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Effect of Footings Shape on Bearing Capacity Factors for Cohessionless Soil

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ABSTRACT

In engineering practice the bearing capacity of soils under footings is generally calculated from Terzaghi's formula which is widely used. The study of bearing capacity under footings subjected to static loads is an essential and introductory step towards understanding the problem of settlement of structures. Bearing capacity factors (Nq) and (Nγ) are dimensionless that depend only on the friction angle of soil. In the present study filed tests have been conducted by the plate load test using nine steel rigid plates (three is circular, three is square and three is rectangular). The plates have a finished thickness of 32 mm. In this research the bearing capacity factors (N\q) and (N\γ) of cohesionless soil has been determined for circular, square and rectangular footings. However, the effect of footings shape has been presented in an empirical formulae for (N\q, N\γ) for circular, square and rectangular footings. In addition, comparison between the obtained results and theoretical bearing capacity factor (N\q, N\γ) has been introduced for different relative densities.

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INTRODUCTION

Theoretical methods for the ultimate bearing capacity can be summarized in the followings:

(A) Terzaghi Method (1943): derived a general bearing-capacity equation from using plasticity theory for shallow foundation of ($D_f \leq B$). This equation was derived based on neglecting the shear resistance of the soil above the horizontal plane through the base of the footing and replacing the soil above this plane with a surcharge ($q = \gamma D_f$) where D_f = depth of footing. From static equilibrium conditions the following equations are obtained:

For strip footing:

$$q_{ult} = cN_c + qN_q + 0.5\,\gamma B\,N_\gamma \qquad (1)$$

For square footings:

$$q_{ult} = 1.3\,cN_c + qN_q + 0.4\,\gamma B\,N_\gamma \qquad (2)$$

For circular footings:

$$q_{ult} = 1.3\,cN_c + qN_q + 0.3\,\gamma BN_\gamma \qquad (3)$$

Where:

q_{ult} = Ultimate soil bearing pressure.
c = Cohesion of soil.
q = γDf.
B = Least lateral dimension of footing (diameter = B for round footings).
γ = Unit weight of soil (use submerged weight for soil below water table).
N_c, N_q and N_γ (bearing capacity factors depend on the angle of internal friction) as shown in Table (1).

(B) Meyerhof's analysis (1963): presented the following formula:

$$q_u = cN_c\, s_c\, d_c\, i_c + q\, N_q\, s_q\, d_q\, i_q + 0.5\,\gamma\, B\, N_\gamma\, s_\gamma\, d_\gamma\, i_\gamma \qquad (4)$$

Where:

q_u = Effective stress at the level of the bottom of foundation.
c = Cohesion of soil.
B = width of foundation.
γ = Unit weight of soil.
S_c, s_q, s_γ = shape factors.
d_c, d_q, d_γ = depth factors.
i_c, i_q, i_γ = inclination factors.
N_c, N_q, N_γ = factors depends on angle of internal friction as shown in Table (2).

(C) Hansen's analysis (1970): is one of the most important analyses for determining the ultimate bearing capacity. He presented the bearing capacity factors (N_c, N_q, N_γ) which depend on the angle of internal friction as shown in Table (3).

(D) Vesic's analysis (1975): presented the bearing capacity factor (N_γ) calculated by $N_\gamma = 2\,(Nq + 1)\tan ø$. N_c, N_q, N_γ factors depend on the angle of internal friction as shown in Table (4).

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Table 1: Terzaghi's bearing capacity factors.

Φ	**N_c**	**N_q**	**N_γ**
0	3.7	2.0	0.0
5	7.3	1.6	0.3
10	9.6	2.7	1.2
20	17.7	7.4	5.0
23	25.1	12.7	9.7
30	37.2	22.5	19.7
34	32.6	36.5	33.0
40	95.7	81.3	100.4
45	172.3	173.3	247.3

Table 2: Meyerhof's bearing capacity factors.

