Australian Journal of Basic and Applied Sciences, 9(17) Special 2015, Pages: 46-53

ISSN:1991-8178

Australian Journal of Basic and Applied Sciences

Journal home page: www.ajbasweb.com

Stability Analysis and Qualitative Behavior of Giving up Smoking Model with Education Campaign

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ARTICLE INFO

Article history:
Received 12 March 2015
Accepted 28 April 2015
Available online 2 May 2015

Keywords:
Smoking, Smokers Reproductive Number, Stability Analysis, Equilibrium Point

ABSTRACT

In this study, we proposed and analyzed a mathematical model to study the dynamics of giving up smoking and the effect of education campaign as a control strategy. The model is analyzed using stability theory of non-linear differential equations and is supported the results with numerical simulations. The results showed that there were two equilibrium points; smoking-free equilibrium point and smoking-present equilibrium point. The qualitative behavior results depend on the smokers reproductive number We obtained the smokers reproductive number by using the next generation method. Stabilities of the model are determined by Routh-Hurwitz criteria. If , then the smoking- free equilibrium point is local asymptotically stable, but If , then the smoking-present equilibrium point is local asymptotically stable. The graphical representations are provided to qualitatively support the analytical results. It concluded that with an increase in the education campaign, the number of everyday smokers will be decrease.

To Cite This Article: Thanawat Rittisak and Surapol Naowarat, Stability Analysis and Qualitative Behavior of Giving up Smoking Model with Education Campaign. ***Aust. J. Basic & Appl. Sci.,*** *9(17): 46-53, 2015*

INTRODUCTION

Tobacco epidemic is one the biggest public health threats to people of all ages in the world. The World Health Organization (WHO,2013) estimates that tobacco causes approximately 5 million deaths annually worldwide, and the number is expected to double by 2025. The reason for that high number is that tobacco use is a major cause of many of the world's top killer diseases including cardiovascular disease, chronic lung disease, and lung cancer. Smoking tobacco is related to more than two dozen diseases and conditions (Zaman,2009). Tobacco use is considered a disease that can spread through social contact in a way very similar to the spread of infectious diseases. Smoking is a prevalent problem among people that require intervention for eradication. Persistent education of the health hazards related to smoking is recommended particularly at early ages in order to prevent initiation of smoking (WHO,2013).

In many infectious diseases, mathematical models have become important tools can be use to understand the spread of smoking and to predict the impact of in analyzing the spread and control of infectious diseases. Due to the increasing in the number of smokers, tobacco use is also a disease to be treated. In order to explore the spread rule of smoking model. In 2002, Castillo-Garsow *et al* proposed a simple model for giving up smoking. They divided total constant population into four classes: potential smokers, occasion smokers, smokers and quit smokers. Sharrami and Gumel (2008) developed model by introducing mild and chain classes. Zaman(2009) proposed the giving up smoking model by taking into account the occasion smokers in the model. Zaman (2011). presented the optimal campaigns in the smoking dynamics, which described by the simplified PLSQ(potential-light-smoker-quit smoker). The model considered two control variables in the form of education and treatment campaigns oriented to decrease the attitude towards smoking. In order to minimize the number of occasion smokers and persistant smokers and maximize the number of quit smokers in the community. Alkhudhari *et al* (2014) proposed a generalization of the giving up smoking model in which quitting smokers can be temporary or permanent. The population with peer pressure effect on temporary quitting smokers and the impact of this transformation on the existence and stability of equilibrium points. Huo and Zhu (2013) derived and analyzed a model which takes into account light smokers class, recovery class, and two relapses to the giving up smoking model. Stability of the model is obtained. Numerical simulations are also provided to

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illustrate our analytic results and to show the effect of controlling the rate of relapse on the giving up smoking model. Alkhudhari *et al* (2014) derived and analyzed a model of smoking in which the population is divided into four classes: potential smokers, smokers, temporary quitters, and permanent quitters. We studied the effect of smoking on temporary quitters. Two equilibrium points are found: smoking-free equilibrium and the other corresponds to the presence of smoking. We examined the local and global stability of both equilibrium points and supported our results by using numerical simulations.

The objective of the study is to determine the effects of education campaign oriented to decrease the attitude towards smoking on the dynamics transmission of giving up smoking model. The structure of this paper is organized as follows. In section 2, we present a formulation model with the influence of education campaign. In section 3, we analyze the model by using the stability differential equations theory, to determine both smoking- free and smoking-present equilibrium point, derive the smokers reproductive number and investigate the stability of the model. In section 4, we simulate the numerical results of the model numerically, which support our analytic results. Finally, we summarize the conclusions of our study in section 5.

