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Linear Quadratic Gaussian (LQG) Controller Design for Servo Motor

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ABSTRACT

This paper presents the development of Linear Quadratic Gaussian controller design for DC servo motor. The accuracy of the control system is directly proportional to servo motor precision has been difficult to achieve due to the ignorance of the sensor noise and plant disturbance. Therefore LQG controller has been design to control speed and position of DC servo motor based on combination of Steady-state Linear Quadratic (LQ) Optimal Control and Steady-state Kalman Filter State Estimation. MATLAB Simulink and M-File approached have been done to simulate the design by solving matrix Riccati equation in order to determine steady-state feedback gain, K and steady-state Kalman gain, G. The step response of closed-loop digital control system using LQG controller and opened-loop control system have been compared to verified that the performance of closed-loop control system better than opened-loop system.

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INTRODUCTION

Servo motor is a motor that is used to control position or speed in closed-loop control systems. It is generally designed for the computer, numeric control machines, industrial equipment, weapon industry, speed control of alternators and to control mechanism of full automatic regulators as the first starter, and also to quickly and correctly starting a system.

In the field of control mechanical linkages and robots, research works are mostly found on DC motors. Therefore the requirement of a servo motor is to turn over a wide range of speeds and also to perform position and speed based on instruction given has been developed.

Properties of DC motor such like inertia, physical structure, shaft resonance and shaft characteristics, electrical and physical constant are variable. Besides that, the velocity and position tolerance of servo motor which are used at the control systems are nearly identical. Therefore, servo motor must be controlled to ensure the stability of the close-loop control system.

The need for a feedback element was required in order to make control system stable. However, the feedback not only increases the cost but also makes the control of position and speed of servo motor becomes complex. Further, the control accuracy is directly proportional the precision of servo motor control. However, the precision of servo motor control has been difficult to achieve due to the ignorance of the plant disturbance and inaccuracies in feedback measurement cause by sensor noise. Therefore, Linear Quadratic Gaussian (LQG) controller has been proposed to overcome the sensor less position and speed control due to this problem.

Linear Quadratic Gaussian control is one of the most fundamental optimal control problems that concerns uncertain linear systems disturbed by additive white Gaussian noise. LQG controller is simply the combination of a Linear Quadratic (LQ) Optimal Control and Kalman Filter State Estimation.

Methodology:

The procedure to designing LQG control of DC servo motor has been divided into three categories parts which were Mathematical Model of DC Servo Motor Design, Linear Quadratic (LQ) Optimal Control Design and Kalman Filter Optimal State Estimation Design

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Mathematical Model of DC Servo Motor:

The DC servomotor that has been considered in this paper was a permanent magnet DC motor as shown in Fig.1.

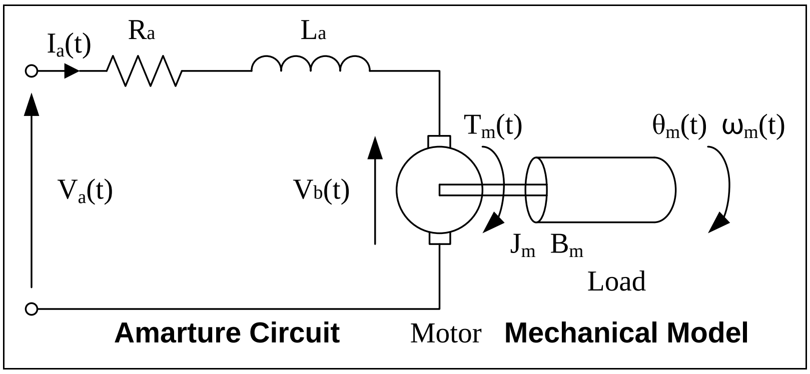

Fig. 1: DC Servo Motor Modelling

In designing a LQG controller for Servo Motor, Motor ID23005 has been used as the system. The parameter were given as follow

Parameter	Value
Table 1: Parameter of DC Motor ID23005e Motor Inertia, J_m	6.286 x 10^{-5} Kg-m^2
Viscous Damping Coefficient,B_m	7.08 x 10^{-5} rad/sec
Back-EMF constant, K_b	0.121 Nm/A
Torque constant, K_t	0.121 V/rad/sec
Armature Resistance, R_a	2.23 ohms
Armature Inductance, L_a	6.40mH

Transfer function for servo motor modelling can be derived by Laplace Transform of Kirchhoff's Law and Newton's Law. The Kirchhoff's Law can be related as

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + K_b \frac{d\theta_m}{dt} \tag{1}$$

While the Newton's Law can be related as

$$K_t i_a(t) = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} \tag{2}$$

Transfer function of servo motor modelling has been determined by using Laplace transform for Kirchoff's law and Newton's Law.

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_t}{s[(R_a + L_a s)(J_m s + B_m) + K_b K_t]} \tag{3}$$

Since from parameter given, K_b=K_t, Therefore K=K_b= K_t, thus the transfer function of servo motor to measured position can be simplified as

$$G(s) = \frac{K}{s[(R_a + L_a s)(J_m s + B_m) + K^2]} \tag{4}$$

Referred to equation (4), transfer function of DC servo motor to measure speed can be derived by using equation

$$\omega_m = \frac{d\theta_m}{dt} \tag{5}$$

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By taking Laplace transformation of equation (5) and substitute into equation (4), transfer function of DC servo motor to measure speed can be defined as

$$G(s) = \frac{K}{(R_a + L_a s)(J_m s + B_m) + K^2} \tag{6}$$

Therefore block diagram for servo motor to measure position and speed can be described in Fig. 2.

