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The Transmission Dynamics of Denque Fever with Optimal Control Theory

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ABSTRACT

Background: In this work the dynamics transmission of denque fever was proposed. The standard method is used to analyze the behaviors of the proposed model. After that the model is modified by adding optimal control functions that includes two time – dependent control functions with one minimizing the contract between the susceptible human and the infected vector and the other, minimizing the population of the infected human. **Results:** The results shown that there were two equilibrium points; disease free and endemic equilibrium point. The qualitative results are depended on a basic reproductive number . We obtained the basic reproductive number by using the next generation method and finding the spectral radius. Routh-Hurwitz criteria is used for determining the stabilities of the model. If , then the disease free equilibrium point is local asymptotically stable: that is the disease will died out, but if , then the endemic equilibrium is local asymptotically stable. **Conclusion:** Simulation results indicate that the numbers of susceptible individuals be decreased after control strategies and also the number of infectious individuals and infectious vector are decreased after control.

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INTRODUCTION

In recent years, the transmission of Denque fever has increased predominantly in urban and semi-urban areas and has become a major international public health concern. Severe dengue (also known as Dengue Haemorrhagic Fever) was first recognized in the 1950s during dengue epidemics in the Philippines and Thailand. Today, severe dengue affects most Asian and Latin American countries and has become a leading cause of hospitalization and death among children in these regions (WHO) (2014). Estimated about 50 to 100 million cases reported. Around 500,000 people are estimated to be infected by hemorrhagic dengue fever each year. Four serotypes of the dengue virus exist and are called DEN1, DEN2, DEN3 and DEN4. Infection by one serotype of the virus confirms permanent immunity to further infections by the infecting strain and temporary immunity to the others. The disease is usually found in tropical region of the world. This disease can be transmitted to human by biting of infected *Aedes Aegypti* mosquitoes (WHO, 2004). This mosquito breeds in artificial water containers such as discarded tires, cans, barrels and flower vases, all of which are usually found in the domestic environment. Dengue fever (DF) is characterized by the rapid development of the illness that may last from five to seven days with headache, joint and muscle pain and a rash (Naowarat, 2011). The symptom of dengue patients may occur from four to twelve days after exposure to an infected mosquito. Mathematical models have become an important tool for understanding the spread and control of disease. Because of this disease is caused by virus, therefore no drug can cure this disease specifically. This paper is organized as follows. In section 2, we present the dynamics of the model for denque fever. The standard method is used to analyze the behaviors of the proposed model. The analysis of optimization problem is presented in section 3. In section4, we give a numerical appropriate method and the simulation corresponding results. Finally, the conclusions be summarized in section 5.

2. Mathematical model:

In this paper, we study the transmission of denque fever through mathematical modeling. By using the standard method to analyze the behaviors of the proposed model which was adopted (Naowarat, 2011). The population are divided into two groups: human population N and population vector N_v .We assume that human population and mosquito population are constant. Human population divided into two disease-state compartments: susceptible individuals $\bar{\mathrm{S}}$, people who can catch the

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disease. Infectious (infective) individuals $\bar{I}$, people who have the disease and can transmit the disease. Population vector or mosquitoes N_v are divided into two groups of mosquitoes: the susceptible mosquitoes population $\overline{S_v}$, and the infected mosquitoes population $\overline{I_v}$. In this study, we assumed that there are numbers of people in the populations that have already infected by the virus while others have not. It is also assumed that the transmission of the virus continues in the population but number of mosquitoes as the vector is constant. People and mosquitoes are categorized in one group at a time. Then we obtained the transmission model as shown by a system of ordinary differential equations as follows;

$$\begin{aligned}
\frac{d\bar{S}}{dt} &= \lambda N - \frac{\beta c}{N+m}\bar{S}\overline{I_v} - \mu\bar{S} \\
\frac{d\bar{I}}{dt} &= \frac{\beta c}{N+m}\bar{S}\overline{I_v} - (\mu + r)\bar{I} \\
\frac{d\overline{S_v}}{dt} &= A - \frac{\beta_v c}{N+m}\overline{S_v}\overline{I_v} - \mu_v\overline{S_v} \\
\frac{d\overline{I_v}}{dt} &= \frac{\beta_v c}{N+m}\overline{S_v}\overline{I_v} - \mu_v\overline{I_v}
\end{aligned} \qquad (1)$$