Model formulation:

For this study, we formulated (PLSQ) model (potential smokers, occasion smokers -everyday smokers-quit smokers) for the dynamics transmission of smoking model . Let $P(t)$, $L(t)$, $S(t)$ and $Q(t)$ denoted the potential smokers, occasion smokers, everyday smokers, and quit smokers, respectively. The diagram of the transmission of the smoking model as shown in Fig. 1

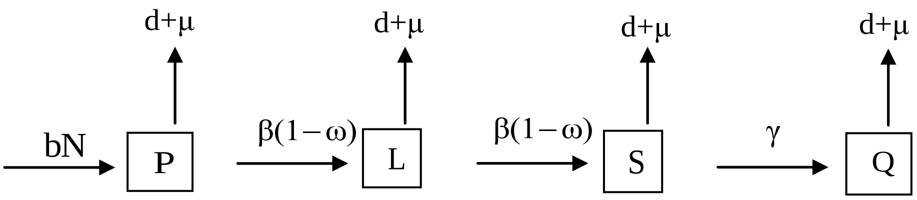

Fig. 1: Diagram of the transmission of the giving up smoking model.

We defined,
$P(t)$ is the number of potential smokers at time t ,
$L(t)$ is the number of occasion smokers at time t ,
$S(t)$ is the number everyday smokers at time t ,
$Q(t)$ is the number quit smokers at time t ,

The dynamical representation of the model consists of a system of non-linear differential equations with four state variables as follows,

$$\frac{dP}{dt} = bN - \beta(1-\omega)LP - (d+\mu)P \qquad (1)$$

$$\frac{dL}{dt} = \beta(1-\omega)LP - \beta(1-\omega)LS - (d+\mu)L \qquad (2)$$

$$\frac{dS}{dt} = \beta(1-\omega)LS - (\gamma+d+\mu)S \qquad (3)$$

$$\frac{dQ}{dt} = \gamma S - (d+\mu)Q \qquad (4)$$

with $P+L+S+Q = N$ (5)
where
b is the birth rate of human population
μ is the natural death rate of human population
β is the transmission coefficient from occasion smokers class to potential smokers class or
from everyday smokers class to occasion smokers class,
γ is the recovery rate from smoking ,
d is the death rate of smokers related to smoking diseases,
ω is the effective of educational campaign ,
N is the total number of human population .

Model analysis:
Bounded of solutions:

From $N = P + L + S + Q$

Consider $\frac{dN}{dt} = \frac{dP}{dt} + \frac{dL}{dt} + \frac{dS}{dt} + \frac{dQ}{dt}$

$= bN - \beta(1-\omega)LP - (d+\mu)P + \beta(1-\omega)LP - \beta(1-\omega)LS - (d+\mu)L +$
$\beta(1-\omega)LS - (\gamma+d+\mu)S + \gamma S - (d+\mu)Q$

$= (b-\mu-d)N$

Thus, $\frac{dN}{dt}=0,$ if $b = d+\mu$. This mean that the population is constant (Zaman *et al*,2008). In this paper we consider that the total population is not constant, so we assumed that $b < d+\mu$. In order to understand the qualitative behavior of the model, let us consider the set Ω and the initial conditions for this system given by:

$$\Omega=\{(P,L,S,Q)\,|\,P\geq 0, L\geq 0, S\geq 0, Q\geq 0, P+L+S+Q\leq N\},$$

with the initial conditions :

$$P(0)=P_0\geq 0, L(0)=L_0\geq 0, S(0)=S_0\geq 0, Q(0)=Q_0\geq 0.$$

Lemma 1.1 All feasible solution of the system (1) are bounded and enter the region

$$\Omega_e=\{(P,L,S,Q)\in\Re^4_+\,|\,P+L+S+Q\leq N_0e^{-Kt}\}$$

where $K=\mu+d-b$ and $b<\mu+d.$

Proof It is shown that all the state variables of the model are non-negative for the positive parameters,

Continuity of the right hand side of the system (1)-(4) and its derivative imply that the model is well - posed for $N(t)>0$ for time $t>0$. The dynamics of the total population are governed by

$$\frac{dN}{dt}=\frac{dP}{dt}+\frac{dL}{dt}+\frac{dS}{dt}+\frac{dQ}{dt}$$

$$=(b-d-\mu))N$$

$$\leq(b-(d+\mu))N$$

$$\leq -KN$$

Thus, we have $0\leq N(t)\leq N_0e^{-Kt}$ as $t\to\infty.$ Therefore, all the feasible solution of the giving up smoking model (1) - (4) are bounded and enter the region Ω_e. This completes the proof of lemma 1.1.

