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The Optimization Problem of Product-Mix and Linear Programing Applications; A single-Case Study in Tea Industry

[1]Hemendra Lal Gunasekaran, [2]Suhaiza Zainali and [3]Ali Haj Aghapour

[1]*Graduate School of Business, University of Malaya, Malaysia.*
[2]*Graduate School of Business, University of Malaya, Malaysia*
[3]*Graduate School of Management, Multimedia University, Malaysia*

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ABSTRACT

This paper propose a practical optimization problem of product-mix based on labor capacity, machine capacity, raw materials and demand constrains in a Sri Lankan tea producing company. The objective is to profit maximization, satisfying all constraints. In this paper, the problem is formulated as a linear programming model. As a case study, a software package (LINGO 9.0) is applied to solve the optimization problem. As with any LP model, the reduced costs for decision variables and dual prices and allowable fluctuation for constraints are used to conduct detailed sensitivity analysis. Eventually, this paper comes up with a set of policy making suggestions which might be helpful for the production planning and detailed scheduling.

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INTRODUCTION

Manufacturing sector all around the World continuously face to shortages of production outputs to satisfy market demands, especially for those adopting lean solutions such as “Just-in-Time”. In fact, “Production Planning& Detailed Scheduling” (PP/DS), based on time efficient methods such as is more likely to result in low capacity utilization and consequently low outputs. But, an economy can only grow if decision makers at the firm level result in boosted output through. Therefore, as a basic function of the management, planning which is the process of outlining the activities is necessary to achieve the goal of the organization (Lewis, Goodman and Fandt, 1995).

However, although planning is an essential managerial task for every functional area, the production process is an area in which operation planning is extensively involved due to its huge social and economic impact in terms of cost and energy and pollution reduction from a CSR perspective (Zhai Guan and Gao, 2010). From management perspective, PP/DS help decision makers to determine how organizational resources should be allocated and what activities should be assigned to individuals and workgroups to yield optimal performance. PP/D Senables firms to move towards cost minimization (Nicholls, 1996; Trick, 1994) or profit maximization (Levin and Wright, 2004) which entitled under the optimization problem. Decision makers are always seeking for the right decisions so as to meet their objectives which mainly revolve on how best to increase profit with scare resources at the tactical level of decision making.(Yingfeng and Dawei, 2011)

As the purpose of optimization is to ensure firms that their productive operations is effective enough to utilize the available resources in an effective manner, companies must develop a production process in which particular production resources are allocated to produce optimal number of units in each product. In this regard, the theory of constraints (TOC) is a production control methodology that improves the throughput of a system by effectively managing constrained resources. A constraint in a manufacturing firm shows that the system could not meet the demands of all the products. So, for exploitation of constraint, the product mix should be determined to maximize product throughputs (Coman and Ronen, 2000).The product-mix problem is defined as the determination of the best quantity of each product to manufacture, over a complete range of products competing for a number of limited resources(Hodges and Moore, 1970). The product-mix problem is a classical optimization problem, which involves the decision to select the volume and mix of products that maximize the profit within the constraints of demand and production resources.

Corresponding Author: Ali Haj Aghapour, Graduate School of Management, Multimedia University, Malaysia.
E-mail: Aghapourali@gmail.com

Decision makers are constantly are struggling with the product-mix problem, where they have to decide how much to produce for each range of possible items that being manufactured by the firm in order to maximize the revenue(Tanhaei and Nahavandi, 2013). In some circumstances, it may be most profitable to manufacture in large batches, building up considerable stocks of the finished products, rather than in small batches to satisfy the demand forecast over a short period (Hodges and Moore, 1970). However, the problem of product mix can be useful for aggregate planning, where proprietary software may be less affordable due to economic reasons, to help the managers in decision making.(Nikumbh et al., 2009)

Like most other developing countries of the World, Sri-Lanka is faced with the general bottlenecks. As a result of the high customers' expectations and the general nature of the scarcity of factors of production, the management of industries can hardly do without a well-articulated course of action in their PP/DS. Tea industry often determines a nation's economic strength and elevates the position of Sri Lanka among all tea producers. Sri Lanka, the third biggest tea producing country, has a production share of 9% in the international market, and one of the world's leading exporters with a share of around 19% of the global demand ("Seri Lanka Tea board", 2011). Therefore, tea industry is also crucial to enhance their economic competitiveness in the world market. The nature of the highly competitive global market has made scientific and reasonable production management increasingly important for tea companies to differentiate themselves from competitors.In order to enhance their competitive position,"*Sri-Lankantea manufacturers have been considering optimization solutions in their own PP/DSs",* an interviewee said.