Where; $N = \bar{S} + \bar{I}$ and $N_v = \overline{S_v} + \overline{I_v}$

$\bar{S}(t)$ is the susceptible human population at time t

$\bar{I}(t)$ is the infected human population at time t

N is the total number of human population ,

A is the recruitment rate of mosquito population

$\bar{S}_\upsilon(t)$ is the susceptible mosquitoes population at time t

$\bar{I}_\upsilon(t)$ is the infected mosquitoes population at time t

N_υ is the total number of mosquitoes population ,

c is the biting rate of the mosquito,

μ is the death rate of human population ,

μ_υ is the death rate of mosquitoes population ,

m is the number of other animals that the mosquitoes can feed on,

r is the recovery rates of human population

At first we have to show both the invariant region and the positivity of the solutions. Therefore ;

$N+N_v = \bar{S} + \bar{I} + \overline{S_v} + \overline{I_v}$, then $\frac{d(N+N_v)}{dt} = \frac{d\bar{S}}{dt} + \frac{d\bar{I}}{dt} + \frac{d\overline{S_v}}{dt} + \frac{d\overline{I_v}}{dt}$ .We consider into three cases; such as $N = N_v, N > N_v$ and $N < N_v$,for $t \to \infty$,then we got $0 \le N < \infty$ .Next for the positivity of the solution; Let's consider the first equation; $\frac{d\bar{S}}{dt} = \lambda N - \frac{\beta c\bar{S}}{N+m}\overline{I_v} - \mu\bar{S}$;

$$\frac{d\bar{S}}{dt} \ge -\left(\frac{\beta c}{N+m}\overline{I_v} + \mu\right)\bar{S};$$

$$\int\frac{d\bar{S}}{\bar{S}} \ge -\int\left(\frac{\beta c}{N+m}\overline{I_v} + \mu\right)dt;$$

$$\bar{S}(t) = S(0)\,e^{-\int\left(\frac{\beta c}{N+m}\overline{I_v}+\mu\right)dt} > 0.$$

In the same method for $\bar{I}(t), \overline{S_v}(t)$ and $\overline{I_v}(t)$, we can see that the solution set of model system is positive for all $t > 0$.

We normalize the system of equations (1) by letting $S = \frac{\bar{S}}{N}, I = \frac{\bar{I}}{N}$, $S_v = \frac{\overline{S_v}}{N_v} = \frac{\overline{S_v}}{A/\mu_v}$,and $I_v = \frac{\overline{I_v}}{N_v} = \frac{\overline{I_v}}{A/\mu_v}$ then we get; The system of equations (2) as fallow:

$$\frac{dS}{dt} = \mu(1-S) - \frac{A\beta cSI_v}{\mu_v(N+m)}$$
$$\frac{dI}{dt} = \frac{A\beta cSI_v}{\mu_v(N+m)} - (\mu+r)I \quad (2)$$
$$\frac{dI_v}{dt} = \frac{\beta_v cN(1-I_v)I}{N+m} - \mu_v I_v$$

Analysis of the model:
2.1 Basic properties of the model:

The equilibrium points for (S^*, I^*, I_υ^*) are found by setting the right hand side of each equations (2) equal to zero. We obtained two equilibrium points as follows;

2.1.1 Disease Free Equilibrium Point (E_0) ***:***

In the absence of the disease in the community, there are $I=0$ and $I_v = 0$, we obtained $S=1$,then the disease free equilibrium is $E_0(1,0,0)$.