Positivity of the solutions:

Since, all state variables and parameters of the model must be non negative. We have the following results on the positivity solution of the system (1)-(4).

Lemma 1.2 Let the initial condition be $\{(P_0,L_0S_0,Q_0)\geq 0\}\in\Omega,$ then the solution set $\{P(t),L(t),S(t),Q(t)\}$ of the system (1)-(4) is positive for all $t>0.$

Proof:

From the first equation of the model system (1)-(4)

$$\frac{dP}{dt}=bN-\beta(1-\omega)LP-(d+\mu)P$$

That is,

$$\frac{dP}{dt}\geq-(d+\mu)P$$

Integrating both sides by separable of variables and applying initial condition. We have,

$$\int\frac{dP}{P}\geq-\int(d+\mu)dt$$

Thus, $P(t)\geq P_0e^{-\int(d+\mu)dt}>0.$

In the same manner, we can show that

$L(t)\geq L_0e^{-\int(d+\mu)dt}>0,$

$S(t)\geq S_0e^{-\int(\gamma+d+\mu)dt}>0,$ and

$Q(t)\geq Q_0e^{-\int(d+\mu)dt}>0.$ Therefore the solution set of the model system (1)-(4) is positive for all $t>0.$

Equilibrium Points:

By using the standard method for analyzing our model, this system has two equilibrium points; smoking- free equilibrium point and smoking-present equilibrium point. We obtained these by setting the right hand side of equations, (1)-(5) to zero. Doing this, we obtain

1. Smoking- Free Equilibrium (SFE) denoted by $E_0(P,LS,Q)=(N,0,0,0)$

In the case of the absence of the smokers, that is $L=0,S=0,Q=0.$ We obtained $P=N.$ Thus, $E_0(P,LS,Q)=(N,0,0,0)$

2. Smoking-Present Equilibrium (SPE) denoted by $E_1(P^*,L^*,S^*,Q^*)$ In the case where the disease is present, that is $S\neq 0$. We obtained

$$P^*=\frac{bN[\beta(1-\omega)^2S^*-\beta(1-\omega)(d+\mu)]}{\beta(1-\omega)[\beta(1-\omega)bN-(d+\mu)]-\beta(1-\omega)(d+\mu)[1+(d+\mu)(1-\omega)]S^*},$$

$$S^*=\frac{bN((\beta(1-\omega))^2-\beta(1-\omega)(d+\mu)^2-\beta(1-\omega)(\gamma+d+\mu)(d+\mu)}{((\beta(1-\omega))^2[2(d+\mu)+\gamma]}, \quad (6)$$

$$L^*=\frac{\beta(1-\omega)(bN)-(d+\mu)^3-\beta(1-\omega)(d+\mu)S^*}{\beta(1-\omega)(d+\mu)+\beta(1-\omega)^2S^*},$$

$$Q^*=\frac{\gamma S^*}{d+\mu}$$

Smokers Reproductive Number:
The smokers reproductive number (basic reproductive number) (R_0) is the threshold condition in epidemiology. It measure the average number of new smokers generated by single smoker in a population of potential smokers (Zaman, 2009). By using the next generation method and used spectral radius (Van den Driessche and Watmough,2002). Doing this, we rewritten the system in matrix form

$$\frac{dx}{dt} = F(x)-V(x) \text{ and } x=(P,L,S,Q)^T$$

We obtained,

$$F(x)=\begin{bmatrix}0\\ \beta(1-\omega)LP\\ 0\\ 0\end{bmatrix}, V(x)=\begin{bmatrix}-bN+\beta(1-\omega)LP+(d+\mu)P\\ \beta(1-\omega)LS+(d+\mu)L\\ -\beta(1-\omega)LS+(\gamma+d+\mu)S\\ -\gamma S+(d+\mu)Q\end{bmatrix}$$

