theorem (3.12) may not be true in general. Consider the following example:

Example 3.14:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ & $\sigma = \{\emptyset, Y, \{a\}, \{a, c\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = a$, $f(b) = b$ & $f(c) = c \Rightarrow f$ is contra semi-continuous and contra gs -continuous, but f is not contra pre- α -continuous, since $\{a\}$ is open in Y , but $f^{-1}(\{a\}) = \{a\}$ is not pre- α -closed in X , since $\text{cl}(\text{pint}(\text{cl}(\{a\}))) = \text{cl}(\text{pint}(\{a, c\})) = \text{cl}(\{a\}) = \{a, c\} \not\subset \{a\}$.

Remark 3.15:

Contra αg -continuous functions and contra pre-continuous functions are in general independent as shown by the following examples:

Example 3.16:

Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ & $\sigma = \{\phi, Y, \{a, b\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = a, f(b) = b$ & $f(c) = c \Rightarrow f$ is contra αg -continuous, but f is not contra pre-continuous, since $\{a, b\}$ is open set in Y , but $f^{-1}(\{a, b\}) = \{a, b\}$ is not pre-closed in X , since $\text{cl}(\text{int}(\{a, b\})) = \text{cl}(\{a\}) = X \not\subset \{a, b\}$.

Example 3.17:

Let $X = Y = \mathfrak{R}$, $\tau = \mu =$ usual topology & $\sigma = \{\phi, \mathfrak{R}, \{Q\}\}$. Define $f : (\mathfrak{R}, \mu) \rightarrow (\mathfrak{R}, \sigma)$ by: $f(x) = x$ for each $x \in \mathfrak{R} \Rightarrow f$ is not contra αg -continuous, since Q is open in Y , but $f^{-1}(\{Q\}) = Q$ is not αg -closed set in X . But f is contra pre-continuous.

Remark 3.18:

Contra $g\alpha$ -continuous functions and contra g -continuous functions are in general independent as shown by the following examples:

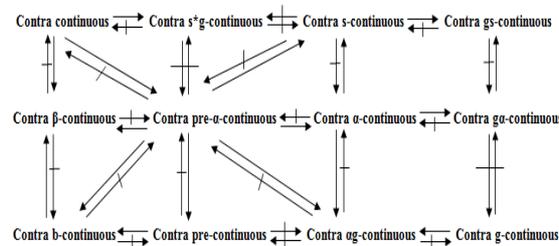
Example 3.19:

Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ & $\sigma = \{\phi, Y, \{b\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = b, f(b) = c$ & $f(c) = b \Rightarrow f$ is contra g -continuous, but f is not contra $g\alpha$ -continuous, since $\{b\}$ is open set in Y , but $f^{-1}(\{b\}) = \{a, c\}$ is not $g\alpha$ -closed set in X .

Example 3.20:

Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ & $\sigma = \{\phi, Y, \{b\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = a, f(b) = c$ & $f(c) = b \Rightarrow f$ is contra $g\alpha$ -continuous, but f is not contra g -continuous, since $\{b\}$ is open set in Y , but $f^{-1}(\{b\}) = \{c\}$ is not g -closed set in X .

The following diagram shows the relationships between contra pre- α -continuous functions and each of contra continuous functions and other weaker forms of contra continuous functions:

**Proposition 3.21:**

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra pre- α -continuous, then $f^{-1}(\text{int}(B)) \subseteq p\text{-}\alpha\text{-cl}(f^{-1}(B))$ for every subset B of Y .

Proof:

Since $\text{int}(B) \subseteq B \Rightarrow f^{-1}(\text{int}(B)) \subseteq f^{-1}(B)$. Since $\text{int}(B)$ is an open set in Y and f is contra pre- α -continuous, then by definition (3.2) $f^{-1}(\text{int}(B))$ is a pre- α -closed set in X such that $f^{-1}(\text{int}(B)) \subseteq f^{-1}(B)$. Therefore by theorem ((2.26),iv) $f^{-1}(\text{int}(B)) \subseteq p\text{-}\alpha\text{-cl}(f^{-1}(B))$ for every subset B of Y .

Theorem 3.22:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following statements are equivalent:

i) f is contra pre- α -continuous.

- ii) For every closed subset F of Y , $f^{-1}(F)$ is a pre- α -open set in X .
- iii) For each $x \in X$ and each closed set F in Y containing $f(x)$, there exists a pre- α -open set V in X such that $x \in V$ and $f(V) \subseteq F$.
- iv) $f(p\text{-}\alpha\text{-cl}(A)) \subseteq \ker(f(A))$ for each subset A of X .
- v) $p\text{-}\alpha\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\ker(B))$ for each subset B of Y .