Φ	**N_c**	**N_q**	**N_γ**
0.0	5.4	1.0	0.0
5.0	6.3	1.7	0.5
10	8.0	2.7	1.0
15	11	4.1	1.3
20	14	6.7	2.8
25	20	12	6.3
30	29	20	16
35	48	34	40
40	76	67	100
45	140	130	260

Table 3: Hansen's bearing capacity factors.

Φ	**N_c**	**N_q**	**N_γ**
0	5.14	1.0	0.0
5	6.49	1.6	0.1
10	8.34	2.5	0.4
15	10.97	3.9	1.2
20	14.83	6.4	2.9
25	20.71	10.7	6.8
30	30.13	18.4	15.14
40	75.25	64.1	79.4
45	133.73	134.7	200.5

$N\gamma = 1.5(Nq-1) \tan (\O)$

Table 4: Vesic's bearing capacity factors.

Φ	**N_c**	**N_q**	**N_γ**
0	5.14	1.00	0.00
5	6.49	1.57	0.45
10	8.35	2.47	1.22
15	10.98	3.94	2.65
20	14.83	6.40	5.39
25	20.72	10.66	10.88
30	30.14	18.40	22.40
35	46.12	33.30	48.03
40	75.31	64.20	109.41
45	133.88	134.88	271.76

Abdel Aziz, (2000), presented small laboratory model test used to obtain the bearing capacity for footing with different sizes subjected to a vertical load on two layers of soil. The top layer possesses high bearing capacity and the second layer possesses low bearing capacity. The thickness of the top layer is varied. The results of this research showed that the thickness of soil layer under footing is an important factor affecting its bearing capacity.

Zhu *et al*., (2007), presented the results of a research program of strip and circular footings resting on dry dense sand. The scale effect on the bearing capacity and the shape factors of the footings are investigated numerically and experimentally. The calculated bearing capacity (q_u) of strip and circular footings can be expressed in terms of footing dimension (B or d) as follows:

$$q_u = 92.1 p_a \left(\frac{\gamma B}{P_a} \right)^{0.65}$$

For strip footings (5)

$$q_u = 50.5 p_a \left(\frac{\gamma d}{P_a} \right)^{0.69}$$

For circular footings (6)

Where:

q_u = The bearing capacity in kpa.

P_a = Atmospheric.

B = Width of strip footing in meter.

d = Diameter of circular footing in meter.

γ = The unit weight of soil = 1.54 t/m^3.

Ueno *et al.*, (2001), presented a method for estimating the ultimate bearing capacity of surface footings on sand. An extended slip line method is developed, in which the dependency of the angle of internal friction on the confining stress is formulated from results of conventional triaxial compression tests for various sands. A modified formula is proposed to consider the size effects on bearing capacity.

Junhwan *et al.*, (2005), analyzed the estimation of limit unit bearing capacity of axially loaded circular footings on sands based on cone penetration test cone resistance. Normalized limit unit bearing capacities were calculated from non-linear finite element and cone penetration resistance analyses for various soil and footing conditions. Effects of the relative density, the lateral earth pressure ratio and the footing size are also addressed. It is observed that the normalized limit unit bearing capacity decreases as relative densities increases.

The Egyptian Code equation (2005) presented a general equation to calculate the ultimate bearing capacity for any shapes, depth, and inclination as follows:-

$$q_{ult} = cN_c \gamma \lambda_c + \gamma_1 \; D_f N_q \lambda_q + \gamma_2 \; B N_\gamma \lambda_\gamma \qquad (7)$$

Where:

c = Cohesion of soil (KN/m^2).

γ_1 = Unit weight of soil above of the foundation level (KN/m^3).

γ_2 = Unit weight of soil under of the foundation level (KN/m^3).

B = Least lateral dimension of footing (m).

D_f = Depth of foundation (m).

N_c, N_q and N_γ = factors depend on angle of internal friction as shown in Table (5).