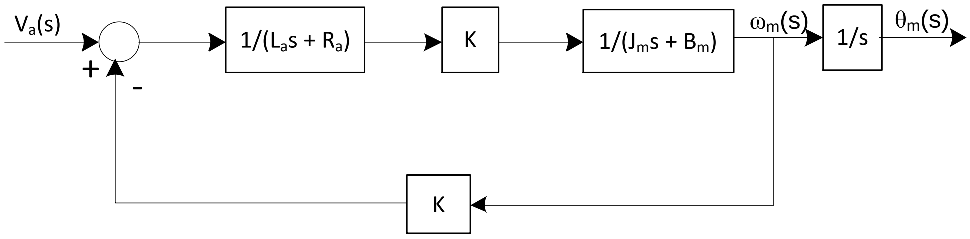

Fig. 2: Block diagram for servo motor to measure position and speed

The state space of the system required to be determined in order to design LQG controller. Continuous State Space Equation can be described as

$$\begin{aligned} \dot{x}(t) &= A_C x(t) + B_C u(t) \\ y(k) &= C_C x(t) + D_C u(t) \end{aligned} \tag{7}$$

where
x(t) = state vector (n-vector)
u(t) = control vector (r-vector)
A_C = n x n system matrix
B_C = n x r input matrix
C_C = m x n output matrix
D_C = m x r connection matrix

In matrix form, state-space equation to control the position of DC servo motor can be described as

$$\dot{x}(t) = \begin{bmatrix} -\frac{R_a}{L_a} & 0 & -\frac{K}{L_a} \\ 0 & 0 & 1 \\ \frac{K}{J_m} & 0 & -\frac{B_m}{J_m} \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} u(t) \tag{8}$$

$$y(k) = [0 \quad 1 \quad 0]x(t)$$

While state-space equation to control the speed can be described as

$$\dot{x}(t) = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K}{L_a} \\ \frac{K}{J_m} & -\frac{B_m}{J_m} \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} u(t) \tag{9}$$

$$y(k) = [0 \quad 1]x(t)$$

After continuous transfer function and state-space equations of the servo motor have been derived manually, the equations were determined by using MATLAB M-File and parameter in Table 1.

Since the LQG controller will be implemented in discrete time, therefore the continuous state space of DC servo motor will be transform into discrete time as well. In LQG controller design, DC servo motor of continuous plant has been transform into discrete time MATLAB M-file.

In MATLAB M-file, continuous state space of DC servo motor was converted into discrete state space by using command "[A, B] = c2d(Ac, Bc, Ts)" with T_S denote for time sampling for DC servo motor.

3. Linear Quadratic (LQ) Optimal Control:

Fig. 3 show the flow chart of the steady-state quadratic optimal control for third order system to determine gain K by using Lagrange multiplier iteration method. The iterations were done using MATLAB M-File approach.

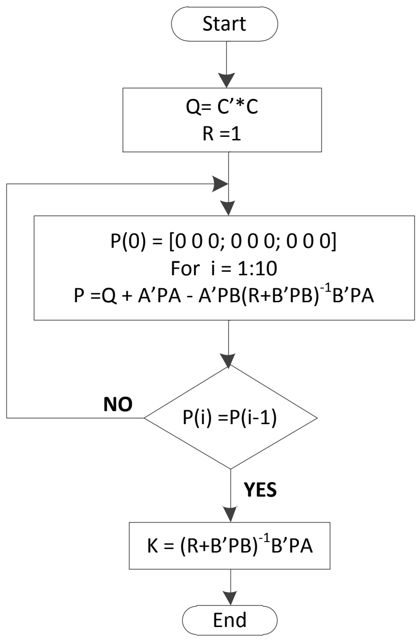

Fig. 3: Flow Chart Steady-State Quadratic Optimal Control to Control Position

Referred to Fig. 3, the iteration was started by selects the weight matrix Q and R such that the performances of the closed loop system can satisfy the desired requirements to minimizes the quadratic function or performance index. Therefore, by using Bryson's Rule, positive-definite Hermitian matrices Q and R were selected as Q = C'C and R = 1.