Find the Jacobian matrix of $F(x)$ and $V(x)$ evaluated at $E_0=(N,0,0,0)$, we obtained,

$$V(E_0)=\begin{bmatrix}d+\mu & \beta(1-\omega)N & 0 & 0\\ 0 & d+\mu & 0 & 0\\ 0 & 0 & \gamma+d+\mu & 0\\ 0 & 0 & -\gamma & d+\mu\end{bmatrix} \qquad F(E_0)=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & \beta(1-\omega)N & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Find FV^{-1}, we get

$$FV^{-1}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & \frac{\beta(1-\delta)N}{d+\omega} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Thus, the spectral radius of FV^{-1} denoted by $\rho(FV^{-1})$

$$\rho(FV^{-1})=\frac{\beta(1-\delta)N}{d+\omega}$$

We obtained the smokers reproductive number as shown,

$$\Re_0=\sqrt{\frac{\beta(1-\delta)N}{d+\omega}}$$

Stability Analysis:
In this section, we show the stability of the model at both smoking- free equilibrium and smoking- present equilibrium. First, we show that the system (1)-(4) is local asymptotically stable. The stability of this system as shown in the follow theorem.

Theorem 1: The smoking-free equilibrium of the system (1)-(5) about the equilibrium E_0, is local asymptotically stable if $R_0<1$, and unstable if $R_0>1$.

Proof. Since $R_0 < 1$, we have the Jacobian matrix of the system (1)-(5) at $E_0(N,0,0,0)$ is

$$J_0 = \begin{bmatrix} -(d+\mu) & -\beta(1-\omega)N & 0 & 0 \\ 0 & -\beta(1-\omega)N-(d+\mu) & 0 & 0 \\ 0 & 0 & -(\gamma+d+\mu) & 0 \\ 0 & \omega & \gamma & -(d+\mu) \end{bmatrix}.$$

The eigenvalues of the Jacobian matrix J_0 are obtained by solving $\det(J_0-\lambda I)=0$. From this, we obtain the characteristic equation ,

$$(\lambda+d+\omega)^2(\lambda+\gamma+d+\mu)(\lambda+d+\mu-\beta(1-\omega)N) = 0$$

From the characteristic equation, we see that two eigenvalues are $\lambda_{1,2}=-(d+\mu)<0, \lambda_3=-(\gamma+d+\mu)<0, \lambda_4=-((d+\mu)-\beta N(1-\omega))$. The fourth eigenvalue will be negative if $(d+\mu)>\beta N(1-\omega)$ by the Routh- Hurwitz criteria(Alen,2006).

Theorem 2: The smokers-present equilibrium of the system (1)-(4) about the equilibrium E_1, is local asymptotically stable if $R_0>1$, and unstable if $R_0<1$.

Proof. Since $R_0>1$, we have the Jacobian matrix of the system (1)-(4) at $E_1(P^*,L^*,S^*,Q^*)$ is

$$J_1 = \begin{bmatrix} -\beta(1-\omega)L^*-(d-\mu) & -\beta(1-\omega)P^* & 0 & 0 \\ \beta(1-\omega)L^* & \beta(1-\omega)P^*-\beta(1-\omega)S^*-(d-\mu) & \beta(1-\omega)L^* & 0 \\ 0 & \beta(1-\omega)S^* & -\varepsilon & 0 \\ 0 & 0 & \gamma & -(d+\mu) \end{bmatrix}$$

Where P^*,L^*,S^*,Q^* are given by equation (6). The characteristic equation of Jacobian matrix at E_1, given by equations (1)-(4), becomes

$$(\lambda+d+\mu)(\lambda^3-X\lambda^2+Y\lambda-Z)= 0$$

where

$$X = A_1+A_2+A_3, Y = A_1A_2+A_1A_3+A_2A_3-A_4A_5-A_8A_9,$$

$$Z = A_1A_2A_3-A_4A_5A_6-A_7A_8A_9$$

with $A_1=-\beta(1-\omega)L^*-(d+\mu)$, $A_2=\beta(1-\omega)P^*-\beta(1-\omega)S^*-(d+\mu)$, $A_3=\beta(1-\omega)L^*-(\gamma+d+\mu)$, $A_4=\beta(1-\omega)S^*$, $A_5=-\beta(1-\omega)L^*$, $A_6=-\beta(1-\omega)L^*-(d+\mu)$, $A_7=\beta(1-\omega)L^*-(\gamma+d+\mu)$, $A_8=\beta(1-\omega)L^*$, $A_9=-\beta(1-\omega)P^*$.

The first eigenvalue is $\lambda=-(d+\mu)<0$. The other three eigenvalues of $\lambda^3-X\lambda^2+Y\lambda-Z=0$ will have negative real part if they satisfy the Routh-Hurwitz criteria (Marsden and McCracken,1976) as follows.