Table 5: Egyptian code bearing capacity factors.

Φ	**N_c**	**N_q**	**N_γ**
0	5.0	1.0	0.0
5	6.5	1.50	0.0
10	8.5	2.5	0.5
13	11.0	4.0	1.0
20	15.0	6.5	2.0
25	20.5	10.5	4.5
30	30.0	18.0	10.0
35	46.0	33.0	23.0
40	75.0	64.0	53.0
42.5	99.0	92.0	83.0

Cerato, and Lutenegger, (2007), investigated scale effect of shallow foundation bearing capacity on granular materials. Model-scale square and circular footing tests were performed on two compacted sand at three relative densities. Results of the model-scale show that the bearing capacity factor N_γ is dependent on the absolute width of footing for both square and circular footings. It also shows that the behavior of most model scale footing test cannot be directly corrected to the behavior of full scale test.

Lyamin *et al.*, (2007), presented the bearing capacity of strip, square, circular and rectangular foundations for frictional soils following an associated flow rule using finite-element limit analysis. The results of the analyses are used to propose values of the shape and depth factors to be used in the traditional bearing capacity equation.

2. Experimental Study:

The plate load tests were carried out and the settlement of sand was measured under different stress levels at surface of plates.

2.1. Field Samples:

In the present study graded sand has been used. Each sample has been placed in a square open box and compacted in layers with different relative densities.

2.2. Loading:

The loads were applied by using steel frame fixed in the ground as shown in Figure (1).

2.3. Used Plates:

In the present study, nine plates were used in testing the specifications of which are shown in Table (6). The plates have a finished thickness of 32 mm and are according to ASTM D1194 and D1196 as shown in Figure (2).

Table 6: Used plates.

Circular Plate (diameter) B (mm)	Square plate (B=L) (mm)	Rectangular plate (B*L) (mm)	Equivalent area (mm^2)	Thickness (mm)	Weight (kg)
305	270.3	238*307	73061.66	32	14.5
455	403.2	360*451.6	162597.05	32	33
610	540.6	458*638	292246.66	32	56

2.4. Test Procedure:

The test procedure was as follows:

a- The soil has been placed in a square open box.

b- The box was filled with different soil layers compacted to different densities which has been determined by sand cone test.

c- The surface of the tested soil was prepared for plate test using fine sand at the surface.

d- The steel plate was placed on the prepared surface.

e- A hydraulic jack was placed on the steel plate.

f- Four dial gauges has been placed on the plate surface.

g- The settlement has been measured by using dial gauges of sensitivity 0.01mm placed on the edge of the steel plate.

h- The settlement has been measured at the surface of the plate as shown in Figure (3).

i- The load was applied in increment by using steel frame. Each load increment was maintained constant until the settlement rate reaches 0.02 mm/min but not less than one hour in any case.

3. Experimental Results:

The settlement in field was recorded for cohesionless soil for different footings shape and size (circular, square and rectangular) under different stresses ranging between 58.9 and 530.1 kN/m^2.

4. Theoretical Analysis:

A convergence study has been performed to determine bearing capacity factors (N_q) and (N_γ) using "SPSS" statistical scientific program. From the experimental results the following empirical formula are presented. The formulas are derived to calculate bearing capacity factors ($N^{\backslash}_q$) and ($N^{\backslash}_\gamma$) for the circular, square and rectangular foundations using regression methods:

$$N^{\backslash}{}_q = 1.045\left[\Phi + a(\frac{L}{B}) + (f_s)^b\right] \qquad (8)$$

$$N^{\backslash}\gamma = 1.2N_q * \tan\Phi \qquad (9)$$

Where:

($N^{\backslash}_q$) and ($N^{\backslash}_\gamma$) = Bearing capacity.
Φ = Angle of internal friction.
B = width of footing.
L = length of the footing.
(L/B) = 1.0 in case of circular and square footings.
f_s = shape of footing (circular=1.2, square=2 and rectangular=3).
a and b= constants listed in the Table (7).