Then, the selected weighting matrices Q and R were used to solve Steady-state Riccati equation to determine steady-state matrix P with initial condition, P(0) = [0 0 0; 0 0 0; 0 0 0] . The iteration was repeated using ten steps of iteration. Steady-state Riccati equation in this iteration was derived from equation.

$$P(k) = Q + A'P(k+1)[I + BR^{-1}B'P(k+1)]^{-1}G \tag{10}$$

By modified the equation using the matrix inversion lemma,matrix P can be determined as follows:

$$P = Q + A'(P^{-1} + BR^{-1}B')^{-1}A \tag{11}$$

It can be describe that, the matrix P is determined by matrices A, B, Q and R. By solving equation (11), Steady-State Riccati Equation was determined as

$$P = Q + A'PA - A'PB(R + B'PB)^{-1}B'PA \tag{12}$$

The value of P was checked after iteration. When the value of P stays constant, the iterations were stopped and the value of optimal state-feedback matrix gain, K was determined. However, if the value of P was different for each iteration, the iteration was continued until the value of P became constant. The steady gain matrix was obtained in terms of P by using equation (13) which was derived using Lagrange multiplier.

$$K = (R + B'PB)^{-1}B'PA \tag{13}$$

The flow chart to determine LQ optimal control to control speed of servo motor was similar to Fig. 3by using initial condition of Steady-state matrix P, P(0) = [0 0; 0 0].

Kalman Filter Optimal State Estimation:

In order to design Linear Quadratic Gaussian Controller, an optimal state estimation technique called Kalman filtering has been considered to estimates the state of a plant from the information that is available concerning the plant.

Fig. 4 shows Kalman filter optimal state estimation to design LQG controller of DC servo motor for third order system based on Discrete Kalman Filter process

State estimation of Kalman filters was also done by Lagrange multiplier iteration by using MATLAB M-File approach as well as LQ optimal control.

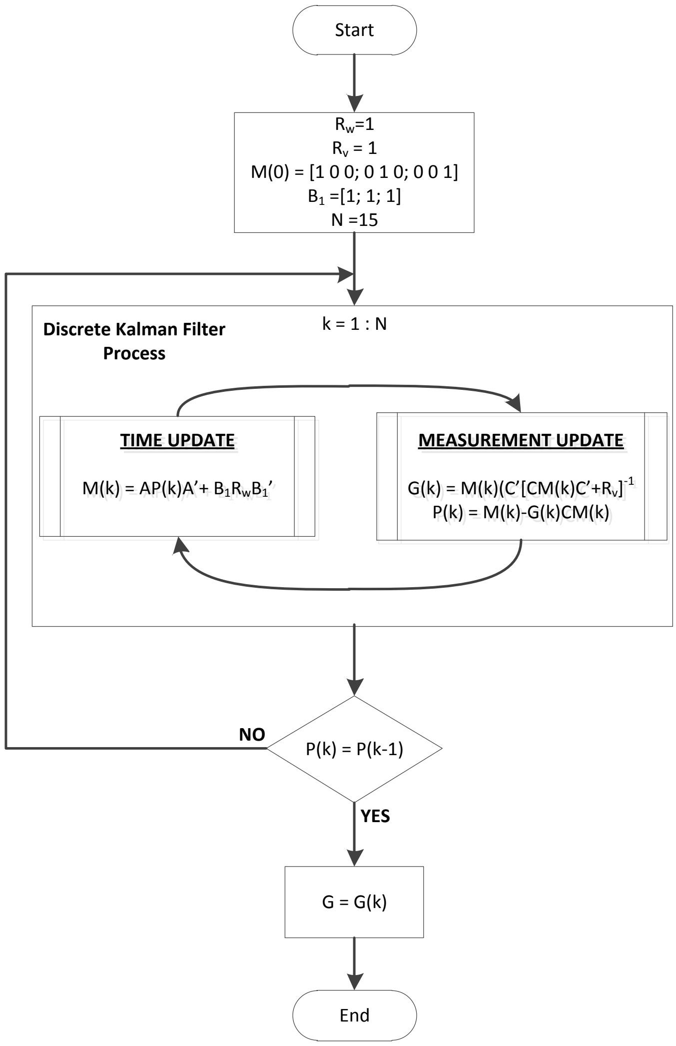

Fig. 4: Flow Chart of Kalman Filter Optimal State Estimation to Control Position

In the Kalman filter optimal state estimation, the discrete white-noise sequence with Gaussian distributions w(k) was assumed to have random input of B_1 = [1; 1; 1]. Besides that, the random inputs w(k) and v(k) are assumed to be uncorrelated and have Gaussian distributions. Thus, the inputs covariance in equation were assumed as

$$cov[w(j), w(k)] = R_w\delta_{jk} \qquad R_w = 1 = \sigma_w^2 \tag{14}$$
$$cov[v(j), v(k)] = R_v\delta_{jk} \qquad R_v = 1 = \sigma_v^2$$

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where σ denotes the standard deviation of the plant disturbances and the sensor errors. The initial condition of covariance of the prediction error, M(k) was assumed to be identity matrix and the iteration was repeated in 15 samples.

Kalman filter estimates the process can be described in two terms which are time update equations and measurement update equations. In time update, as covariance of the prediction errors M(k) was update by using equation (15).After that, the value of M(k) was used to update the measurement of Kalman gain, G(k) and covariance error vector, P(k) by equation (16).

$$M(k+1) = AP(k)A^T + B_1 R_w B_1^T \quad (15)$$

$$G(k) = M(k)C^T[CM(k)C^T + R_v]^{-1} \quad (16)$$
$$P(k) = M(k) - G(k)CM(k)$$

After the iteration has been done in 15 samples of iteration, the value of P(k) was. If P(k) was constant, the iterations were stopped and the value of Kalman gain, G(k) was measured. However, if the value of P changed after iteration, iteration will be done until the value of P became constant.