2.1.2 Endemic Equilibrium Point (E_1) ***:***

In case the disease is presented in the community, $I > 0$ and $I_v > 0$, we obtained, $E_1(S^*, I^*, I^*_\upsilon)$ where;

$$S^* = \frac{\beta + M}{(\beta + MR_0)},$$
$$I^* = \frac{R_0 - 1}{(\beta + MR_0)},$$
$$I_\upsilon = \frac{\beta(R_0 - 1)}{R_0(\beta + M)}$$

Where;

$$\beta = \frac{b\beta_v N}{\mu_v(N+m)}, M = \frac{r+\mu}{\mu}$$

and
$$R_0 = \frac{c^2\beta\beta_v cNA/(\mu_v)}{(N+m)^2\mu_v(r+\mu)} \quad (3)$$

2.2 Basic Reproductive Number: (R_0)

We obtained a basic reproductive number by using the next generation method (van den Driessche and Watmough, 2002)[9]. By rewriting the equations (2) in matrix form ;

$$\frac{dX}{dt} = F(X) - V(X) \quad (4)$$

Where $F(X)$ is the non-negative matrix of new infection terms and $V(X)$ is the non-singular matrix of remaining transfer terms. And setting;

$$F = \left[\frac{\partial F_i(E_0)}{\partial X_i}\right] \text{ and } V = \left[\frac{\partial V_i(E_0)}{\partial X_i}\right] \quad (5)$$

Where F and V are the Jacobean matrix of $F(X)$ and $V(X)$ at E_0 for all $i, j = 1, 2, 3$. The basic reproductive number (R_0) is the number of secondary case generate by a primary infectious case (Andeson and May, 1991; van den Driessche and Watmough, 2002) or basic reproductive number is a measure of the power of an infectious disease to spread in a susceptible population. It can be evaluated through the formula;

$$\rho\left(FV^{-1}\right). \quad (6)$$

Where FV^{-1} is called the next generation matrix and $\rho\left(FV^{-1}\right)$ is the spectral radius (largest eigenvalues norm) of FV^{-1}. Then we get the reproduction number R_0 where ,

$$R_0 = \frac{c^2\beta\beta_v cNA/(\mu_v)}{(N+m)^2\mu_v(r+\mu)} \quad (7)$$

Finally, Routh-Hurwitz criteria is used for determining the stabilities of the model. . If $R_0 < 1$, then the disease free equilibrium point is local asymptotically stable: that is the disease will died out, but if $R_0 > 1$, then the endemic equilibrium is local asymptotically stable. Optimal control is the standard method for solving dynamic optimization problems ,when those problems are expressed in continuous time (Lenhart and workman,2006). In this paper ,we use this method as part of control measures for denque fever epidemics. Into the system of equations (2), we include two controls a and b that represent, the effort that reduces the contract between the infectious vector and the susceptible individuals and also to reduces the infectious human, respectively. The mathematical system with controls is given by the nonlinear differential equations subject to non-negative initial conditions as the following;

$$\frac{dS}{dt} = \mu(1-S) - \frac{A\beta c(1-a)}{\mu_v(N+m)}SI_v$$
$$\frac{dI}{dt} = \frac{A\beta c(1-a)}{\mu_v(N+m)}SI_v - (\mu+r)bI$$
$$\frac{d\overline{I_v}}{dt} = \frac{\beta_v cN}{N+m}(1-I_v)bI - \mu_v I_v \quad (8)$$

With initial conditions ; $S(0) \geq 0, I(0) \geq 0$ and $I_v(0) \geq 0$.