Table 7: Empirical formula constants.

Constant	Circular Plate	Square Plate	Rectangular Plate
a	4.105	3.725	3.525
b	4.500	2.000	1.500

One-way analysis of variance (One-Way ANOVA) is used to calculate the correlation coefficient of the resulting equation. The One-Way ANOVA procedure produces a one-way analysis of variance for a quantitative dependent variable by a single factor (independent) variable. Analysis of variance is used to test the hypothesis that several means are equal. This technique is an extension of the two-sample test. The number of cases, mean, and standard deviation, standard error of the mean, minimum, maximum, and 95%-confidence interval for the mean were calculated.

It should be mentioned that many trails were done to increase the accuracy of the derived formula (correlation coefficient = 0.95).

($N^{\backslash}_q$) and ($N^{\backslash}_\gamma$) = bearing factor were obtained from the empirical formula (8) and (9) for different friction angles are shown in Tables (8), (9) and (10), respectively.

The obtained values of ($N^{\backslash}_q$) and ($N^{\backslash}_\gamma$) are to be used in the traditional bearing capacity equation (Terzaghi).

Table 8: Author's bearing capacity factors for circular footing.

Φ	**$N^{\backslash}_q$**	**$N^{\backslash}_\gamma$**
0	6.66	0.00
5	11.89	1.25
10	17.11	3.62
15	22.34	7.19
20	27.56	12.04
25	32.79	18.36
30	38.01	26.35
35	43.24	36.35
40	48.46	48.82
45	53.69	64.47

Table 9: Author's bearing capacity factors for square footing.

Φ	**$N^{\backslash}_q$**	**$N^{\backslash}_\gamma$**
0	8.07	0.00
5	13.30	1.40
10	18.52	3.92
15	23.75	7.64

20	28.97	12.66
25	34.20	19.15
30	39.42	27.32
35	44.65	37.54
40	49.87	50.25
45	55.10	66.16

Table 10: Author's bearing capacity factors for rectangular footing.

Φ	**$N^{\backslash}_{q}$**	**$N^{\backslash}_{\gamma}$**
0	10.18	0.00
5	15.41	1.62
10	20.63	4.37
15	25.86	8.32
20	31.08	13.58
25	36.31	20.33
30	41.53	28.79
35	46.76	39.31
40	51.98	52.37
45	57.21	68.69

5. Results:

From the present study, the following relationships are obtained:

a- Figures (4 and 5) show the ($N^{\backslash}_{q}$) and ($N^{\backslash}_{\gamma}$) bearing capacity factors obtained from the empirical formula (8), (9) for circular, square and rectangular footings. From these figures it can be shown that the ($N^{\backslash}_{q}$) and ($N^{\backslash}_{\gamma}$) bearing factors for rectangular footings are higher than that square than that circular footing.

b- Figures (6 - 8) show a comparison between the theoretical and the obtained empirical bearing capacity factors ($N^{\backslash}_{q}$) under different relative densities. These figures show close agreement between the experimental and theoretical results up to angle of internal frictions (40^{o}) after which lower values have obtained from the empirical formula (8).

c- Figures (9 - 11) show a comparison between the theoretical and the obtained empirical bearing capacity factor ($N^{\backslash}_{\gamma}$) under different relative densities.

These figures show close agreement between the experimental and theoretical results up to angle of internal frictions (40^{o}) after which lower values have obtained from the empirical formula (9).

Fig. 1: Loading frame.

Fig. 2: The nine rigid plates.

Fig. 3: Settlement readings using dial gauges.