With this regard, Dulwan Tea Company of Sri Lanka- a nick name- was selected as the case company for this study. Although many tea manufacturers are operating in Sri Lanka, Dulwan Tea Company, established in 1974, is be considered as not only one of the leading manufacturers but also a leading tea exporting company in the Sri Lankan tea industry. They use their own tea leaves which are being grown in their tea plantations. More than 2,500 flavoured and non-flavoured tea products are being produced and globally exported by this company. This brand is available in over 90 countries including the United Kingdom, Poland, Canada, South Africa, Australia and New Zealand. Therefore, how to optimize the production process yielding maximum profit is a crucial and challenging problem faced by Decision makers at Dulwan Tea Company especially as optimization problems of production are crucial for firms to improve their competitiveness and profitability due to efficiency improvement (Yingfeng and Dawei, 2011). However, the research objective can be technically phrased as follow;

- To formulate a mathematical model that would suggest a viable product- mix to ensure maximum profit for Dulwan Tea Company as well as evaluating performance of the proposed product-mix mode.
- To highlight the peculiarities of using linear programming technique at a single operating procedure and prove that despite the obstacles, the application of the technique in determining the product-mix enables Dulwan Tea Company to be more profitable than otherwise.

Bearing these objectives in mind, a post optimal analysis of the developed profit maximization model would be attempted to help manufacturing decision makers to; firstly, adjusting their decisions in the face of increases or decreases in demand, resource surplus or shortage and availability of raw materials. Secondly, despite the obstacles, evaluate whether the applications of LP in determining the product-mix and constrain allocations lead theDulwan Tea Company tobe more profitable than otherwise. Finally, from the findings of the study, suggestions of how linear programming method could be widely applied in business decision making process.

This paper is organized as follows; Section 2 provides a briefed literature review of prior work on the optimization problem of product mix using LP as well as describing our problem scenarios. In Section 3,after designing our single-case research design, we introduce the "LINGO" Optimization Modeling Software to formulate the proposed LP model. In section 4, we first define our problem notations, and then provide details of the mathematical formulations for the model. We then illustrate numerical results of the optimization model in Section 5. We discuss the linear programing applications resulted from running the software in section 6. Last but not the least; we conclude this paper with some managerial implications and suggestions for policy making.

Literature Review:

The product-mix strategy plays a vital role in the manufacturing sector(Kahane, 1977;Bengtsson and Olhager, 2002; Ramirez-Beltran, 1995).A product mix, also known as product assortment, is the set of product items that a particular seller or producer offers for sale to buyers. The optimal product-mix composition enables the managers to solve the major problems of profit maximizing(Chou and Hong, 2000; Moon, 1989) and cost minimizing(Ramirez-Beltran, 1995).In fact, the product-mix adjustments guide managers to reduce the volume of some products, to maximize the revenue by producing the product with high profitability. Problems identified in the manufacturing sector, such as resource allocation (Righter, 1989), optimal portfolio selection in service sector (Kahane, 1977), and satisfying the market-demand (Byrd and Moore, 1978) are overcome by the optimal product-mix strategy. However, there are several solution techniques that have been applied to the product-mix problems. These common solution techniques include the optimization and the analytic hierarchy process (AHP) techniques. Optimization techniques such as the linear programming (LP) and integer programming (IP) are commonly applied in various problems.

In applying product-mix strategy, an allocation problem, when there are a number of activities to be performed, alternative ways of doing them, and limited resources or facilities for performing each activity in the most effective way, the management is faced with the problem of how best to combine these activities and resources in an optimal manner so that the overall efficiency is maximized. According to Cooper and Henderson (1956), this is known as optimization problem, and can be approached using mathematical programming. They further refer to linear programming as a uni-objective constrained optimization technique. This is because, according to them, it seeks a single objective of either minimizing or maximizing unknown variables in a model. In line with this, Gupta and Hira (1992) argue that linear programming deals with linear optimization of a function of variables known as objective function subject to set of linear equations and /or inequalities known as constraints. The objective function may be profit, cost, production capacity or any other measure of effectiveness which is to be obtained in the best possible or optimal manner. The constraints may be imposed by different resources such as market demand, production process and equipment storage capacity, raw material available, and so on. They further posit that programming implies planning and by linearity is meant a mathematical expression in which the expressions among the variables are linear.