The flow chart of Kalman filter optimal optimal state for second order servo motor was similar to Fig. 4 by using initial condition of covariance of the prediction error, M(k) = [1 0; 0 1] and B1 =[1; 1].

Linear Quadratic Gaussian (LQG) of DC Servo Motor:

LQG controller for DC servo motor was implemented by using block diagram in Fig. 5.

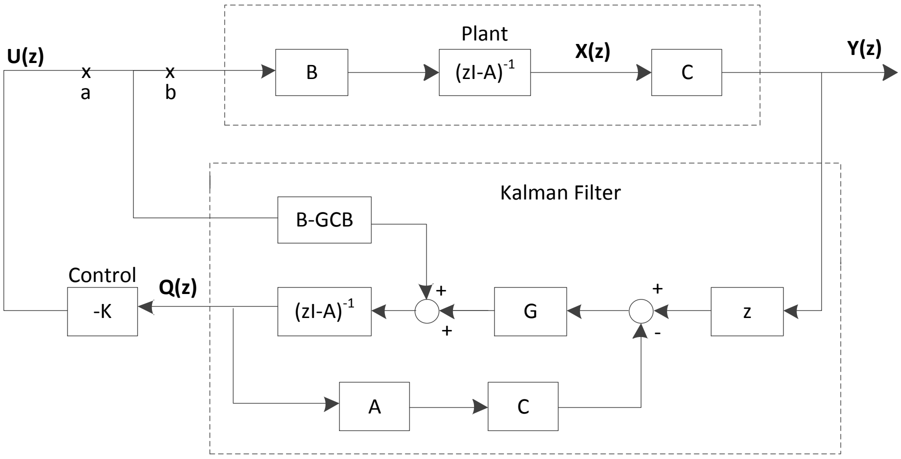

Fig. 5: LQG Control System

By using simplification of block diagram, block diagram of control-observer between LQ optimal control and Kalman filter in LQG digital controller can be described in Fig. 6

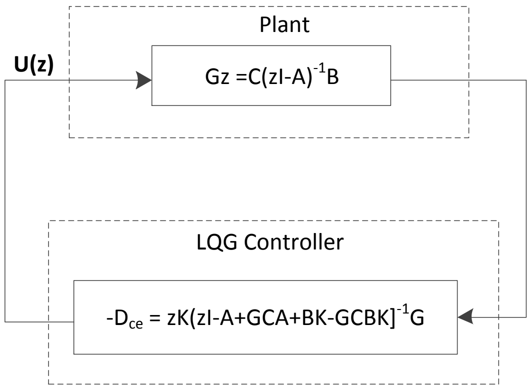

Fig. 6: Control-Observer Combination of Digital Controller

Then, LQG Digital controller can be realized in the system as

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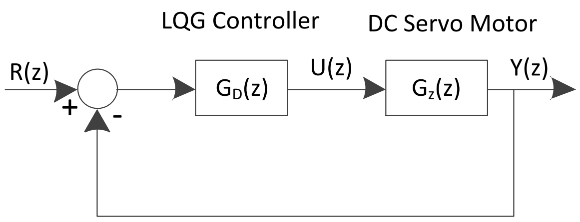

Fig. 7: LQG Controller of DC Servo Motor

Next, transfer function of digital controller was determined by using equation (17)

$$G_D(z) = D_{ce}(z) \tag{17}$$
$$G_D(z) = zK[zI - A + GCA + BK - GCBK]^{-1}G$$

Where
$G_D(z) = zK[zI - L]^{-1}G$ with $L = A - GCA - BK + GCBK$

RESULT AND DISCUSSION

The output response of DC servo motor before implementation of LQG controller and after implementation has been analyzed to compare the result. Besides that, the transient response of control system has also been observed to ensure the objectives of this project successfully achieved.

Analysis of Open-loop DC servo motor to Control Position and Speed:

Based on the transfer function that has been determined from M-file, the discrete transfer function of DC servo motor to control position was implemented in Simulink as shown in Fig. 8.

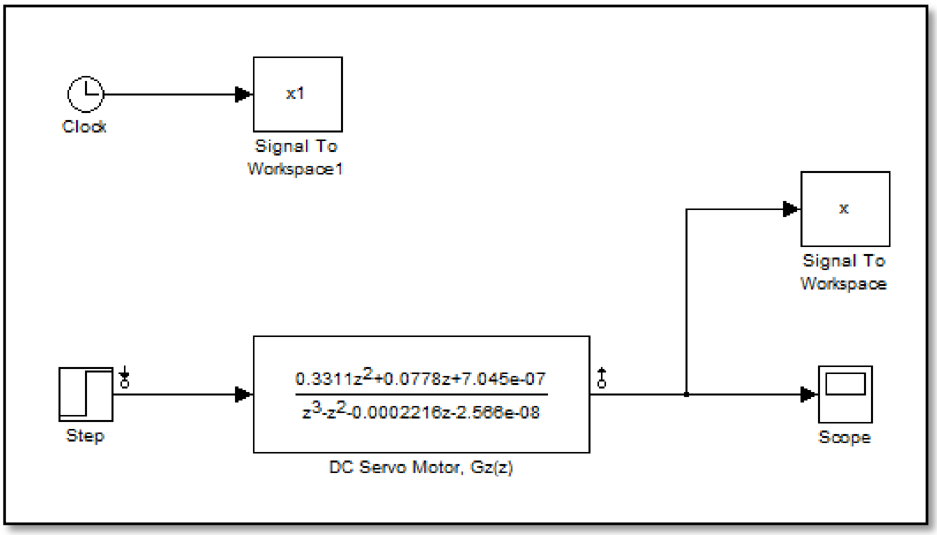

Fig. 8: Block Diagram of DC Servo Motor to Control Position

Fig. 9 show the step response of open-loop DC servo motor to control position.