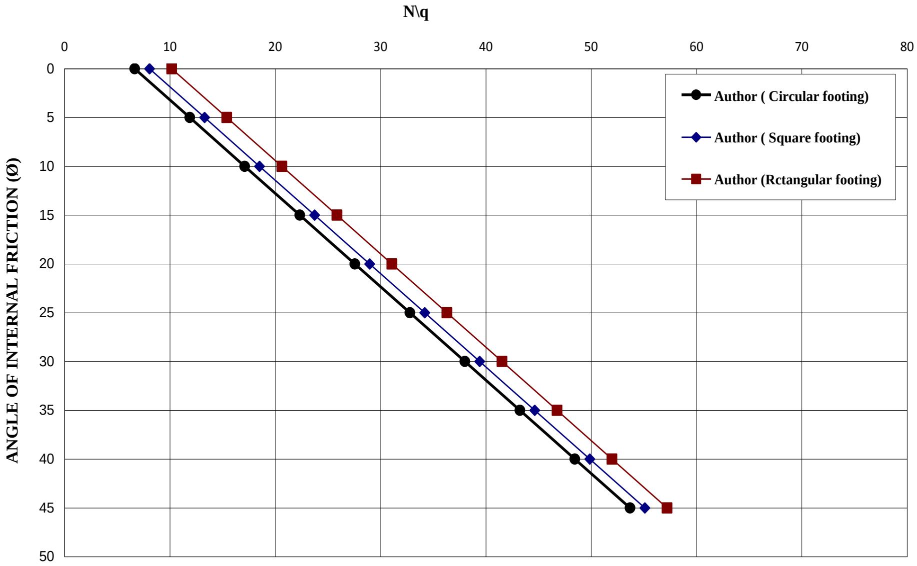

Fig. 4: The relationships between experimental $N^{\backslash}_{q}$ factor and the angle of internal friction (Ø) under different relative densities for circular, square and rectangular footings.

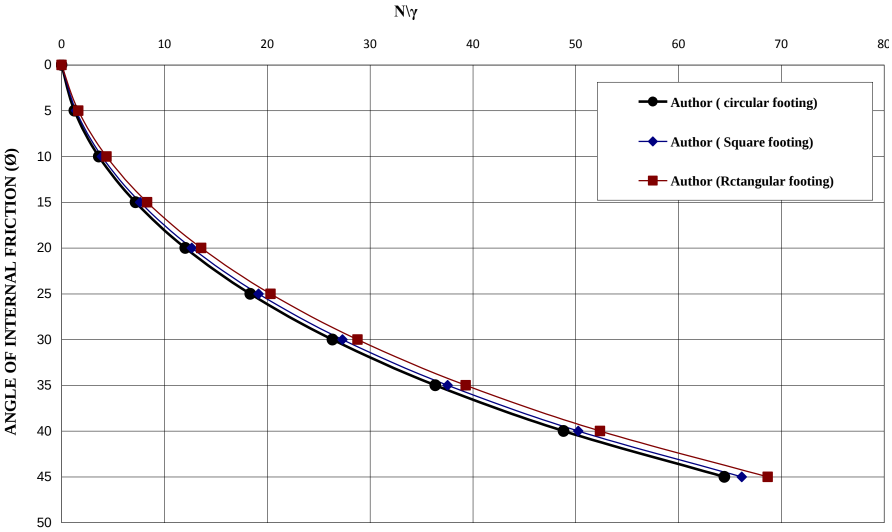

Fig. 5: The relationships between experimental $N^{\backslash}_{\gamma}$ factor and the angle of internal friction (Ø) under different relative densities for circular, square and rectangular footings.