However, the LP technique has been frequently employed by several researchers (Hodges and Moore, 1970;Byrd and Moore, 1978;Manivel and Murugan, 2009)to determine the product-mix composition. LP is a model that consists of linear relationship representing a firm's decision(s), given an objective and resource constraints. LP is a technique or tool in operational research study for solving optimization problems. LP is applied in situation in which each product is manufactured using one unique operating procedure (Letmathe and Balakrishnan,2005). In 1993, Dantzig developed an efficient method, the simplex algorithm (Dantzig, 2002), for solving LP problems. Since the development of the simplex algorithm, LP has been used to solve optimizations problems in industries as diverse as banking, education, forestry, petroleum, and trucking.Wayne (1994)

Hodges and Moore (1970) employed LP to determine the product-mix in their study. Their effort is to formulate the next twelve-month production plan. First, the product-mix was calculated assuming the demand is known exactly. Then, they continue with the technique known as linear programming under uncertainty. The objective function is to maximize profit, while obtaining the optimal product-mix for next year production planning under the uncertainty of demand constraint. In this study, unspecified eleven products are used as an example to get the product-mix under the uncertainty of demand. Later, Byrd and Moore (1978) use LP in their study to determine the product-mix of 29 unnamed products. The objective function here is to maximize profit under the capacity constraint, demand constraint and raw materials availability. As a result of this study, the management of the company considered (an American company) to narrow down their production (specializing on high-profit items only) and removed the low profit items from the list of production. A substitute product was to be offered to the company's customers instead. Linear programming was used by Letmathe and Balakrishnan (2005)to determine the product-mix for the profit maximization while considering the emission allowance (constraint) as well as the resource constraints (raw material, machine capacity, and labour capacity), the cost constraint and the demand constraint were considered in addition to the emission constraint. For this study, they used a real world example of twelve product items. Finally, the appropriate product-mix was achieved which gains the maximum profit with satisfying the emission allowance and the other constraint functions. Murugan and Manivel (2009) determine the optimal product-mix for the purpose of profit maximization using LP. Raw material cost, labour cost, and overhead cost constraints are taken into consideration in their study. They (constraints) are concerned with textile and non-textile products in the clothes of Village Industries Commission in India. The product-mix model suggests doubling the quantum of production of silk khadi. That is, 1.926 times of the existing production volume would help maximize the profit to the tune of INR 24.752 million. The profit is about INR 2.442 million higher than that of the present profit. Furthermore, their results suggest more concentration on production of praying materials, in large quantity and to reduce production of other non-textile products.

From the aspect of capacity, limited labour hours and limited machine hours are normally considered in previous literature. By and large, the management of manufacturing companies must deal with scarce and limited resources to gain their goals and objectives. Due to the limitations on the resource capacity of a company, meeting the customers demand becomes difficult and hence reduces the opportunity to earn profits. (Tanhaei and Nahavandi, 2013)

Bengtson and Olhager(2002) determine the optimal product-mix with six products. In this study, the product-mix model was formulated subject to the capacity constraints of both labour and machine. Input of raw material for each production process is considered as a limited resource. Thus, raw material is also a constraint in the manufacturing process. In a study on an American manufacturing company, Byrd and Moore (1978) consider raw material as a critical constraint to determine the optimal product-mix composition. Maximum possible cost which can be incurred in the production process of a company is considered under cost constraints. Bayou and Reinstein(2005) considered the cost as a constraint in their profit maximization study. Their research effort is to rank the four product items which are considered in their study. Demand restrictions or market restrictions are normally considered as another constraint in the production planning. A market restriction was considered as a constraint by Koenigsberg (1961)in his study of product-mix determination in a plywood manufacturing company.

However, with respect to strong empirical and theoretical evidence as well as the emphasise of both line and sale decision makers of the Dulwan Tea Company during interview sessions, the current research has attempted to solve the product mix problem of a real- world case with 25 discrete variables in accordance with LP and TOC in order to gain profit maximization in subject to 4 constrains of machine capacity, labor capacity, raw materials availability and demand limitations.

Research Methodology:

Dulwan Tea Companyis chosen for this case-study for two main reasons. First, it uses the trial-and-error method in arriving at major management decisions even when the R&D department feels that a linear programming approach would have given a better result. Secondly, Dulwan Tea Company produces twenty five different products which makes the determination of the quantity combinations of the products produced (product mix) an important and major management decision.