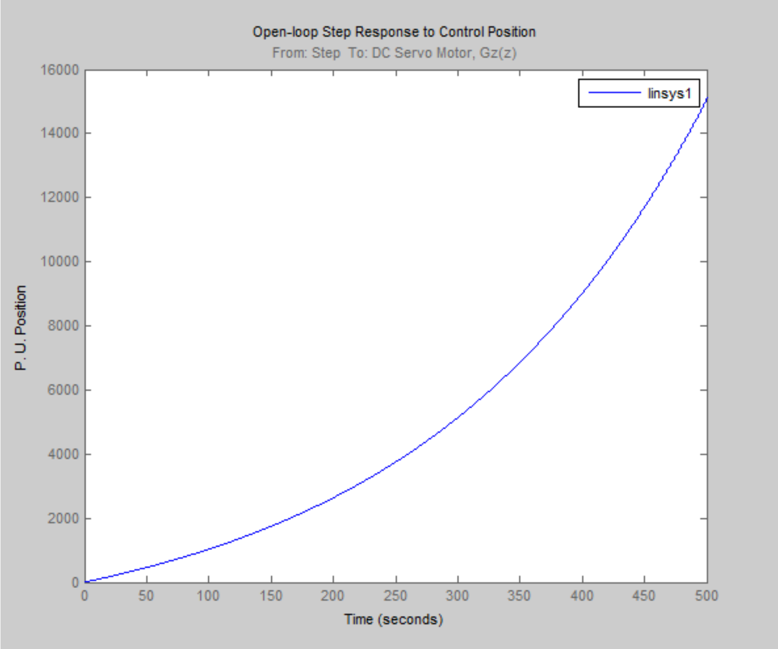

Fig. 9: Open-loop Step Response to Control Position

Based on Fig. 9, although open-loop DC servo motor has zero overshoot which generally indicates the system stability, however based on Fig. 9, the step response of DC servo motor intend to approach infinity which make the system is unstable.

Fig. 10 show the location of pole and zero for open-loop third order DC servo motor

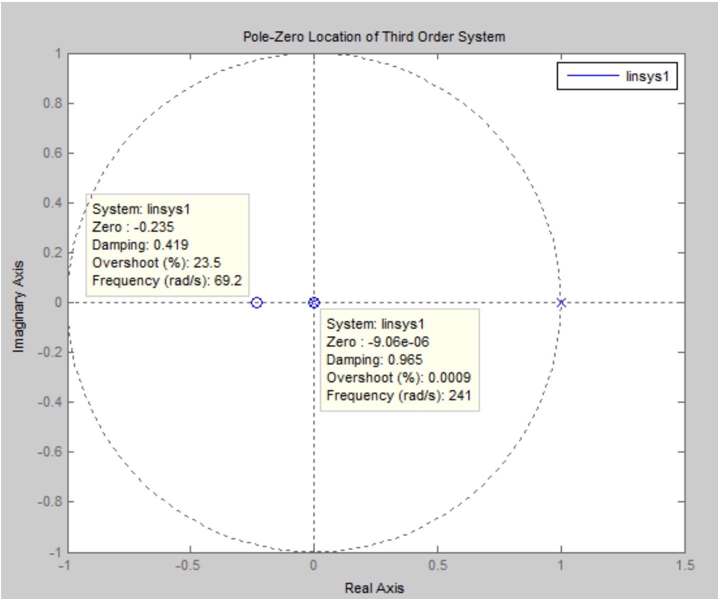

Fig. 10: Pole-Zero Location of Open-loop DC Servo Motor to Control Position

Based on Fig. 11, since the location of open-loop zero located in the unit circle, therefore the system may become stable during closed-loop system .

Fig. 11 show the block diagram of open-loop DC servo motor to control the speed.

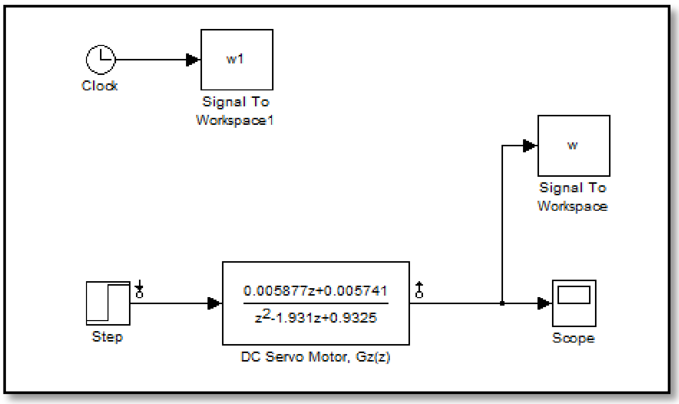

Fig. 11: Block Diagram of DC Servo Motor to Control Speed

The step response of open-loop DC servo motor to control speed was shown in Fig. 12.