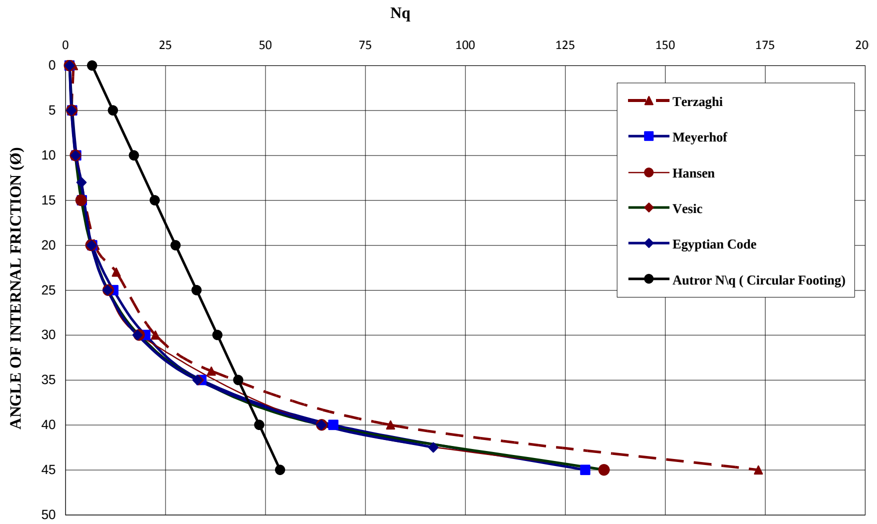

Fig. 6: Compression between theoretical and experimental bearing capacity factor (Nq) under different relative densities for circular footings.

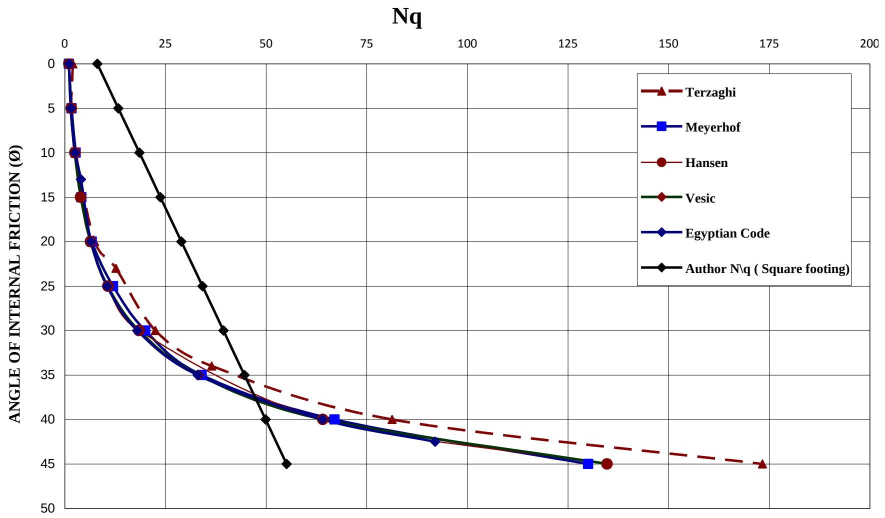

Fig. 7: Compression between theoretical and experimental bearing capacity factor (Nq) under different relative densities for square footings.