This case-studyis designed to cover one month, 1st to 31st May, 2012. Researchers have investigated the overall quantity combination of the twenty five products produced by Dulwan Tea Company, Sri Lanka during the research period and the allocation of resources to the various products. This has been made possible by the records (e.g. bill of materials, process specifications, and sale report) kept by the Production and the Sales Departments relating to the different items produced by the firm. The researchers also had personal interview with Production Line Manager, R&D Manager and Sale Manager in order to gain some primary source of data especially in the step of problem definition.

Furthermore, the activities involved in this research design can be categorized into several steps which are presented in Figure 3.1. In the first step, identifying a problem related to the study area, investigating that problem and defining the problem identified are involved. In the second step, similar previous studies on product-mix problem and their solution methods are studied for the purpose of identifying the appropriate technique to solve the problem. The next step deals with the data collection. For this purpose, a case company is selected and data are collected to solve the problem identified. Developing the LP model to determine the optimal product-mix composition is consisted in the fourth step.

The next step, the model was implemented in "LINGO" by LINDO Systems. "LINGO" Optimization Modeling Software is a powerful tool for building and solving mathematical optimization models. "LINGO" package provides the language to build optimization models and the editor program including all the necessary features and built-in "solvers" in a single integrated environment. "LINGO" is designed to model and solve linear, nonlinear, quadratic, integer and stochastic optimization problems (“Lindo”, 2012). Studying the solution and determining the optimal product-mix composition are considered on the solution step. Finally, the model is evaluated by comparing its output to the actual data as well as performing the sensitivity analysis to come up with some managerial insights driven from the outputs.

Problem Definition and Mathematical Formulation:

The mathematical model presented in this paper shows how important resource constraints can be included in OPS on a regular and routine basis. In fact, the model can be used by firms to determine the optimal product mix and production quantities in the presence of production and demand constraints with several “what-if” scenarios. However, the model is comprehensive and incorporates several diverse issues such as:

- Multiple products, each with its own resource usage per unit of the product
- Product demands per month
- Finite resource availability
- Distinct resource costs

For each issue listed above, we present a real life situation to prove their practical relevance and applicability. The objective in the mathematical model is to maximize the firm’s profit duo to developing a maximization LP model as the nature of business prolongsa situation in which a unique production procedure is being used to product each production item (Letmathe & Balakrishnan, 2005). Therefore, the model assumes that each product has its own unique operating procedure, which we define as the parameters of a production process that uses a known amount of each resource and results in a specific production yield of that product.

The resulting mathematical model in this scenario is a linear program. The outputs of the model give the firm precise information regarding which products should be produced, and in what quantities. Furthermore, the model shows which production and demand constraints limit the firm’s profits so that the firm can carefully study these problem areas. Since the model calculates the amount of different products in advance, the results can also be used for comparison between optimal and actual amounts of production.

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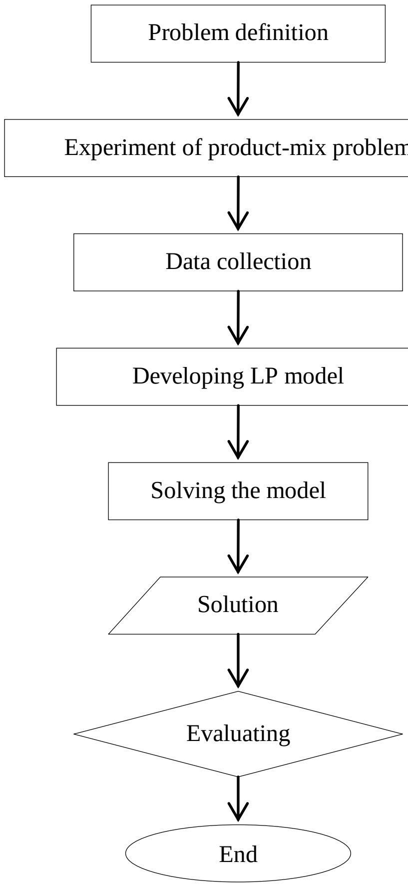

Fig. 1: Research Design.

Notation:

We first define the notations used in our model, and then use these notations to describe model formulation. All resource requirements and availabilities are measured in terms of the units for that specific resource (for example, machine time could be in hours, while raw material could be in kilograms).The notation used in the model is given as follows.