1. $X<0$, 2. $Z<0$, 3. $XY<Z$.

Numerical results:

The parameters used in the numerical simulation results are given in Table.1

Table 1: Parameter values in numerical simulations at smoking - free state.

Parameters	Descriptions	Values
b	Birth rate of human population	0.081 day^{-1}
N	Total number of population	700
μ	Natural death rate of population	0.081 day^{-1}
β	Transmission coefficient	0.00011
d	Smoking-related death rate	0.999 day^{-1}
γ	Recovery rate of human population	0.0001 day^{-1}
ω	Education campaign	0.001

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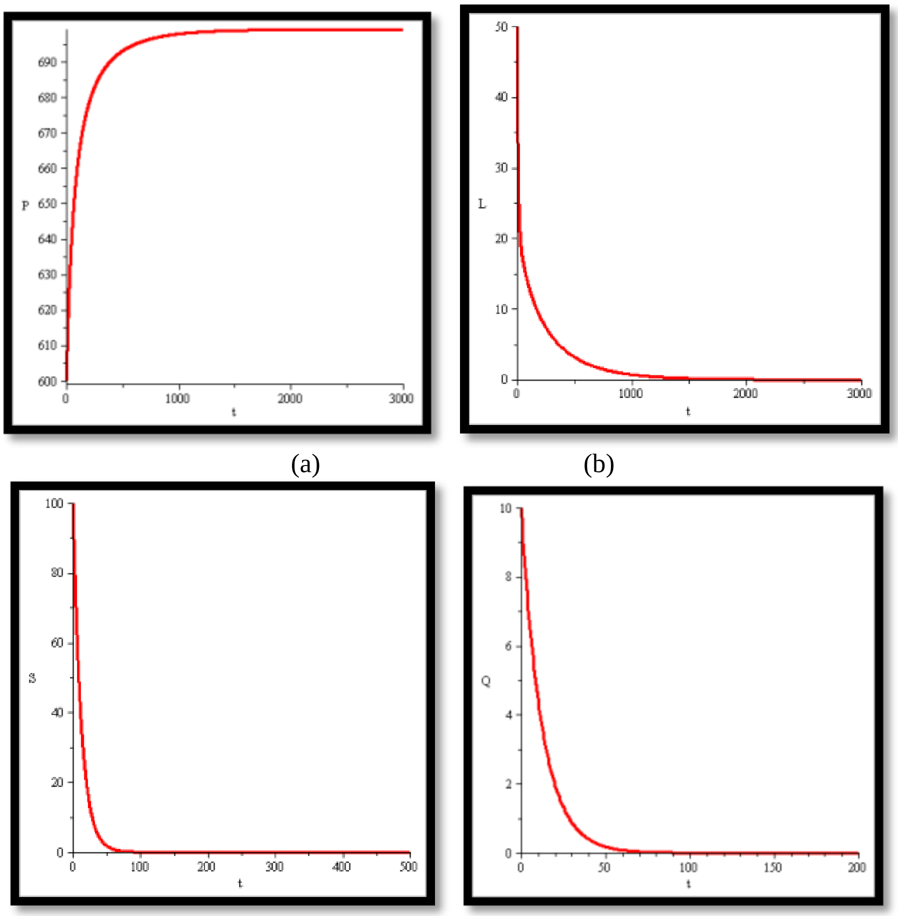
Stability of smoking- free state:

Using the values of parameters as shown in Table. 1. We obtained the eigenvalues and the smokers reproductive number as follows;

Since all eigenvalues are to be negative and the smokers reproductive number is less than one, the disease equilibrium, E_0, will be local asymptotically stable, as shown in Fig. 2

$\lambda_1 = -0.08121,\ \lambda_2 = -0.08111,\ \lambda_3 = -0.08111,\ \lambda_4 = -0.0027,\ R_0 = 0.966708 < 1.$



(a) (b)

(c) (d)

