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Effect of Education Campaign on Transmission Model of Conjunctivitis

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ABSTRACT

A deterministic mathematical model for the dynamics model of conjunctivitis is proposed and analyzed. The developed model is taken into account the effect of education campaign. The standard stability method is used for model analysis. The disease free equilibrium and endemic equilibrium are investigated and the stabilities of the model at each equilibrium points are determined. Basic reproductive number is obtained using the next generation matrix and spectral radius. The results shown that when the effectiveness of education campaign increase, the infected human decrease and if the effectiveness of education campaign decrease, the infected human increase. We performed numerical simulations for representative set of parameters to verify and discuss results obtained analytically.

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INTRODUCTION

Conjunctivitis is an infection of the conjunctiva. It is often called red eye or pink eye because the white of the eye has red or pink more than the other side. Symptoms of conjunctivitis can vary but typically include redness or swelling of the white of the eye. Conjunctivitis may be caused by a viral or bacterial infection. It also infection due to an allergic reaction such as pollen or smoke, chlorine in swimming pools or cosmetics (CDC, 2014). In this study, we are interested in Conjunctivitis caused by Hemorrhagic Conjunctivitis (HC). Symptoms of HC are itching and redness of the eye(s). Chow ell *et al* (2006) proposed an epidemic model of Acute Hemorrhagic Conjunctivitis (AHC) that consists of susceptible, infectious, reported, and recovered class. The model taken into account the impact of underreporting and behavior changes on the transmission rate and is applied to an epidemic of AHC in Mexico. Simulation results estimated that a primary infectious case generated about 3 secondary cases $(R_0 = 2.64)$. Sangsawang *et al* (2012) proposed and analyzed an AHC model. The basic reproductive number is obtained and the stability of the model is determined. Disease free and endemic equilibrium points are found. The numerical simulation results are shown the qualitative behavior of the dynamics AHC model to support the theory of mathematical model.Suksawat and Naowarat, (2014) proposed the model of effect of rainfall on the transmission model of Conjunctivitis. The numerical results are shown for supporting the analytic solutions. They concluded that the rainfall is effected on the spread of this disease, to bring the epidemic under control more quickly, it is most important to educate the people in the community on the correct prevention method.

Unyong and Naowarat. (2014) proposed a simple model of Conjunctivitis. The investigation of the model and stability of Conjunctivitis model with nonlinear incident term is taken into account. The new incidence term is used that shows the number of infected people in fraction. The qualitative results are depended a basic reproductive number.

The objective of the study is to determine the effects of education campaign oriented to decrease the attitude towards Conjunctivitis on the dynamics transmission of giving up Conjunctivitis model. The structure of this paper is organized as follows. In section 2, we present a formulation model with the influence of education campaign. In section 3, we analyze the model by using the stability differential equations theory, to determine both and endemic equilibrium point, derive the basic reproductive number and investigate the stability of the model. In section 4, we simulate the numerical results of the model numerically, which support our analytic results. In section 5, we discuss the results. Finally, we summarize the conclusions of our study in section 6.

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1. *Model Formulation:*

In our model, we assume that the human population is constant because the birth rate the death rate of human population are equal. The total number of human population denote by N. The human population is divided into three classes; the susceptible human (S), the infected human (I) and the recovered human (R). The diagram of three classes of human population and the crucial parameters are used which represented the dynamics model of Conjunctivitis as shown in Fig. 1.

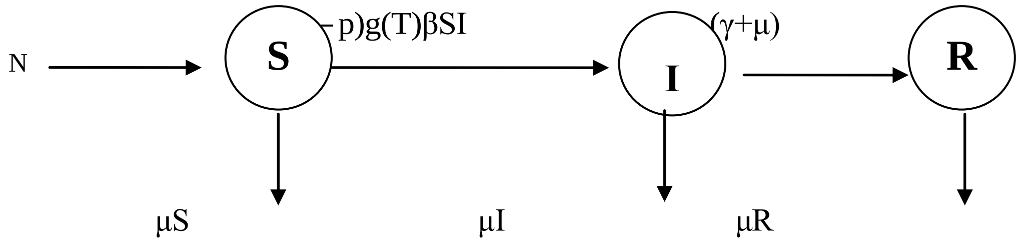


Fig. 1: Flow chart of the dynamical transmission of Conjunctivitis.