Parameters (for which values are known):

i = types of product, where i = 1, 2,…, n

p_i = profit per unit of product i,

t_i = machine time in minutes required for per unit of product i,

h_i = labour time in minutes required for per unit of product i,

r_i = raw material requirements for per unit of product i,

C = total capacity of machine time in term of minutes,

H = total available labour time (in minutes)

U_i = the upper demand limit for product i,

R = total amount of available raw material (in kilograms)

n = total product types

Decision variables (for which values are obtained from the model solution):

x_i = the number of units of product i to be manufactured

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Profit Maximization Linear Programing Model:

As noted earlier, the main focus of this work is to determine the quantity of each product that will maximize the profit of the company with the given constrains. The model assumes that each product has only a single operating procedure for its production that requires an exact amount of resources to produce each unit of each product (Letmathe and Balakrishnan, 2005). Therefore, linear programming was applied to determine an optimal product-mix.

From mathematical perspective, where there are variables (product) and j constraints (resource), the linear programming takes the general form with a linear objective function, a set of linear inequality constraints and a set of non-negativity restrictions as its major ingredients. In our model, C_j, H_j, R_j, are given production constrains, U_i is maximum monthly demand for product i , and the product $x_1, x_2, \ldots, x_n$ are called variables.

The problem is to find the values of the decision variables which maximize the objective function Z subject to the j constraints and the non-negativity restriction on the x_i variable. The resulting set of decision variables which maximize the objective function is called the optimal solution. However, the linear programming problem developed here is a mathematical program in which the objective function is to maximize profit which may express as follows:

Profit Maximization (Z):

$$\sum_{i=1}^{n} p_i x_i \text{ where } i = 1, 2, \ldots, n \tag{1}$$

Subject to the following constraints

Raw material availability:

$$\sum_{i=1}^{n} r_i x_i \le R \text{ where } i = 1,2,\ldots,n \tag{2}$$

Machine capacity:

$$\sum_{i=1}^{n} t_i x_i \le C \text{ where } i = 1,2,\ldots,n \tag{3}$$

Labor capacity:

$$\sum_{i=1}^{n} h_i x_i \le H \text{ where } i = 1,2,\ldots,n \tag{4}$$

Demand limitations:

$$Xi \le Ui \text{ where } i = 1, 2,\ldots, n \tag{5}$$

Non negativity constraint:

$$x_i \ge 0, \tag{6}$$

Data Analysis using LINDO:

As mentioned earlier, the data collected from Dulwan Tea Company, Sri Lanka on her main product line were analyzed to determine the best sales package that would yield maximum profit to the company. The various estimated values of the optimization model for the case company are presented in several below tables in order to simplify the findings.

But, we initially need to calculate and then insert the inputs. Table 1 illustrates the input data for one unit of each product which are used for formulating the model.

Available raw material (tea leaf) for the period concerned (one month) is at most 175,000 kilograms given by either the Production Manager or production records. Leaf tea products do not contain small tea bags and they contain only particular weight of loose tea. Hence, g is not applicable to calculate the quantity of raw material for each leaf tea product. However, the below formula was used to calculate the raw material needed for one unit of each product.

$$q = \frac{egf}{1000} \tag{7}$$

Where, q = quantity of raw material for one unit of each product (in kilograms)

e = Weight of one pack (leaf tea) or one tea bag (bagged tea)

f = number of packs in one unit
g = number of tea bags in one pack; not applicable when calculate the leaf tea products

Table 1: Input data.

Product	Raw material(kg)	Machine capacity(min)	Labour capacity(min)	Maximum average demand	Profit of a unit ($)
1	1.76	0.4	0.46	4,000	3.8
2	1.2	0.66	0.75	15,000	2.91
3	3	0.24	0.27	2,100	5.73
4	4.8	0.88	1	1,000	11.17
5	0.6	0.22	0.25	20,000	1.57
6	2.4	1.33	1.52	1,200	5.67
7	1.92	0.88	1	5,000	4.72
8	1.44	0.3	0.34	12,000	3.37
9	1.44	0.48	0.55	6,200	3.39
10	2.4	6.6	7.52	3,000	7.02
11	0.6	0.34	0.39	14,000	2.24
12	0.6	0.83	0.95	8,200	3.54
13	0.6	0.45	0.51	16,000	1.95
14	1.5	0.48	0.55	5,000	4.6
15	0.75	0.66	0.75	12,000	2.52
16	0.24	0.24	0.27	22,000	3.27
17	0.75	0.9	1.03	6,500	3.66
18	0.3	0.3	0.34	18,000	1.88
19	0.9	0.9	1.03	4,000	3.71
20	0.6	0.83	0.95	7,500	3.4
21	0.3	0.3	0.34	9,000	3.32
22	1.2	0.4	0.46	7,500	1.7
23	2	1.14	1.3	3,500	4.19
24	1.08	0.48	0.55	4,500	2.65
25	0.45	0.45	0.51	10,000	2.38
Resource availability	175,000	168,000	192,000		