Optimal control for the dynamics of denque fever model:

In this section we use the optimal control theory to analyze the behavior of the system of equations (8) . The objective one is to minimize the susceptible human and the infected vector and the other is to minimize the population of the infected human. Mathematically, for a fixed terminal time t_f , the problem is to minimize the objective functional**;**

$$J(a,b)=\int_0^{t_f}\left[S(t)+I(t)+\frac{B_1}{2}a^2(t)+\frac{B_2}{2}b^2(t)\right]dt \quad (9)$$

The parameter $B_1 \geq 0$ and $B_2 \geq 0$ denote weights that balance the size of the terms for a fixed terminal time t_f. Hence we are interested in finding an optimal control pair a^* and b^* ,such that:

$$J\left(a^*,b^*\right)=\min\left\{J(a,b):(a,b)\varepsilon U\right\} \quad (10)$$

Where,

$$U=\{(a,b):0\leq a,b\leq 1, t\in[0,t_f], \text{a and b are Lebesque measurable}\}$$

Next, by applying the Pontryagin's Maximum Principle (Kirschner et al.,1997),we derive necessary conditions for our optimal control and corresponding state variables, including the two control functions. Therefore we have three corresponding adjoint variables where λ_1 corresponds to S , λ_2 corresponds to I , λ_3 corresponds to I_v .

3.1 The Hamiltonian adjoint equations:

The Hamiltonian equation is formed by allowing each of the adjoint variables that correspond to each of the state variables accordingly and combining the result with the objective functional as below:

$$H=S(t)+I(t)+\frac{B_1}{2}a^2(t)+\frac{B_2}{2}b^2(t)+\sum_{i=1}^{3}\lambda_i f_i \quad (11)$$

Where f_i is the right hand side of the differential equation of the i^{th} state variables. The adjoint equations are formed by taking the derivative of the Hamiltonian with respect to each of the state variables as follow; By applying the Pontryagin's maximum principle (Pontryagin, 1962) and the existence result of optimal control from ([] Raj joshi, 2006), we obtain the following theorem:

Theorem 1 There exists an optimal control $\left(a^*,b^*\right)\in U$ and corresponding solutions $S^*, I^* and I_\upsilon^*$ that minimize $J(a,b)$ over U .And there exists adjoint functions $\lambda_1,\lambda_2 and \lambda_3$ verifying;

$$\lambda_1'=-\frac{\partial H}{\partial S}=-1+\lambda_1\mu+\left(\frac{Ac\beta(1-a(t))I_\upsilon}{\mu_v(N+m)}\right)(\lambda_1-\lambda_2) ,$$

$$\lambda_2'=-\frac{\partial H}{\partial I}=-1+\lambda_2(\mu+r)b-\lambda_3\frac{c\beta N}{N+m}(1-I_v)b ,$$

$$\lambda_3'=-\frac{\partial H}{\partial I_v}=\lambda_3\left(\frac{b(t)c\beta N}{N+m}+\mu_v\right)+(\lambda_1-\lambda_2)\frac{Ac\beta(1-a(t))S}{\mu_v(N+m)} ,$$

With the transversality conditions $\lambda_1(t_f)=\lambda_2(t_f)=\lambda_3(t_f)=0$, and the optimize control $\left(a^*,b^*\right)$ is given by;

$$a^*=\min\left\{1,\max\left\{0,\frac{((\lambda_2-\lambda_1)(Ac\beta I_v^*)S^*}{B_1\mu_v(N+m)}\right\}\right\}$$

$$b^*=\min\left\{1,\max\left\{0,\frac{(\lambda_2(\mu+r)(N+m)-\lambda_3 c\beta N(1-I^*{}_\upsilon))I^*}{B_2(N+m)}\right\}\right\}.$$

Proof:

The existence of optimal control can be proved by using the results from (Pontryagin, 1962). The adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum Principle such that

$$\lambda_1'=-\frac{\partial H}{\partial S}=-1+\lambda_1\mu+\left(\frac{Ac\beta(1-a(t))I_\upsilon}{\mu_v(N+m)}\right)(\lambda_1-\lambda_2) ,$$

$$\lambda_2'=-\frac{\partial H}{\partial I}=-1+\lambda_2(\mu+r)b-\lambda_3\frac{c\beta N}{N+m}(1-I_v)b ,$$

$$\lambda_3'=-\frac{\partial H}{\partial I_v}=\lambda_3\left(\frac{b(t)c\beta N}{N+m}+\mu_v\right)+(\lambda_1-\lambda_2)\frac{Ac\beta(1-a(t))S}{\mu_v(N+m)} ,$$