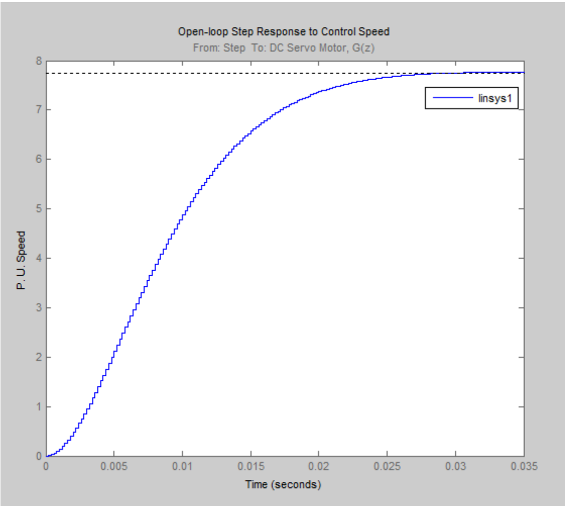

Fig. 12: Open-loop Step Response to Control Speed

Based on Fig. 12, eventhough open-loop DC servo motor had lower maximum overshoot however the system was unstable due to higher steady-state error.

Fig. 13 show location of the pole and for second order DC servo motor. Based on Fig. 15, since the open-loop pole was located in the unit circle, therefore the system may become stable during closed-loop system.

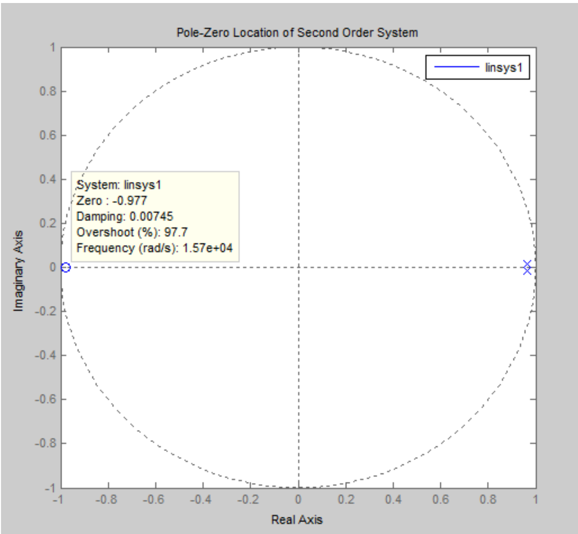

Fig. 13: Pole-Zero Location of Open-loop DC Servo Motor to Control Speed

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Analysis of Closed-loop DC servo motor to Control Position and Speed:

Fig. 14 shows the block diagram of digital control system to control position. In this control system LQG controller has been used to control the position of DC servo motor with unity feedback.

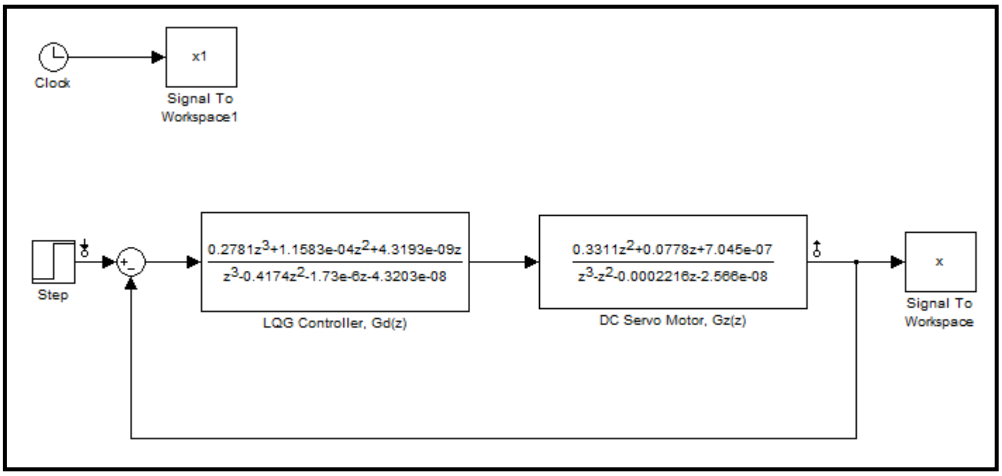

Fig. 14: Block diagram of Digital Control System to Control Position

Unit step input was used in designed this control system to guarantee the system easy to generate and sufficiently enable to provide useful information on both the transient response and the steady-state response characteristic of the system.

Fig. 15 show the step response of close-loop digital control system to control the position of DC servo motor.