Available machine time (machine capacity) of the company for the production process is 168,000 minutes in a month given by either the Production Manager or production records. However, this amount was computed using the formula three presented below:

$$C = (ajd)60 \tag{8}$$

Where, C = available machine time in the month (in minutes)
a = number of working hours per day (8 hours)
d = number of working days in the month (25 days)
j = number of machines in the process (14 machines)
Machine time for producing one unit of each product was calculated using the following formula:

$$t = \frac{60}{\left(\frac{b}{l}\right)} \tag{9}$$

Where, t = machine time for one unit of each product (in minutes)
b = one hour output of packs or tea bags in each product
l = number of packs or tea bags in one unit of each product

Available labor time (labor capacity) of the company for the production process is 192,000 minutes in a month. The following method was used to compute this amount:

$$H = (awd)60 \tag{10}$$

Where, H = available labour time in the month (in minutes)
a = number of working hours per day (8 hours)
d = number of working days in the month (25 days)
w = number of workers in a day (16 workers)

Machine time to labour time ratio (14:16) was used to calculate the labour time for one unit of each product. Therefore one machine hour equals to 1.14 labour hours (16/14). Thus, labour time for each product was calculated by multiplying the machine time needed for one unit of each product by 1.14. Therefore, its labour time equals:

$$t \times 1.14 \tag{11}$$

Where, t = machine time for produce one unit of each product (in minutes)

Finally, number of units in each product should be less or equal to their demand upper limits. The monthly average demands have been obtained through discussion with the sale manger or sales reports.

The optimization model was implemented in the “LINGO” environment. Model implementation is possible in two basic ways. The first way is to use the "LINGO" language of mathematical modelling. It is very useful where the number of decision variables and constraints is extremely big. The second way is to enter the model into the "LINGO" editor in the explicit form, that is, a full function of the objective with all the constraints, parameters, etc. It is an intuitive approach consistent with the standard form of linear programming, and it is practical in small optimization models (Sitek and Wikarek, 2012). With respect to the size of model, the construction and implementation of the model was being accomplished through the second method (i.e. necessary data were manually inserted). However, the resulting “LINGO” formulation of the model is being presented as follows:

Profit maximization (objective function):

$$3.80X_1+2.91X_2+5.73X_3+11.17X_4+1.57X_5+5.67X_6+4.72X_7+3.37X_8+3.39X_9+\&.02X_{10}+2.24X_{11}+3.54X_{12}+1.95X_{13}+4.60X_{14}+2.52X_{15}+3.27X_{16}+3.66X_{17}+1.88X_{18}+3.71X_{19}+3.40X_{20}+3.32X_{21}+1.70X_{22}+4.19X_{23}+2.65X_{24}+2.38X_{25} \quad (12)$$

Subject to

Raw material availability constrain:

$$1.76X_1+1.20X_2+3X_3+4.80X_4+0.60X_5+2.40X_6+1.92X_7+1.44X_8+1.44X_9+2.40X_{10}+0.60X_{11}+0.60X_{12}+o.60X_{13}+1.50X_{14}+0.75X_{15}+0.24X_{16}+0.75X_{17}+0.30X_{18}+0.90X_{19}+0.60X_{20}+0.30X_{21}+1.20X_{22}+2X_{23}+1.08X_{24}+0.45X_{25} \leq 175000 \quad (13)$$

Labor capacity constrain:

$$0.40X_1+0.66X_2+0.24X_3+0.88X_4+0.22X_5+1.33X_6+0.88X_7+0.30X_8+0.48X_9+6.60X_{10}+0.34X_{11}+0.83X_{12}+0.45X_{13}+0.48X_{14}+0.66X_{15}+0.24X_{16}+0.90X_{17}+0.30X_{18}+0.90X_{19}+0.83X_{20}+0.30X_{21}+0.40X_{22}+1.14X_{23}+0.48X_{24}+0.45X_{25} \leq 168000 \quad (14)$$

