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Vibration Control of Flexible Beam using Self-tuning Pole Placement Control Scheme

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ABSTRACT

This paper demonstrated the experimental studies of active vibration control (AVC) using self-tuning pole placement control scheme to suppress vibration of the flexible beam via real-time computer control. An active vibration control using real-time self-tuning pole placement control (STPPC) is developed to monitor and control the vibration of a flexible beam system. The controller is designed with a simple model structure which is 2nd order auto-regressive with exogenous input (ARX) model. The performance of self-tuning controller is investigated by exciting the beam with continuous disturbance force and changing its disturbance magnitude. The proposed controller is developed with real time computer control which run in personal computer (PC) based real-time control and its graphical user interface (GUI) has been developed in such a way that user can perform online monitoring and manipulation of control parameters that are part of the AVC of a flexible beam system. The effectiveness of self-tuning controller in suppressing the vibration of a flexible beam is observed and compared with the fixed controller.

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INTRODUCTION

Vibration reduction is a critical problem related to flexible structures especially in the application of aerospace application and robotics system, which often employs flexible structures that generally light weight and have relatively low damping for the fundamental and initial model. Furthermore, the frequency associated with these models are low, the vibration control of nodes become an important issue in light flexible structures. Active vibration control has been used as a solution for flexible structures to achieve sufficient vibration suppression for required precision accuracy.

With the emergence of smart materials such as a piezoelectric patch, the studies in active vibration control become more attractive. This is because smart materials offer low energy consumption, can be small in size, have fast response and can be integrated with the structures (Preumont, 2011). In the case of active vibration control of flexible structures, such a piezoelectric material is normally bonded onto the structure which acts an actuator or sensor. Hence, it will add complexity to the analysis and modeling of the system. Control strategies of flexible structures often depend on good modeling of the system dynamics. Many analytical model based approaches have been proposed to establish the physical model of the system behavior for a structure embedded with PZT such as finite element analysis, dynamic analysis of the modal response and etc. (Narayanan and Balamurugan, 2003; Tehrani *et al.*, 2011; Wang *et al.*, 2011b; Zhi-cheng *et al.*, 2009). However, those approaches are effective under high precision system because of the difficulty in simulating the properties for such complicated system and sometimes, hindered by factors such as assuming perfect bonding between the structure and its actuator, and high computation time (Ezhilarasi *et al.*, 2006). Furthermore, the assumption is contradictory to reality because of some special difficulties which involve, for example unmodeled dynamics of the flexible beam, component degradation, changing payload, changing structure parameters, etc. can destabilize a conventional fixed parameter control strategy (Kumar *et al.*, 2006).

When conventional control schemes fail to provide adequate performance because of the difficulties stated above, an adaptive or self-tuning control strategy could be adopted. Self-tuning controller represent an important class of adaptive controllers. The objective of self-tuning is to control systems with unknown constants or slowly varying parameters (Åström and Wittenmark, 1973; Wellstead and Zarrop, 1991). Recently, extensive efforts have been made to implement self-tuning control strategies to active vibration control of flexible structures.

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Zilletti *et al.* (2010) proposed a simple approach self-tune decentralized velocity feedback to control the vibration of a flexible beam. The simulation results showed that the vibration of the structure has been reduced. An important assumption in this study was a perfect force actuator being used. Mahmoodi *et al.* (2010) developed an adaptive controller based on modified positive position feedback (MPPF) control with unique features by extending its domain of capabilities from controlling tonal vibrations to broad band disturbances. MPPF was shown to experimentally reduce the vibration of a cantilever beam. Tjahyadi *et al.* (2006) investigated multi-mode vibration suppression of a flexible cantilever beam using adaptive resonant control where the structure's natural frequencies are estimated online and continuously update the adaptive resonant controller's parameters. Experimental results showed that the proposed controller produced better vibration attenuation over its fixed controller counterpart for a range of unmodelled dynamics.