The optimal control pair $\left(a^*,b^*\right)$ are obtained by finding the derivative of the Hamiltonian equation with respect to the control variables, equating to zero, and solving equation. Then we get;

$$\frac{\partial H}{\partial a} = B_1 a(t) + \lambda_1 \frac{Ac\beta}{\mu_v (N+m)} SI_\upsilon - \lambda_2 \frac{Ac\beta}{\mu_v (N+m)} I_\upsilon S$$. Let;

$$B_1 a(t) + \lambda_1 \frac{Ac\beta}{\mu_v (N+m)} SI_\upsilon - \lambda_2 \frac{Ac\beta}{\mu_v (N+m)} I_\upsilon S = 0$$

Then the optimal value for a is;

$$a^* = \frac{(\lambda_2 - \lambda_1)(Ac\beta I_v^*)S^*}{B_1 \mu_v (N+m)}$$

And $\frac{\partial H}{\partial b} = B_2 b(t) + \frac{\lambda_3 c\beta N(1-I_v)I}{N+m} - \lambda_2 (\mu + r) I$. Let ; $B_2 b(t) + \frac{\lambda_3 c\beta N(1-I_v)I}{N+m} - \lambda_2 (\mu + r) I = 0$

Hence the optimal value for b is;

$$b^* = \frac{(\lambda_2 (\mu + r)(N+m) - \lambda_4 c\beta N(1-I_\upsilon))I^*}{B_2 (N+m)}$$

By the bounds in U of the control, the optimal control pair $\left(a^*, b^*\right)$ is given by;

$$a^* = \min\left\{1, \max\left\{0, \frac{(\lambda_2 - \lambda_1)(Ac\beta I_v^*)S^*}{B_1 \mu_v (N+m)}\right\}\right\}$$
$$b^* = \min\left\{1, \max\left\{0, \frac{(\lambda_2 (\mu + r)(N+m) - \lambda_4 c\beta N(1-I_v^*))I^*}{B_2 (N+m)}\right\}\right\}. \quad (12)$$

For supporting analytic results we need to resolve the optimal control model numerically.

Numerical simulations:

In this section we present the results obtained by solving numerically from the following optimality system;

$$\frac{dS}{dt} = \mu(1-S) - \frac{A\beta c(1-a^*)}{\mu_v (N+m)} SI_v$$
$$\frac{dI}{dt} = \frac{A\beta c(1-a^*)}{\mu_v (N+m)} SI_v - (\mu + r) b^* I$$
$$\frac{d\overline{I_v}}{dt} = \frac{\beta_v cN}{N+m}(1-I_v) b^* I - \mu_v I_v$$
$$\lambda_1' = -\frac{\partial H}{\partial S} = -1 + \lambda_1 \mu + \left(\frac{Ac\beta(1-a^*)I_\upsilon}{\mu_v (N+m)}\right)(\lambda_1 - \lambda_2) \text{ , (13)}$$
$$\lambda_2' = -\frac{\partial H}{\partial I} = -1 + \lambda_2 \left(\mu + r\right) b^* - \lambda_3 \frac{c\beta N}{N+m}(1-I_v) b^* \text{ ,}$$
$$\lambda_3' = -\frac{\partial H}{\partial I_v} = \lambda_3 \left(\frac{b^* c\beta N}{N+m} + \mu_v\right) + (\lambda_1 - \lambda_2) \frac{Ac\beta(1-a^*)S}{\mu_v (N+m)} \text{ ,}$$

With $S(0) = S_0, I(0) = I_0, and I_\upsilon(0) = I_{\upsilon_0}$ and $\lambda_i\left(t_f\right) = 0, \left(i = 1,2,3\right)$