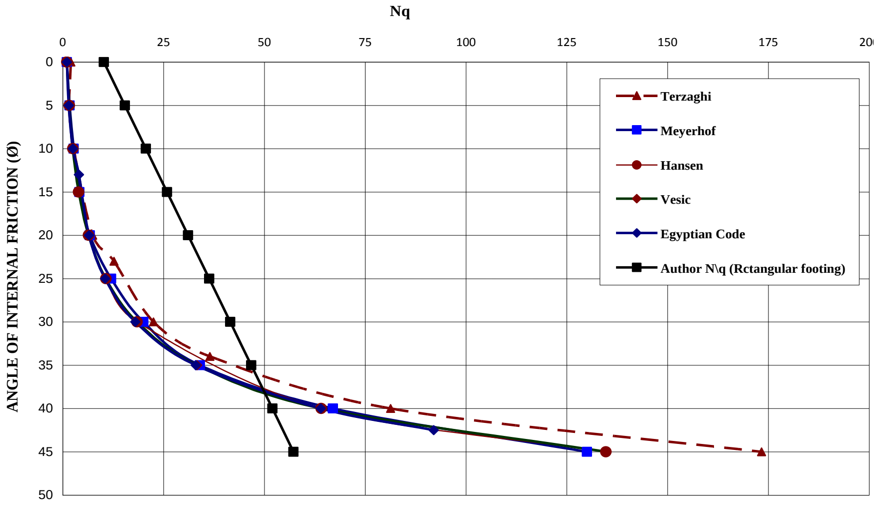

Fig. 8: Compression between theoretical and experimental bearing capacity factor (Nq) under different relative densities for rectangular footings.

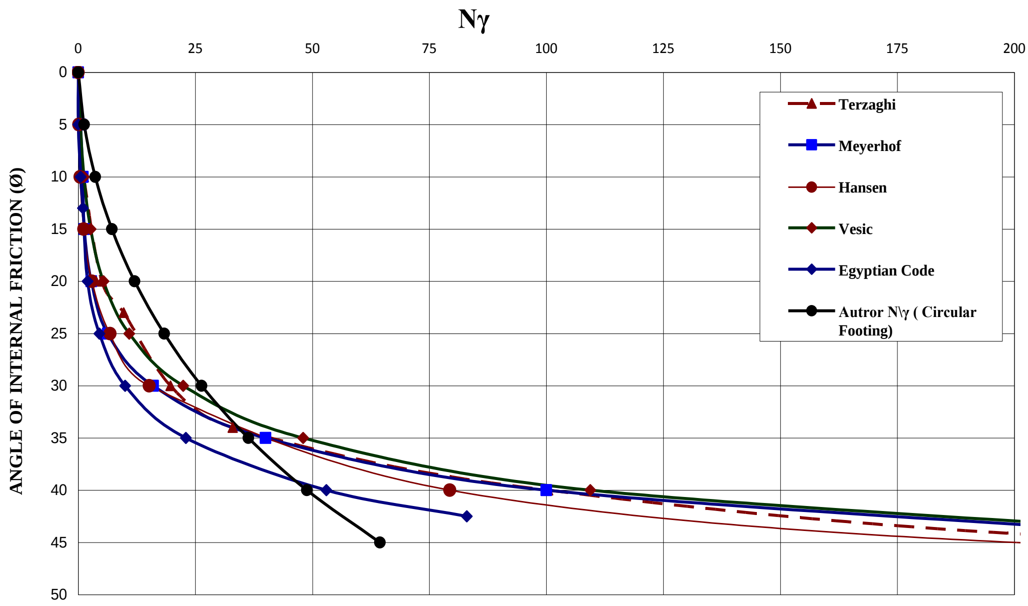

Fig. 9: Compression between theoretical and experimental bearing capacity factor N_γ under different relative densities for circular footings.