Machine capacity constrain:

$$0.46X_1+0.75X_2+0.27X_3+X_4+0.25X_5+1.52X_6+X_7+0.55X_9+7.52X_{10}+0.39X_{11}+0.95X_{12}+0.51X_{13}+0.55X_{14}+0.75X_{15}+0.27X_{16}+1.03X_{17}+0.34X_{18}+1.03X_{19}+0.95X_{20}+0.34X_{21}+0.46X_{22}+1.30X_{23}+0.55X_{24}+0.51X_{25} \leq 192000 \quad (15)$$

Table 2: LINDO solution for the Model.

variable	Value	Reduced cost
Objective function value 619161.7 $		
X1	4000	0
X2	15000	0
X3	1712.33	0
X4	1000	0
X5	20000	0
X6	1200	0
X7	5000	0
X8	12000	0
X9	6200	0
X10	3000	0
X11	14000	0
X12	8200	0
X13	16000	0
X14	5000	0
X15	12000	0
X16	22000	0
X17	6500	0
X18	18000	0
X19	4000	0
X20	7500	0
X21	9000	0
X22	0	0.814
X23	3500	0
X24	4500	0
X25	10000	0

RESULTS AND DISCUSSION

As mentioned earlier, the data collected from Dulwan Tea Company, Sri Lanka on her main product line were analyzed to determine the best sales package that would yield maximum profit to the company. The various estimated values of the optimization model for the case company are presented in several below tables in order to simplify then discuss the solution. However, the optimal solution is shown in Table 3, regardless the value.

Results of running the linear interactive optimizer LINDO software on the LP using Simplex method indicated that alldecision variables $\{X_1, X_2, X_3, \ldots X_{25}\}$ contribute into the objective function ($Xn > 0$) except for X_{22} to yield the value of 619,161.7. Additionally, this simply shows that X_{16}contributed the most to improve the value of the objective function of the LP model as it was aligned the highest value with 1712.33 followed by X_5 and X_{18} with 20000 and 18000, respectively.

The reduced cost rowgives us information about how changing the objective function coefficient for a non-basic variable will change the optimal LP solution. In accordance with the model output, the non-basic variable X_{22}has a reduced cost of $ 0.814. This implies that if we increase X_{22}'s objective function coefficient (in this case, the unit profit of X_{22}) by exactly $ 0.814, then there will be alternative optimal solutions, at least one of which will have X_{22}as the basic variable. If we increase it by more than $ 0.814, then any optimal solution to the LP will have X_{22} as a basic variable with $X_{22} \geq 0$. Thus, the reduced cost of X_{22} is the amount by which X_{22}"misses the optimal basis". Accordingly, this application of linear programming would have indicated to management that the company should either stop producing produce X_{22} or keep watching it since a slight increase in its production will change the LP's optimal solution.

In continue, it is often important for decision makers to determine how a change in a constraint's right-hand side value changes the LP's optimal value (i.e. Z). In this regard, shadow price for constraints may be used to evaluate this impotent concern. The result of this specific sensitivity analysis is presented in Table 4.

Table 3: Shadow price and post optimality.

Row	Slack	Dual price
76	0	2.1
77	49647.02	0
78	57229.79	0
79	0	0.11
80	0	0.4
81	387	0
82	0	1.11
83	0	0.31
84	0	0.64
85	0	0.7
86	0	0.35
87	0	0.37
88	0	1.99
89	0	0.98
90	0	2.28
91	0	0.69
92	0	1.46
93	0	0.95
94	0	2.77
95	0	2.09
96	0	1.25
97	0	1.82
98	0	2.14
99	0	2.69
100	7500	0
101	1	0
102	0	0.39
103	0	1.44
104	0	0.56

Technically, shadow price of the $j_n$constraint shows the amount by which the optimal Z value will be improved if the j_n constrain allocation increases by one unit. The shadow price of each constraint is indicated in the dual price of the model output. For instance, if the right-hand side value of the raw material constraint (constraint 76) is increased by one unit (i.e. one kilogram of tea), it can add $ 2.10 more to the profit with no change in the current basis optimal product mix. If the management wants to increase more profit, they should consider spending on products x_{16} and x_{21}as per unit resource spent increase the profit by $ 2.77 and 2.69 respectively. On the other hand, slack value shows constrains with the dual price = 0 is not adding to the maximization value because of resource shortage.For example, slack or surplus amount in constraint 77 is shown as 49647.02, and this indicates the additional or unused machine time in minutes.