Since ,there were initial condition for the state variables and terminal conditions for the adjoint functions and the optimality system is two-point boundary value problem, with separated boundary conditions at $t = 0$ and t_f . Then we use the semi-implicit finite difference method to solve the optimality system (13). We partition the interval $\left[t_0, t_f\right]$ at the point $t_i = t_0 + ih\left(i = 0,1,2,...,n\right)$, where h is the time step such that $t_n = t_f$. And we define the state and adjoint variable $S\left(t\right), I(t), I_\upsilon(t), \lambda_1, \lambda_2, \lambda_3$ and the control a and b in terms of nodal points $S_i, I_i, I_{\upsilon_i}, \lambda_1^i, \lambda_2^i, \lambda_3^i, a^i$ and b^i . Then we use a combination of forward and backward difference approximation as follows:

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$$\frac{S_{i+1}-S_i}{h}=\mu\left(1-S_{i+1}\right)-\frac{Ac\beta(1-a^i)S_{i+1}I_{v_i}}{\mu_v(N+m)}$$

$$\frac{I_{i+1}-I_i}{h}=\frac{Ac\beta(1-a^i)S_{i+1}I_{v_i}}{\mu_v(N+m)}-\left(\mu+r\right)b^iI_{i+1}$$

$$\frac{I_{\upsilon_{i+1}}-I_{\upsilon_i}}{h}=\frac{c\beta N(1-I_{\upsilon_{i+1}})b^iI_{i+1}}{(N+m)}-\mu_\upsilon I_{\upsilon_{i+1}}$$

And also by using above technique, we approximate the time derivative of the adjoint variables by their first-order backward-difference as the

$$\frac{\lambda_1^{n-i}-\lambda_1^{n-i-1}}{h}=-1+\mu\lambda_1^{n-i-1}+\frac{(Ac\beta(1-a^i)I_{\upsilon_{i+1}})}{\mu_v(N+m)}(\lambda_1^{n-i-1}-\lambda_2^{n-i}),$$

following; $\frac{\lambda_2^{n-i}-\lambda_2^{n-i-1}}{h}=-1+\lambda_2^{n-i-1}\left(\mu+r\right)b^i-\frac{\lambda_3^{n-i}c\beta N(1-I_{v_{i+1}})b^i}{N+m}$

$$\frac{\lambda_3^{n-i}-\lambda_3^{n-i-1}}{h}=\lambda_3^{n-i-1}(\frac{c\beta Nb^iI_{i+1}}{N+m}+\mu_v)+(\lambda_1^{n-i-1}-\lambda_2^{n-i-1})S_{i+1}\frac{Ac\beta(1-a^i)}{\mu_v(N+m)}$$

The algorithm for the approximation method to obtain the optimal control as follows;

Algorithm:

Step 1: $S\left(0\right)=S_0, I\left(0\right)=I_0, I_\upsilon(0)=I_{\upsilon_0}, \lambda_i\left(t_f\right)=0\left(i=1,2,3\right)$ and $a(0)=b(0)=0$

Step 2: For $i=0,\ldots,\mathrm{n}-1,$ do

$$S_{i+1}=\frac{S_i+h\mu}{1+h(1+\frac{Ac\beta I_{v_i}(1-a^i)}{\mu_v(N+m)})}$$

$$I_{i+1}=\frac{(\frac{hAc\beta(1-a^i)S_{i+1}I_{v_i}}{\mu_v(N+m)})+I_i}{1+h(\mu+r)b^i}$$

$$I_{\upsilon_{i+1}}=\frac{(\frac{hc\beta Nb^iI_{i+1}}{N+m}+I_{\upsilon_i})}{(1+h(\frac{c\beta Nb^iI_{i+1}}{N+m}+\mu_\upsilon))}$$

$$\lambda_1^{n-i-1}=\frac{\lambda_1^{n-i}+h(1+\frac{(Ac\beta I_{\upsilon_{i+1}})(1-a^i)\lambda_2^{n-i}}{\mu_v(N+m)})}{h[\mu+\frac{Ac\beta(1-a^i)I_{\upsilon_{i+1}}}{\mu_v(N+m)}]+1}$$