Fig. 2: Time series of (a) Potential smokers (P), (b) Occasion smokers (L), (c) Everyday smokers (S) and (d) Quit smokers (Q) with the values of parameters;

$$N = 700,\ b = 0.081,\ \gamma = 0.00011,\ \beta = 0.00056,\ \mu = 0.081,$$

$$d = 0.0001,\ \omega = 0.80, \text{and}\ \Re_0 = 0.966708 < 1.$$

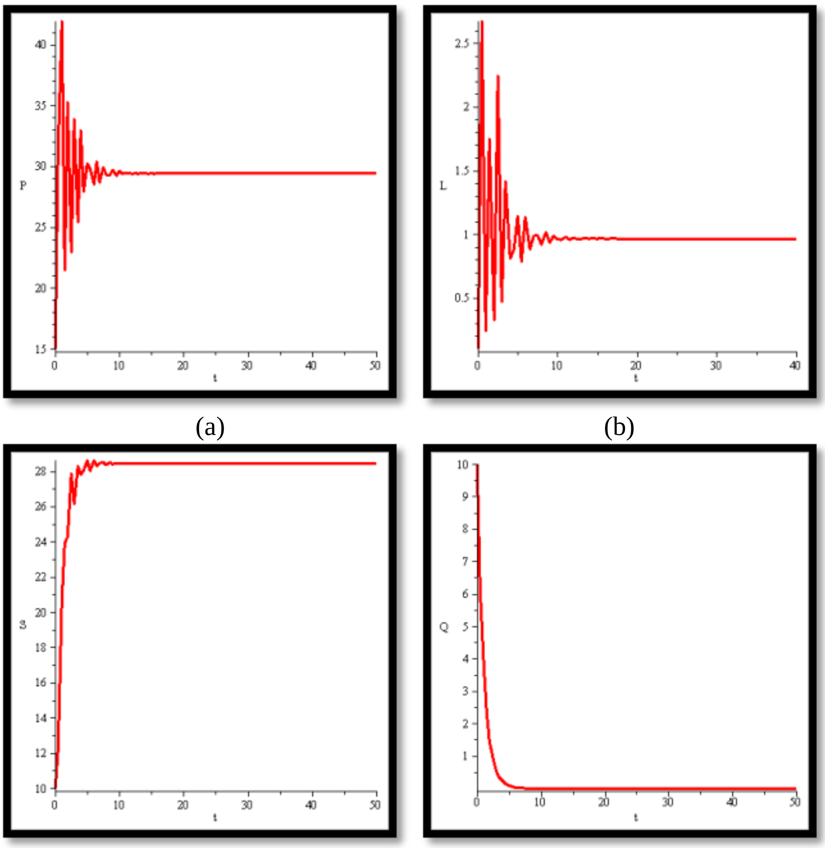
$E_0 = (700, 0, 0, 0)$. We see that the solutions approach to the smoking- free equilibrium

Stability of endemic state:

We change the value of the education program to $\omega = 0.001$ and keep the other values of parameters to be those given in Table. 1. We obtain the eigenvalues and the basic reproductive number as follows;

$\lambda_1 = -23.544, \lambda_2 = -4.65607, \lambda_3 = -0.964073, \lambda_4 = -0.964, R_0 = 724.69 > 1.$

Since all eigenvalues are to be negative and the basic reproductive number is greater than one, the endemic equilibrium state, E_1, will be local asymptotically stable as shown in Fig. 3.



(a) (b)

(c) (d)

Fig. 3: Time series of (a) Potential smokers (P) , (b) Occasion smokers (L) , (c) Everyday smokers (S) and (d) Quit smokers (Q) with the values of parameters;

$E_1 = (28.7384, 1.01099, 28.594, 0.00628).$

$N = 700,\ b = 0.081,\ \gamma = 0.00011,\ \beta = 0.00011,\ \mu = 0.081,$

$d = 0.999,\ \omega = 0.001, \text{and}\ \Re_0 = 724.69 > 1.$

The state variables approach to endemic equilibrium

Conclusion:

In this study, we presented the mathematical model of giving up smoking model with the effect of education campaign as a control measure and analyzed the analytical results by using standard modeling method. The smokers reproductive number is obtained through the use of spectral radius of the next generation matrix. The smokers reproductive number is $\Re_0 = \sqrt{\dfrac{\beta(1-\delta)N}{d+\omega}}$. The smokers reproductive number is the threshold condition for determining the stability of the equilibrium points of the model which are shown in Fig. 2 and Fig. 3. Our simulation results shown that $\Re_0$ will be increase when the effective of education campaign decrease. We found the value of $\Re_0$ were $0.966708, 724.69$ when $\omega = 0.8, 0.001$, respectively. It seen that the infected humans will increase when the effective of education campaign is decreased.

ACKNOWLEDGEMENTS

The author is grateful to the Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University for providing the facilities to carry out the research. Finally, Surapol Naowarat would like to thank Prof. Dr. I-Ming Tang for valuable suggestions.

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