Proof:

(i) \Leftrightarrow (ii). It is obvious.

(ii) \Rightarrow (iii). Let F be a closed set in Y s.t $f(x) \in F$. To prove that, there is a pre- α -open set V in X s.t $x \in V$ and $f(V) \subseteq F$. Since $f(x) \in F$ and F is closed in Y , then by hypothesis $f^{-1}(F)$ is a pre- α -open set in X s.t $x \in f^{-1}(F)$. Let $V = f^{-1}(F) \Rightarrow f(V) = f(f^{-1}(F)) \subseteq F$.

(iii) \Rightarrow (ii). Let F be any closed set in Y . To prove that $f^{-1}(F)$ is a pre- α -open set in X . Let $x \in f^{-1}(F) \Rightarrow f(x) \in F$. By hypothesis there is a pre- α -open set V in X s.t $x \in V$ and $f(V) \subseteq F \Rightarrow x \in V \subseteq f^{-1}(F)$. Thus by theorem ((2.26),vii) $f^{-1}(F)$ is a pre- α -open set in X .

(ii) \Rightarrow (iv). Let A be any subset of X . Assume that $y \notin \ker(f(A))$, then by lemma ((1.5),i) there is a closed set F in Y s.t $y \in F$ and $f(A) \cap F = \emptyset$. Thus, we have $A \cap f^{-1}(F) = \emptyset$ and $p\text{-}\alpha\text{-cl}(A) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(p\text{-}\alpha\text{-cl}(A)) \cap F = \emptyset$ and $y \notin f(p\text{-}\alpha\text{-cl}(A))$. Thus $f(p\text{-}\alpha\text{-cl}(A)) \subseteq \ker(f(A))$ for each subset A of X .

(iv) \Rightarrow (v). Let B be any subset of Y . By (iv) and lemma ((1.5),iii) we have $f(p\text{-}\alpha\text{-cl}(f^{-1}(B))) \subseteq \ker(f(f^{-1}(B))) \subseteq \ker(B)$. Therefore $p\text{-}\alpha\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\ker(B))$ for each subset B of Y .

(v) \Rightarrow (i). Let V be any open set of Y . Then by (v) and lemma ((1.5),ii), we have $p\text{-}\alpha\text{-cl}(f^{-1}(V)) \subseteq f^{-1}(\ker(V)) = f^{-1}(V)$ and $p\text{-}\alpha\text{-cl}(f^{-1}(V)) = f^{-1}(V)$. Therefore $f^{-1}(V)$ is a pre- α -closed set in X . Thus f is contra pre- α -continuous.

Definition 3.23:

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pre- α -irresolute if the inverse image of every pre- α -open set in Y is a pre- α -open set in X .

Definition 3.24:

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called contra pre- α -irresolute if the inverse image of every pre- α -open set in Y is a pre- α -closed set in X .

Remark 3.25:

pre- α -irresolute functions and contra pre- α -irresolute functions are in general independent as shown by the following examples:

Example 3.26:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, c\}\}$ & $\sigma = \{\emptyset, Y, \{b, c\}\} \Rightarrow \tau^{\text{pre-}\alpha} = \{\emptyset, X, \{a\}, \{a, c\}, \{a, b\}\}$ & $\sigma^{\text{pre-}\alpha} = \sigma$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = a$, $f(b) = b$ & $f(c) = c \Rightarrow f$ is contra pre- α -irresolute, but f is not pre- α -irresolute, since $\{b, c\}$ is pre- α -open set in Y , but $f^{-1}(\{b, c\}) = \{b, c\}$ is not pre- α -open set in X .

Example 3.27:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ & $\sigma = \{\emptyset, Y, \{a, b\}\} \Rightarrow \tau^{\text{pre-}\alpha} = \{\emptyset, X, \{a\}, \{a, c\}, \{a, b\}\}$ & $\sigma^{\text{pre-}\alpha} = \sigma$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = a$, $f(b) = b$ & $f(c) = c \Rightarrow f$ is pre- α -irresolute, but f is not contra pre- α -irresolute, Since $\{a, b\}$ is pre- α -open set in Y , but $f^{-1}(\{a, b\}) = \{a, b\}$ is not pre- α -closed in X .

Proposition 3.28:

Every contra pre- α -irresolute function is contra pre- α -continuous.