However, although the shadow price analysis is mostly applied to figure out how much profit will be added for one additional unit of resource, decision makers, sometime, might like to know how much resource they are actually allowed to allocate with no change in the current optimal solution. To find the answers, decision makers must technically calculate allowable fluctuation in the current optimal solution. This information was given in the right-hand side ranges section of the model output, mentioned in Table 5. Right-hand side allowable increase and allowable decrease in the model output should be considered to obtain the limitations to make the changes with current basis remaining optimal.

Table 4: Resource budget.

Constraint	Current RHSV	Allowable increase	Allowable decrease
76	175000	2	6998
77	168000	Infinity	49647.02
78	192000	Infinity	57229.79
79	4000	3976.14	1.14
80	15000	5831.67	1.67
81	2100	Infinity	387
82	1000	1457.92	0.42
83	20000	11663.33	0.33
84	1200	2915.83	0.83
85	5000	3644.79	1.04
86	12000	4859.72	1.39
87	6200	4859.72	1.39
88	3000	2915.83	0.83
89	14000	11663.33	3.33
90	8200	11663.33	3.33
91	16000	11663.33	3.33
92	5000	4665.33	1.33
93	12000	9330.67	2.67
94	22000	29158.33	8.331
95	6500	9330.67	2.67
96	18000	23326.67	6.67
97	4000	7775.56	2.22
98	7500	11663.33	3.32
99	9000	23326.67	6.67
100	7500	Infinity	7500
101	3500	Infinity	1
102	4500	6479.63	1.85
103	10000	15551.11	4.44
104	1713	387	0.67

For instance, if we consider the constraint number 76 (raw material), the current right-hand side of this constraint, calledR_1, is 175000. The current basis remains optimal if R_1 is decreased by up to 6998 (the allowable decrease for R_1) or increased by up to 2 (the allowable increase for R_1). Thus, the current basis remains optimal if;

$$168002 = 175000 - 6998 \leq R_1 \leq 175000 + 2 = 175002$$

Therefore, although one unite increase in constrain 76 can leverage profit by 2.1 $, it shouldn't exceed allowable lower and upper limitations. If it is increased more than two units then current basis will be changed. This result can be used as a guide to decrease the available additional monthly production resources such as raw material, machine time and labour time. A large amount of extra profit can be added, if the managers make the decision to cut off the excess raw material, labours and additional machines capacity.

Managerial implications:

In an effort to improve the corporate policy making in a manufacturing company(Byrd and Ted Moore, 1978), themodel described here can also be used by decision makers to quickly analyse several ''what-if'' scenarios for the given production items which is far from the concern of this paper. But, based on the findings of the study, some policies that would help decision makers in designing appropriate PP/DS are highlighted as;

- Decision makers of Dulwan Tea Company, Sri Lanka should ensure that the resources available to them are used in such a manner as to maintain the optimal values of the model in order to maximize profit.
- In the event of the management's desire to improve the established maximum profit within the limits of the model, only the funds allocated to the scarce resources should be increased by decreasing those of the abundant resources within the range.
- Considerable amount of unused machine capacity and labour capacity are remaining in the process. It indicates the unnecessary cost in the production process. Hence, the management should highly take action to eliminating the unnecessary cost in order to improve production productivity.

Conclusion:

This paper deeply discussed a mathematical model that can be used by the Dulwan Tea Company to determine the optimal product mix and production quantities in the presence of several different types of production constraints, in addition to demand limitation. The model, which assumes that each product has just one operating procedure, is a linear program. Based on real-world data, using simplex algorithm, the solutions of the proposed model identify the products that the firm should produce, their production quantities and resource levels required for all resources (raw materials, machine capacity, human capacity and demand limitation) and discuss the acceptable fluctuation level in production constrain duo to performing the sensitivity analysis.

Another issue becomes how the managerial cadre of the productive firms at Dulwan Tea Company could be exposed to the rigorous steps involved in arriving at the optimal values of the linear programming model. From the researchers' personal observations in the course of this study, Dulwan Tea Company has relatively few persons skilled in the Operations Research techniques who also possess a broad understanding of business environment and knowledge of the managerial roles and functions. As such, the firm should rely on outside consultants to bring this and other techniques to bear on management's decision problems. This can go a long way to assisting the management, at least, in the short run. However, in the long run, many companies would, probably, gain from having permanent employees who can suggest opportunities to utilize these new techniques. They should be people who can effectively and efficiently interpret the results of mathematical analysis to top managers in the company's particular context, as well as possess the necessary competence in the utilization of computers for easy handling of the complex mathematical techniques involved.

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