$$\lambda_2^{n-i-1}=\frac{\lambda_2^{n-i}+h\left(1+\frac{c\beta Nb^i(1-I_{\upsilon_{i+1}})\lambda_3^{n-i}}{(N+m)}\right)}{1+h\left(\mu+r\right)b^i}$$

$$\lambda_3^{n-i-1}=\frac{\lambda_3^{n-i}-\frac{h(\lambda_1^{n-i-1}-\lambda_2^{n-i-1})Ac\beta(1-a^i)S_{i+1}}{\mu_v(N+m)}}{1+(\frac{h(c\beta Nb^iI_{i+1}}{(N+m)}+\mu_v)}$$

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$$M^{i+1} = \frac{(\lambda_2^{n-i-1} - \lambda_1^{n-i-1})(Ac\beta I_{v_{i+1}})S_{i+1}}{B_1\mu_v(N+m)}$$

$$T^{i+1} = \frac{(\lambda_2^{n-i-1}(\mu+r)(N+m) - \lambda_3 c\beta N(1-I_{v_{i+1}}))I_{i+1}}{B_2(N+m)}$$

$$a^{i+1} = \min\left\{1, \max\{0, M^{i+1}\}\right\}$$

$$b^{i+1} = \min\left\{1, \max\{0, \mathrm{T}^{i+1}\}\right\}$$

End for

Step 3:

$$S^*(t_i) = S_i, I^*(t_i) = I_i, I_\upsilon^*(t_i) = I_{\upsilon_i} a^*(t_i) = a^i \text{ and } b^*(t_i) = b^i$$

End for

The simulations at endemic state were carried out using the following values taken from table1, with initial condition; $S(0) = 0.0125, I(0) = 0.001, and I_\upsilon(0) = 0.005$ and results show below;

Table 1: Parameters values used in numerical simulation at endemic state.

Value	Description	Parameters
0.0000456 day^{-1}	Death rate of human population	μ
0.2500 day^{-1}	Death rate of mosquito population	μ_υ
0.5000 day^{-1}	The biting rate of the mosquito population,	c
400-5000	The recruitment rate of mosquitoes	A
0.0000	the number of other animals that the mosquitoes can feed on,	m
0.1428 day^{-1}	the recovery rates of human population	r
0.7500	the sufficient rate of correlation from human to vector	β
1.0000	the sufficient rate of correlation from vector to human	β_υ
10000	Number of human populations	N

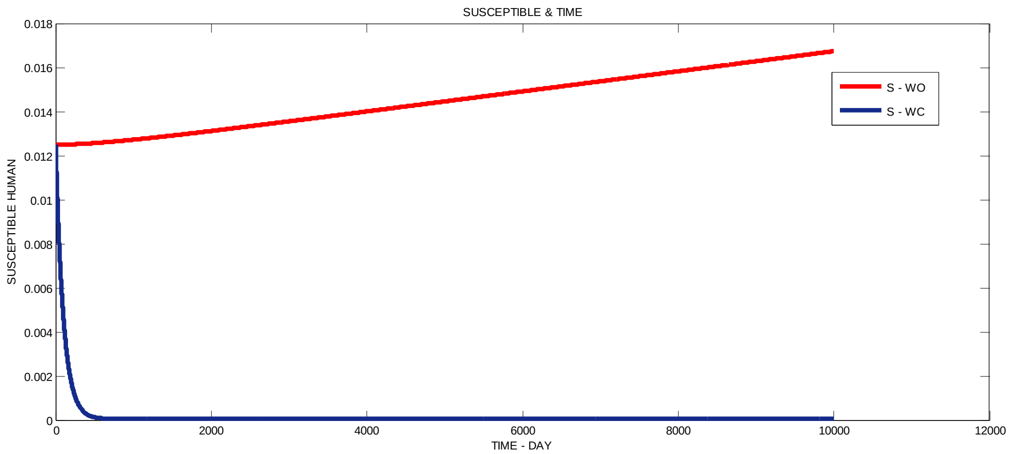

Fig. 1: Represent time series of susceptible individuals (S) with and without controls. It's show that the number of susceptible individuals (S) with controls (blue line) are decreased more than with out control.