Proof:

It is Obvious.

Remark 3.29:

The converse of proposition (3.28) may not be true in general. Consider the following example:

Example 3.30:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, c\}\}$ & $\sigma = \{\emptyset, Y, \{a\}\} \Rightarrow \tau^{\text{pre-}\alpha} = \tau$ and $\sigma^{\text{pre-}\alpha} = \{\emptyset, Y, \{a\}, \{a, c\}, \{a, b\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by: $f(a) = b$, $f(b) = a$ & $f(c) = c \Rightarrow f$ is contra pre- α -continuous, but f is not contra pre- α -irresolute since $\{a, b\}$ is a pre- α -open set in Y , but $f^{-1}(\{a, b\}) = \{a, b\}$ is not pre- α -closed set in X .

Definition 3.31:

Let A be a subset of a topological space (X, τ) . Then $\bigcap \{U \in \tau^{\text{pre-}\alpha} : A \subseteq U\}$ is called the pre- α -kernel of A and is denoted by $\text{pre-}\alpha\text{-ker}(A)$.

Lemma 3.32:

Let A and B be subsets of a topological space (X, τ) . Then:

- i) $x \in \text{pre-}\alpha\text{-ker}(A)$ if and only if $A \cap F \neq \emptyset$ for any pre- α -closed subset F of X containing x .
- ii) $A \subseteq \text{pre-}\alpha\text{-ker}(A)$ and if A is pre- α -open in X , then $A = \text{pre-}\alpha\text{-ker}(A)$.
- iii) If $A \subseteq B$, then $\text{pre-}\alpha\text{-ker}(A) \subseteq \text{pre-}\alpha\text{-ker}(B)$.

Proof:

i) \Rightarrow Suppose that \exists a pre- α -closed set F in X s.t $x \in F$ and $A \cap F = \emptyset \Rightarrow A \subseteq F^c$ s.t F^c is a pre- α -open set in X and $x \notin F^c \Rightarrow x \notin \text{pre-}\alpha\text{-ker}(A)$, this is a contradiction.

Conversely, if $x \notin \text{pre-}\alpha\text{-ker}(A) \Rightarrow \exists$ a pre- α -open set U in X s.t $A \subseteq U$ and $x \notin U \Rightarrow x \in U^c$ s.t U^c is a pre- α -closed set in X and $A \cap U^c = \emptyset$, this is a contradiction.

ii) It is Obvious.

iii) It is Obvious.

Theorem 3.33:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following statements are equivalent:

- i) f is contra pre- α -irresolute.
- ii) For every pre- α -closed subset F of Y , $f^{-1}(F)$ is a pre- α -open set in X .
- iii) For each $x \in X$ and each pre- α -closed set F in Y with $f(x) \in F$, there is a pre- α -open set U in X such that $x \in U$ and $f(U) \subseteq F$.
- iv) $f(\text{p-}\alpha\text{-cl}(A)) \subseteq \text{pre-}\alpha\text{-ker}(f(A))$ for each subset A of X .
- v) $\text{p-}\alpha\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{pre-}\alpha\text{-ker}(B))$ for each subset B of Y .

Proof:

(i) \Leftrightarrow (ii). It is obvious.

(ii) \Rightarrow (iii). Let F be a pre- α -closed set in Y s.t $f(x) \in F$. To prove that, there is a pre- α -open set U in X s.t $x \in U$ and $f(U) \subseteq F$. Since $f(x) \in F$ and F is pre- α -closed, then by hypothesis $f^{-1}(F)$ is a pre- α -open set in X s.t $x \in f^{-1}(F)$. Let $U = f^{-1}(F) \Rightarrow f(U) = f(f^{-1}(F)) \subseteq F$.

(iii) \Rightarrow (ii). Let F be any pre- α -closed set in Y . To prove that $f^{-1}(F)$ is a pre- α -open set in X . Let $x \in f^{-1}(F) \Rightarrow f(x) \in F$. By hypothesis there is a pre- α -open set U in X s.t $x \in U$ and $f(U) \subseteq F \Rightarrow x \in U \subseteq f^{-1}(F)$. Thus by theorem ((2.26),vii) $f^{-1}(F)$ is a pre- α -open set in X .