The dynamical transmission model consists of system of non-linear differential equations is given as the following.

$$\frac{d\bar{S}}{dt} = \pi N - (1-p)g(T)\beta\bar{S}\bar{I} - \mu\bar{S} \quad (1)$$

$$\frac{d\bar{I}}{dt} = (1-p)g(T)\beta\bar{S}\bar{I} - (\gamma+\mu)\bar{I} \quad (2)$$

$$\frac{d\bar{R}}{dt} = \gamma\bar{I} + \mu\bar{R} \quad (3)$$

With $\bar{S} + \bar{I} + \bar{R} = N$ (4)

Where:

π is the birth rate of human populations,

N is the total number of human populations.

p is the effectiveness of education campaign ,

g(T) is a measure of the rainfall on the rainy and dry season,

β is the probability that virus transmitted from infected human to susceptible human,

γ is the recovery rate of infected human populations,

μ is the natural death rate of human populations.

2. *Model Analysis:*

Equilibrium Points:

The model will be analyzed to investigate the equilibrium points by using the standard method for analyzing our model. The system has two possible equilibrium points

In the case of the absence of the disease, that is I=0. We obtained

$E_0 = (N,0,0)$

1. The endemic equilibrium point (E_1):

In the case of the present of the disease, that is $I^* > 0.$.

We obtained $E_1(S^*, I^*, R^*)$

$$I^* = \frac{(1-p)g(T)\beta\pi N - \mu(\gamma+\mu)}{(\gamma+\mu)(1-p)g(T)\beta},$$

$$S^* = \frac{\pi N}{(1-p)\mu + g(T)\beta I^*}, \quad R^* = \frac{\gamma I^*}{\mu}.$$

Basic Reproductive Number:

The basic reproductive number (R_0) is defined as the expected number of secondary cases produced by a single infection in a completely susceptible population (Jones,2007).By using the next generation method and used spectral radius(Van den Driessche and Watmough,2002). We rewritten the system in matrix form

$$\frac{dx}{dt} = F(x) - V(x)$$

Here $F(x)$ gives the rate of appended of new infections in a compartment and V(x) gives the transfer of individuals.

We obtained

2. The disease free equilibrium point (E_0):

$$x = \begin{bmatrix} S \\ I \\ R \end{bmatrix}, F(x) = \begin{bmatrix} 0 \\ (1-p)G(t)\beta SI \\ 0 \end{bmatrix} \text{ and } V(x) = \begin{bmatrix} -\pi N + (1-p)G(t)\beta SI + \mu S \\ \gamma I + \mu I \\ -\gamma I + \mu R \end{bmatrix}$$

Find the Jacobian max of F(x) and V(x) evaluated at $E_0 = (S, I, R) = (N,0,0)$

We obtained,

$$F(E_0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & (1-p)g(T)\beta N & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$V(E_0)=\begin{bmatrix}\mu & (1-p)g(T)\beta N & 0\\ 0 & \gamma+\mu & 0\\ 0 & -\gamma & \mu\end{bmatrix}$$

Find FV^{-1}, we obtained,

$$FV^{-1}-\lambda I=\begin{bmatrix}0-\lambda & 0 & 0\\ 0 & \left(\frac{(1-p)g(t)\beta N}{\mu+\gamma}\right)-\lambda & 0\\ 0 & 0 & 0-\lambda\end{bmatrix}$$

Find spectral radius of FV^{-1} denoted by $\rho\left(FV^{-1}\right)$

$$\rho\left(FV^{-1}\left(E_0\right)\right)=\frac{(1-p)g(t)\beta N}{\mu+\gamma}$$

We obtained the basic reproductive number as shown,

$$R_0=\frac{(1-p)g(T)\beta N}{\mu+\gamma}$$

Stability Analysis:

In this section, the stability of equilibrium can be analyzed using the Jacobian matrix of the model at the disease free equilibrium. Referring to the results of Vanden Driessche and Watmough (2002),the stability of this system as shown in the follow theorem.