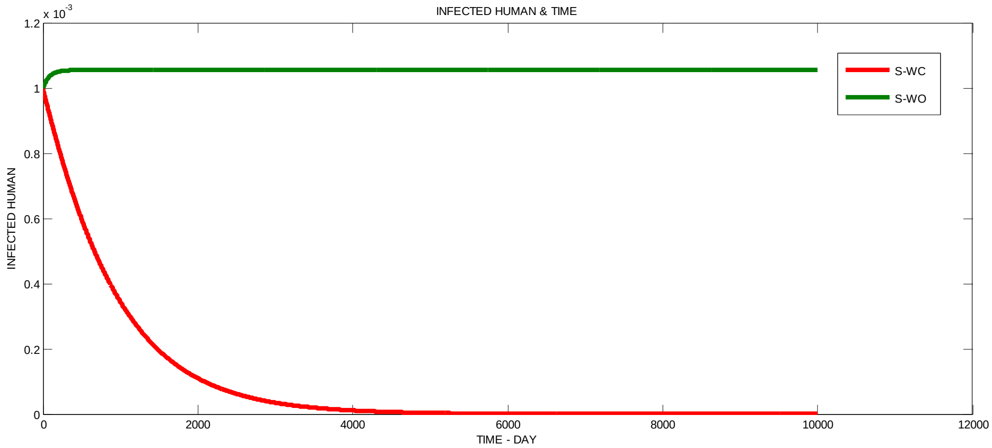

Fig. 2: Represent time series of infectious individuals (I) with and without controls.

It's show that the number of infected individuals (S) with controls(red line) are decreased more than with out control.

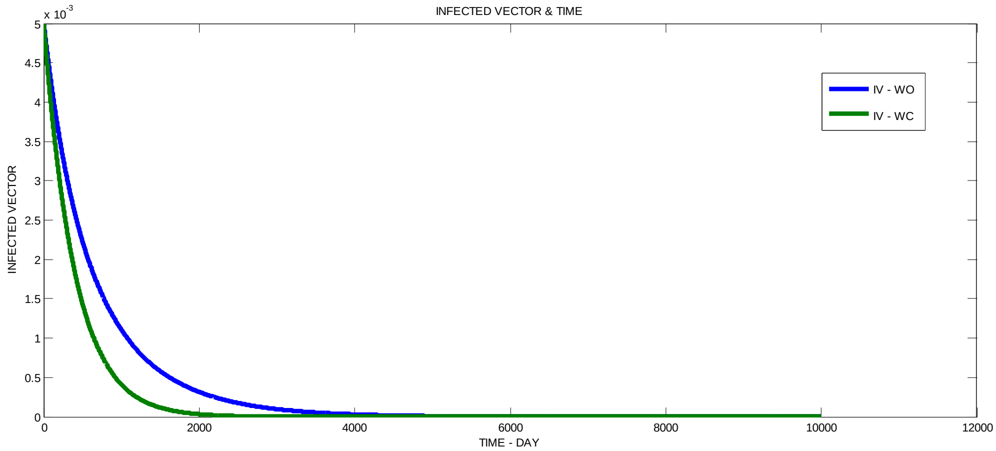

Fig. 3: Represent time series of infected vector (I_{υ}) with and without controls. It's show the number of infected vector (I_{υ}) with controls (green line) are also decreased.

Conclusion:

In this paper, the modified model for transmission of denque fever were proposed and analyzed. To reduce the number of infected human and the infected vector and the other, maximizing the population of the susceptible human. The optimal control strategy has been applied. By using the Pontryagin's maximum principle, the explicit expression of the optimal controls was obtained. Simulation results indicate that the numbers of susceptible individuals (S) be decreased after control, and also the number of infectious individuals (I) and infectious vector (I_v) compare with and without control. After control strategies are decreased.

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