(ii) \Rightarrow (iv). Let A be any subset of X . Assume that $y \notin \text{pre-}\alpha\text{-ker}(f(A))$, then by lemma ((3.32),i) there is a pre- α -closed set F in Y s.t $y \in F$ and $f(A) \cap F = \emptyset$. Thus, we have $A \cap f^{-1}(F) = \emptyset$ and $p\text{-}\alpha\text{-cl}(A) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(p\text{-}\alpha\text{-cl}(A)) \cap F = \emptyset$ and $y \notin f(p\text{-}\alpha\text{-cl}(A))$. Thus $f(p\text{-}\alpha\text{-cl}(A)) \subseteq \text{pre-}\alpha\text{-ker}(f(A))$ for each subset A of X .

(iv) \Rightarrow (v). Let B be any subset of Y . By (iv) and lemma ((3.32),iii) we have $f(p\text{-}\alpha\text{-cl}(f^{-1}(B))) \subseteq \text{pre-}\alpha\text{-ker}(f(f^{-1}(B))) \subseteq \text{pre-}\alpha\text{-ker}(B)$. Therefore $p\text{-}\alpha\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{pre-}\alpha\text{-ker}(B))$ for each subset B of Y .

(v) \Rightarrow (i). Let V be any pre- α -open set of Y . Then by (v) and lemma ((3.32),ii), we have $p\text{-}\alpha\text{-cl}(f^{-1}(V)) \subseteq f^{-1}(\text{pre-}\alpha\text{-ker}(V)) = f^{-1}(V)$ and $p\text{-}\alpha\text{-cl}(f^{-1}(V)) = f^{-1}(V)$. Hence $f^{-1}(V)$ is a pre- α -closed set in X . Therefore f is contra pre- α -irresolute.

However the following theorem holds. The proof is easy and hence omitted.

Theorem 3.34:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$

are functions. Then:

- i) If f is contra pre- α -continuous and g is continuous, then $g \circ f$ is contra pre- α -continuous.
- ii) If f is pre- α -irresolute and g is contra pre- α -continuous, then $g \circ f$ is contra pre- α -continuous.
- iii) If f is pre- α -irresolute and g is contra continuous, then $g \circ f$ is contra pre- α -continuous.
- iv) If f is contra pre- α -irresolute and g is pre- α -continuous, then $g \circ f$ is contra pre- α continuous.
- v) If f is contra pre- α -irresolute and g is pre- α -irresolute, then $g \circ f$ is contra pre- α -irresolute.

Definition 3.35:

A topological space (X, τ) is called:

- i) pre- α -space if every pre- α -closed set is closed.
- ii) pre- α -locally indiscrete if every pre- α -open set is closed.
- iii) $T_{\text{pre-}\alpha}$ -space if every pre-closed set is pre- α -closed.

Theorem 3.36:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then:

- i) If f is pre- α -continuous and (X, τ) is pre- α -locally indiscrete, then f is contra continuous.
- ii) If f is pre- α -irresolute and (X, τ) is pre- α -locally indiscrete, then f is contra continuous.
- iii) If f is contra pre- α -continuous and (X, τ) is pre- α -space, then f is contra continuous.
- iv) If f is contra pre-continuous and (X, τ) is $T_{\text{pre-}\alpha}$ - space, then f is contra pre- α -continuous.
- v) If f is contra pre- α -continuous and (X, τ) is pre- α -locally indiscrete, then f is continuous.

Proof:

- i) Let U be an open set in Y . Then, by assumption $f^{-1}(U)$ is pre- α -open set in X . Since (X, τ) is pre- α -locally indiscrete, then $f^{-1}(U)$ is closed in X . Hence f is a contra continuous function.
- ii) Let U be an open set in Y . Since every open set is pre- α -open and since f is pre- α -irresolute, then $f^{-1}(U)$ is pre- α -open set in X . Since (X, τ) is pre- α -locally indiscrete, then $f^{-1}(U)$ is closed in X . Hence f is a contra continuous function.
- iii) Let U be an open set in Y . Then, by assumption $f^{-1}(U)$ is a pre- α -closed set in X . Since (X, τ) is pre- α -space, then $f^{-1}(U)$ is closed in X . Hence f is a contra continuous function.

iv) Let U be an open set in Y . Then, by assumption $f^{-1}(U)$ is pre-closed set in X . Since (X, τ) is $T_{\text{pre-}\alpha}$ -space, then $f^{-1}(U)$ is pre- α -closed in X . Hence f is a contra pre- α -continuous function.

v) Let U be a closed set in Y . Since f is contra pre- α -continuous, then $f^{-1}(U)$ is a pre- α -open set in X . Since (X, τ) is pre- α -locally indiscrete, then $f^{-1}(U)$ is closed in X . Hence f is a continuous function.

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