Theorem 1:

The disease free equilibrium of the system about the equilibrium E_0,is local asymptotically stable if $R_0>1$ and unstable if $R_0<1$

Proof. The Jacobian matrix of the model (Eqs. 1-3) evaluated at $E_0(N,0,0)$ is obtained as

The local stability of an equilibrium point is determined from the Jacobian matrix of the system of ordinary differential equation (1),(2),(3) evaluated at the equilibrium point (E_0). The Jacobian matrix is

$$J_0=\begin{bmatrix}-\mu & -(1-p)\beta N & 0\\ 0 & (1-p)\beta N-(\gamma+\mu) & 0\\ 0 & \gamma & -\mu\end{bmatrix}$$

The eigenvalues of the Jacobian matrix J_0 are obtained by solving $\det(J_0-\lambda I)=0$. From this, we obtained the characteristic equation as follows:

$$(\lambda+\mu)(\lambda+\mu)(\lambda+A)=0$$

Where

$$A=(1-p)\,\beta N-(\gamma+\mu)$$

Theorem 2:

The endemic equilibrium of the system Eqs. (1)-(3) about the equilibrium E_1, is local asymptotically stable if $R_0>1$, and unstable if $R_0<1$.

Proof:

$$J_1=\begin{bmatrix}-(1-p)\beta I-\mu & -(1-p)\beta S & 0\\ (1-p)\beta I & (1-p)\beta S-(\gamma+\mu) & 0\\ 0 & \gamma & -\mu\end{bmatrix}$$

$$\det\left(J_1-\lambda I\right)=\begin{bmatrix}(1-p)\beta I^*-\mu-\lambda & -(1-p)\beta S^* & 0\\ (1-p)\beta I & (1-p)\beta S^*-(\gamma+\mu)\lambda & 0\\ 0 & \gamma & -\mu-\lambda\end{bmatrix}$$

We obtained the characteristic equation

$$(\lambda+\mu)(\lambda^2-C_1\lambda-C_2)=0$$

where

$C_1=B_1+B_4$, $C_2=B_1B_4+B_2B_3$

$B_1=(1-p)\beta I^*-\mu$, $B_2=(1-p)\beta I^*$, $B_3=(1-p)\beta I^*$,

$B_4=(1-p)SI^*-(\gamma+\mu)$

one eigenvalue $\lambda_1-\mu<0$,

The two roots of $\lambda^2-C_1\lambda-C_2=0$ will be negative real part if they satisfy the Routh-Hurwitz criteria.

1) $C_1<0$,

2) $C_2<0$.

4. Numerical Results:

In this section, we present the numerical simulation of the our model. The parameter values that we used in the numerical simulation are given in Table.1

Table 1: Parameters values used in numerical simulation at disease free state.

Parameters	Description	Values	References
π	the birth rate of human populations	0.000456 day^{-1}	Suksawat and Naowarat (2014)
β	the probability that virus transmitted from infected human to susceptible human,	0.004	
μ	the natural death rate of human populations,	0.0000456 day^{-1}	
γ	the recovery rate of infected human populations,	0.33 day^{-1}	
g(T)	a measure of the rainfall on the dry season	0.01	
p	the effectiveness of education campaign,	0.09	
N	the total number of human populations.	1000	

By solving the system of differential equations. The numerical results showed at the disease free equilibrium point and the endemic equilibrium point as follows:

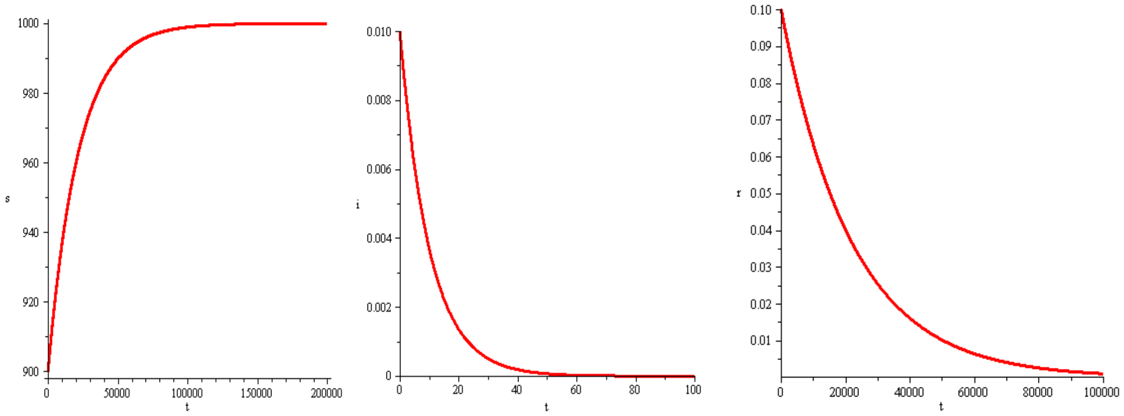
$\pi=0.0000456$, $\beta=0.004$, $\gamma=0.33$, $\mu=0.0000456$, $g(T)=0.01$, $N=1{,}000$,$R_0=0.0096<1$