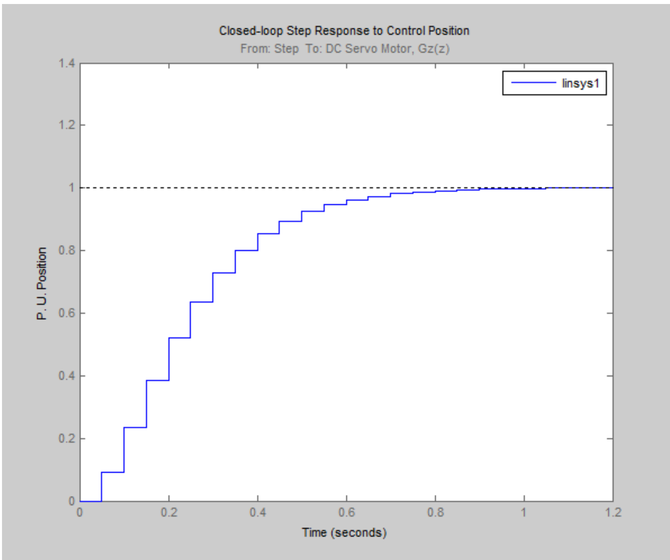

Fig. 15: Closed-loop Step Response to Control Position

Based on Fig. 15, the closed-loop digital control system to control position has zero maximum overshoot which directly indicates the relative stability of the system.

Final steady-state value of the response in Fig. 15 was 1, which defined zero steady-state error of the system. The stability of closed-loop control system was defined by the location of closed-loop poles in the in the z-plane as shown in Fig. 16.

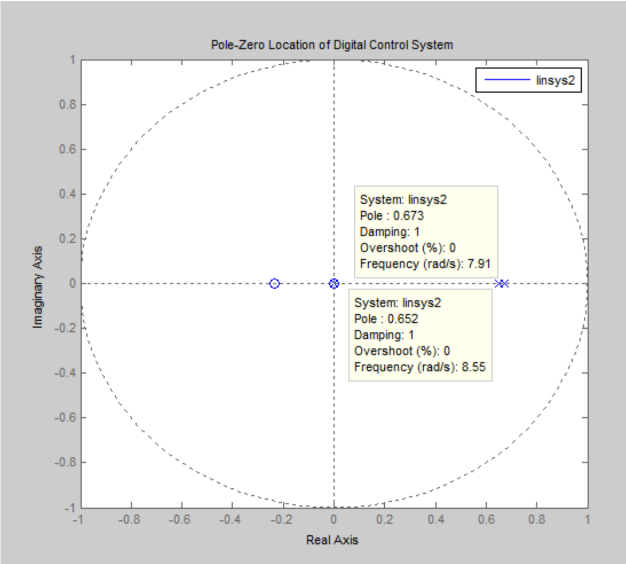

Fig. 16: Pole-Zero Location of Closed-loop to Control Position

Based on 16, since all closed-loop poles lies within unit circle in z-plane, thus the system was stable. Although, the location of closed-loop zero also lies in the unit circle, but closed-loop zero does not affect the absolute stability of the system and therefore may be located anywhere in the z-plane. Therefore, the closed-loop digital control system to control position using LQG controller may verified as stable.

Fig. 17 show the step response of closed-loop digital control system to control the speed of DC servo motor.

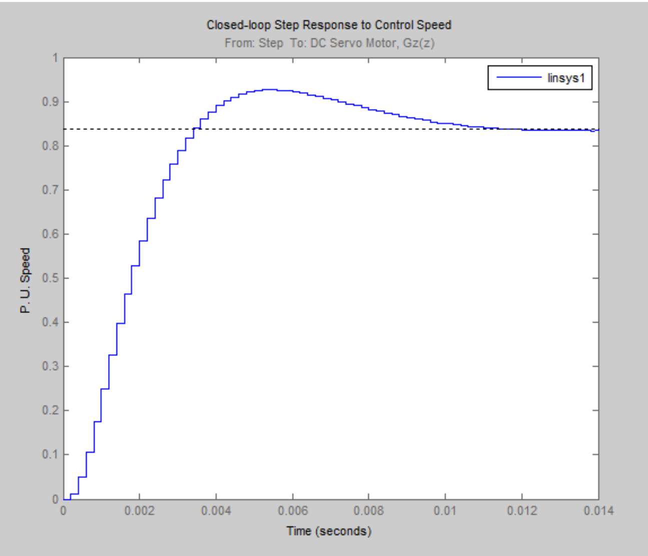

Fig. 17: Closed-loop Step Response to Control Speed

Based on Fig. 17, allthough this closed-loop had higher overshoot compared to its open-loop system, however lower settling time shows that system required shorter time for the response to reach steady state value.time for the response to reach steady state value.

Besides that, the final value of steady-state in closed-loop control system has been improved to reduce the value of steady-state error. The stability of closed-loop control system was defined by the location of closed-loop poles in the in the z-plane.