Several researchers have applied adaptive pole placement control scheme in suppressing unwanted vibration of flexible structures. Among them, Zhang and Li (2012), and Kumar and Khan (2007) have developed an adaptive pole placement techniques in active vibration control. Zhang and Li (2012) studied the performance of adaptive pole placement control on a cantilevered beam. The robustness of the proposed controller was analyzed under various mechanical, electrical, and thermal conditions. The investigation was done through simulation where model was developed using finite element method (FEM). The pole placement controller was adaptively updated at different thermal conditions. Simulation results showed that adaptive pole placement control is effective in controlling the vibration of the cantilever beam at various thermal conditions. Kumar and Khan (2007) proposed adaptive pole placement controller for active vibration control of an inverted L structures. The controller is designed based on 4th order ARMAX model. Experimental results showed that the proposed controller is effective in suppressing the unwanted vibration caused by external force.

Even though several researchers have studied the performance pole-placement control in vibration control of flexible structures, however the implementation of self-tuning pole placement and its real-time implementation still rarely found in the literature. Therefore, it would be worth investigating the performance of self-tuning pole placement control scheme in real-time computer control which provide online monitoring and control of the actual system.

In this study, active vibration control is performed using online self-tuning pole placement control (STPPC) of a flexible beam system which is designed with a simple model structure auto-regressive with exogenous input (ARX) model. This simple structure offers several advantages which are easy to implement in real time and reduce computation time where the overall computational task can be performed effectively. The controller is executed on real-time personal computer (PC) based control. The implementation of self-tuning algorithm in LabVIEW programming is briefly explained. Its graphical user interface (GUI) was developed in such a way that user can perform online monitoring and manipulation of control parameters that are part of the active vibration control (AVC) of a flexible beam system. Results showing the transient performance between the self-tuning controller and a fixed controller due to disturbance change on a flexible beam. The effectiveness of self-tuning controller in suppressing the vibration of a flexible beam is observed and compared with a fixed controller.

2. Self-tuning control scheme:

In self-tuning control, it is assumed that the regulator parameters are adjusted continuously. This implies that the regulator parameters follow the changes in the process. When the process is known, the design procedure specifies a set of desired controller parameters. The adaptive controller should converge to these parameter values even when the process is unknown. A regulator with this property is called self-tuning, since it automatically tunes the controller to the desired performance (Åström and Wittenmark, 1973; Wellstead and Zarrop, 1991). The schematic diagram of STPPC is shown in Fig. 1. The plant output, $y(t)$, is in response of control input, $u(t)$, plus response to system disturbances, $d(t)$.

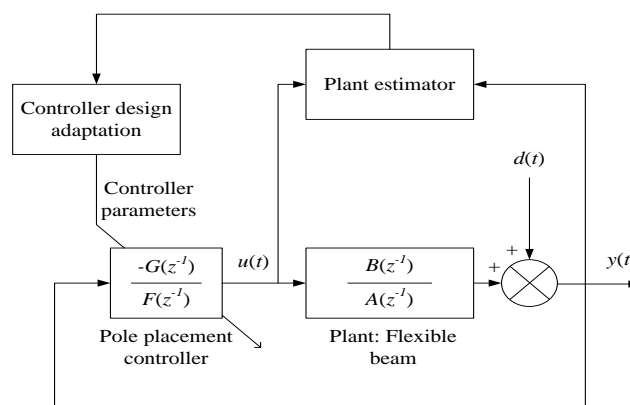


Fig. 1: Self-tuning pole placement control structure

The disturbance consists of measurement noise and excitation from external vibration force. Plant estimator estimates the unknown plant parameters using on-line parameter estimation scheme such as recursive least square, stochastic approximation, extended least squares, maximum likelihood, etc. These parameters are fed to the controller parameter modifier. In this study ARMAX model is considered which is represented by:

$$A(z^{-1})y(t) = B(z^{-1})u(t) + C(z^{-1})d(t) \quad (1)$$

where $u(t)$ is the input signal, $y(t)$ is the output signal, $d(t)$ is the noise disturbance and $A(z^{-1})$, $B(z^{-1})$ and $C(z^{-1})$ are the system polynomial define as:

$$A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + \dots + a_{n_a}z^{-n_a}$$

$$B(z^{-1}) = b_1z^{-1} + b_2z^{-2} + \dots + b_{n_b}z^{-n_b}$$

$$C(z^{-1}) = 1 + c_1z^{-1} + c_2z^{-2} + \dots + c_{n_c}z^{-n_c}$$

where n_a and n_b are the orders of the auto-regressive (AR) with exogenous terms; n_c is the order of the disturbance model.