Stability of the endemic state:

We changed the values of $p = 0.82$, $g(T) = 0.33$ (rainy season) and keep the values of the other values of parameters to be those given in Table 1, we obtained the eigenvalues and basic reproductive number are:

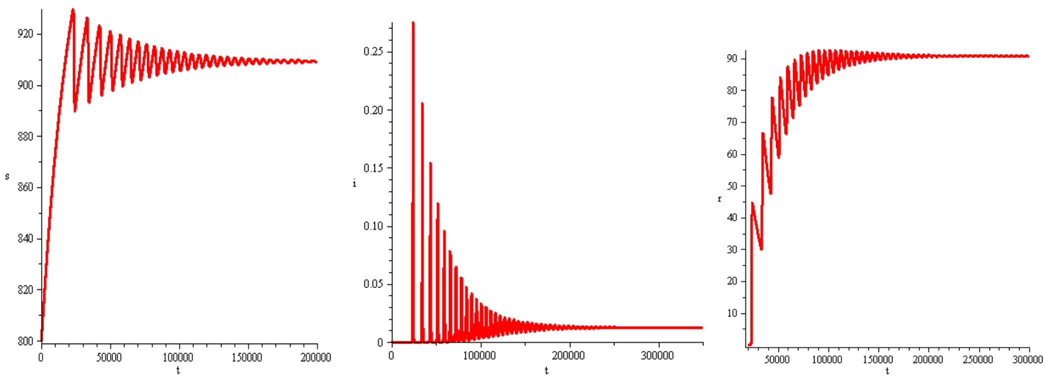
$\lambda_1 = -0.67013$ $\lambda_2 = -0.000045$ $\lambda_3 = -0.0000112$ $R_0 = 0.0096 > 1$

Since all of eigenvalues are to be negative and the basic reproductive is to be greater than one, the endemic equilibrium point will be local asymptotically stable, as shown in Fig. 3



(a) (b) (c)

Fig. 2: Time series of (a) Susceptible human, (b) Infected human , (c) Recovered human. The values of parameters are in the text. We see that the solutions converge to the disease free state $E_0(1{,}000,0,0)$



(a) (b) (c)

Fig. 3: Time series of (a) Susceptible human (S) , (b) Infected human (I),(c) Recovered human (R). The values of parameters are in the text and $R_0 > 1$. We see that the solutions converge to the disease free state $E_1(909.216, 0.0125, 90.770)$

5. Discussion:

In this study, we proposed the dynamics model of Conjunctivitis with taken into account the education campaign. We analyzed the model by standard method which we determined equilibrium points and investigated the stability of the model. The basic reproductive number is obtained through the next generation method. The basic reproductive number is

$$R_0 = \frac{(1-p)g(T)\beta N}{\mu+\gamma}$$

In epidemiology, the basic reproductive number is the number of secondary case generate by a primary infectious case (van den Driesch and Watmough,2002).For mathematician, the basic reproductive number is the threshold parameter for determining the stability of the model at each equilibrium points. The stability of the system is investigated using the Roth-Hurwitz criteria. The qualitative behaviors of this model are shown as Fig. 2 and 3. We found that when p = 0.82, the value of R_0 is 0.0996 and when p = 0.09 ,the value of R_0 is 1.099.

5. Conclusion:

It concluded that when the value of effectiveness of education campaign decrease, the number of infected human increase. The basic reproductive number is greater than one, meaning that the Conjunctivitis will occur in the community. On the other hand, when the value of effectiveness of the education campaign decrease, the number of infected human decrease. The basic reproductive number is less than one, meaning that the Conjunctivitis will died out the community.

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