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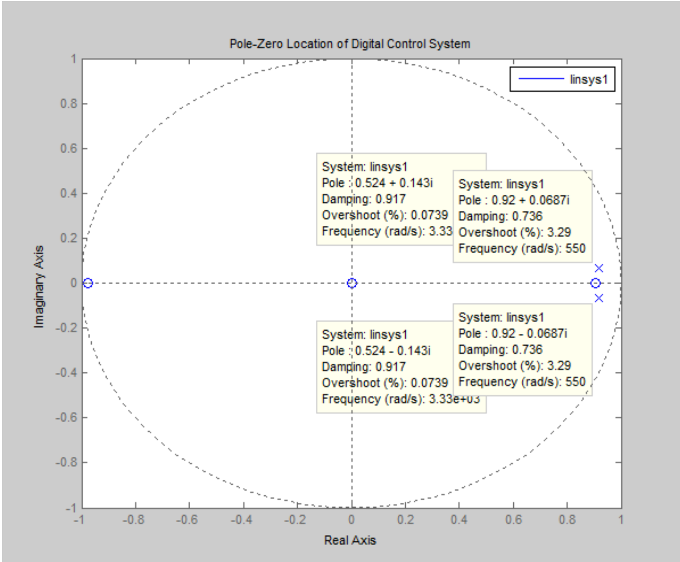

Fig. 18: Pole-Zero Location of Closed-loop to Control Speed

Based on 18, since all closed-loop poles lies within unit circle in z-plane, thus the system was stable.

Comparison of Open-loop and Closed-loop System:

Fig. 19 show the comparison of open-loop and closed-loop control system step response to control position.

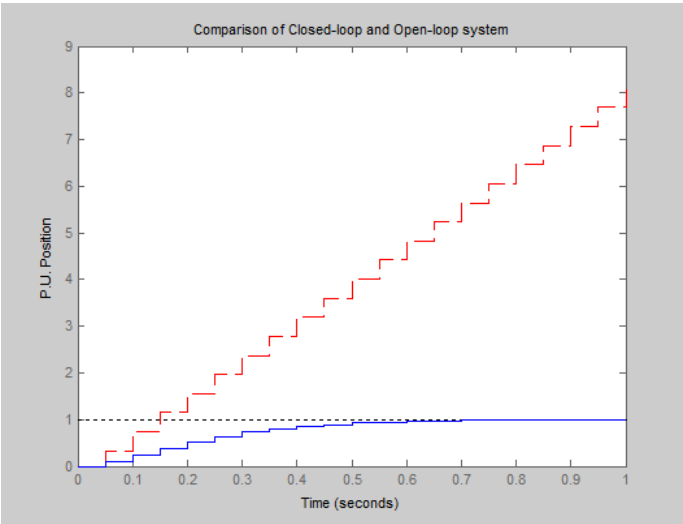

Fig. 19: Comparison of Open-loop and Closed-loop system to Control Position

The blue graph in Fig. 19 indicates the closed-loop control system using LQG controller while red graph indicates the open-loop control system without controller.

Based on comparison in Fig. 19, since the closed-loop system has better transient specification rather than open-loop system, it can be described that the performance of DC servo motor using LQG controller (closed-loop system) was much better compared to the performance of DC servo motor without controller (open-loop system)

Fig. 20 show the comparison of open-loop and closed-loop control system step response to control the speed of DC servo motor.

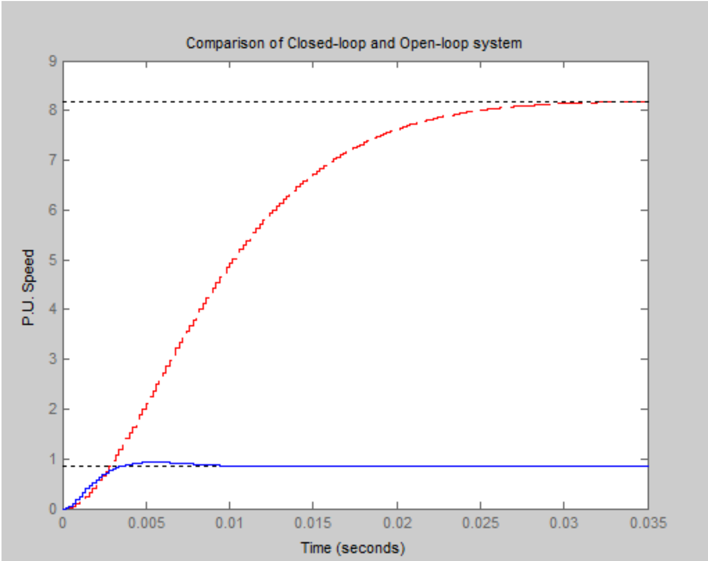

Fig. 20: Comparison of Open-loop and Closed-loop system to Control Speed

Based comparison in Fig. 20, since closed-loop system can improved the performance of steady-state error in open-loop system, therefore it can be described the performance of DC servo motor using LQG controller (closed-loop system) was better compare to the performance of DC servo motor without controller (open-loop system).

Conclusion:

As the conclusion, Linear Quadratic Gaussian (LQG) controller to control the speed and position of DC servo motor has been achieved. The controller has been designed based on parameter of BDC motor ID23005, LQ optimal control and Kalman Filter state estimation by using MATLAB software approached. Based on the comparison that has been done, it can be conclude that the closed-loop control system that used LQG controller to control the position of DC servo motor has better specification of transient response compare to the system without controller. This conclusion also applied to control the speed of DC servo motor. Besides that, it has been proved that the closed-loop DC servo motor has a better performance rather than open loop system in order to control position and speed.

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