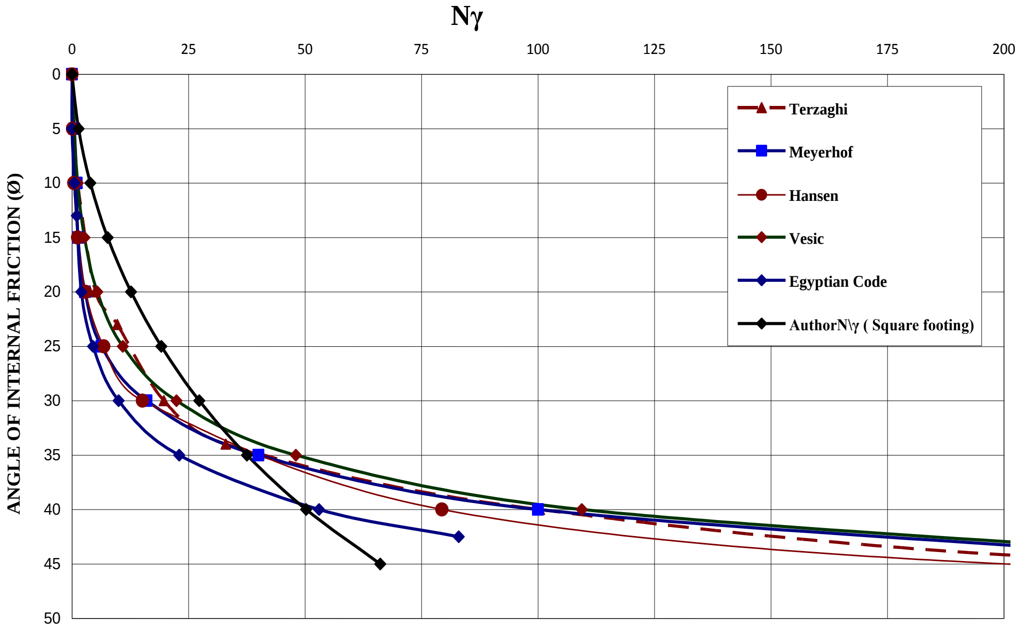

Fig. 10: Compression between theoretical and experimental bearing capacity factor N_γ under different relative densities for square footings.

Conclusions:

From the present study the followings are concluded:

a. An empirical formula to calculate bearing capacity factor ($N^\backslash_q$) and ($N^\backslash_\gamma$) has been presented taking into consideration the effect of foundations shape.

b. Close agreement between the empirical and the theoretical results up to angle of internal frictions up to (40^o) is obtained.

c. For angle of internal friction more than (40^o) lower values have been obtained from ($N^\backslash_q$) and ($N^\backslash_\gamma$) bearing capacity factors.

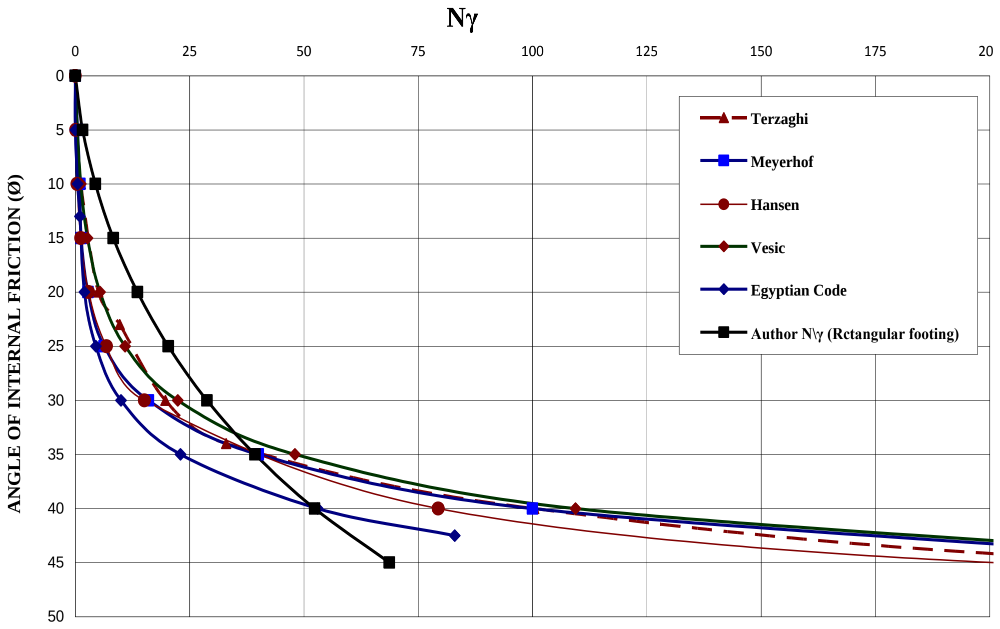

Fig. 11: Compression between theoretical and experimental bearing capacity factor N_γ under different relative densities for rectangular footings.

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