As described by Clarke (1996), a system can also be self-tuned by RLS without estimating the polynomial $C(z^{-1})$. Hence it will reduce the complexity of the identification process as well as minimize burden to the computation time which is very important in practical implementation. Therefore by referring to Figure 6.1, Eq. (1) is now reduced to:

$$A(z^{-1})y(t) = B(z^{-1})u(t) + d(t) \quad (2)$$

and

$$u(t) = -\frac{G(z^{-1})}{F(z^{-1})}y(t) \quad (3)$$

where,

$$G(z^{-1}) = g_0z^{-1} + g_1z^{-2} + \dots + g_{n_g}z^{-n_g}$$

$$F(z^{-1}) = 1 + f_1z^{-1} + f_2z^{-2} + \dots + f_{n_f}z^{-n_f}$$

n_g and n_f are the orders of polynomials $G(z^{-1})$ and $F(z^{-1})$ respectively.

By substituting Eq. (3) into Eq. (2), the closed loop system Eq. of Figure 6.1 can be obtained as:

$$\frac{y(t)}{d(t)} = \frac{F(z^{-1})}{A(z^{-1})F(z^{-1}) + B(z^{-1})G(z^{-1})} \quad (4)$$

where $d(t)$ is a disturbance signal.

The characteristic polynomial of Eq. (4) defines the position of the closed loop poles of the system that will dominate the dynamic performance. Therefore, the closed-loop poles of the system will be shifted to the locations defined by the desired characteristic equation, $T(z^{-1})$ given by:

$$T(z^{-1}) = A(z^{-1})F(z^{-1}) + B(z^{-1})G(z^{-1}) \quad (5)$$

where,

$$T(z^{-1}) = 1 + t_1z^{-1} + t_2z^{-2} + \dots + t_{n_t}z^{-n_t}$$

The closed-loop system is stable if all the roots of $T(z^{-1})$ are within the unit circle. The controller parameters, F and G , can be solved from the Diophantine equation in Eq. (5). Hence, the control output at instant of time, t , is given by:

$$u(t) = -f_1(t)u(t-1) - f_2(t)u(t-2) - \dots - f_{n_f}(t)u(t-n_f) - g_0(t)y(t) - g_1(t)y(t-1) - \dots - g_{n_g}(t)y(t-n_g) \quad (6)$$

3. Controller Implementation:

The control algorithm was implemented using LABVIEW programming software. LABVIEW offers a simple graphical user interface approach which is able to execute algorithm via PC-based control. Fig. 2 summarizes the algorithm for STPPC. It consists of 4 main parts: (A) input and output interfacing (DAC and ADC), (B) parameter estimation, (C) controller design and adaptation mechanism, (D) controller, and (E) disturbance excitation signal. The sampling rate for input and output interface was set to 1 kHz which met the rule proposed by Shannon (Young and Jakeman, 1980).

In order to deal with time varying system, RLS is chosen as the parameter estimation algorithm where the parameters are updated each time new measurements are obtained. For online self-tuning control, the model order must be as simple as possible. This will reduce the computation burden while the process is in operation (Virk and Al-Dmour, 1999). Hence, a second order ARX model is used. The STPPC can be described by the following steps:

1. RLS technique is used to estimate the polynomials A and B in Eq. (2) of a flexible beam system (Fig. 2 part B). In this study, the second order polynomials are chosen.
2. The controller polynomial, F and G are determined such that the desired closed loop response is obtained. These polynomials are solved using the Diophantine matrices at each sample using user desired characteristic equation (Fig. 2 part C). The controller parameter, f_1 , g_0 and g_1 can be calculated from Diophantine matrices represent as:

$$\begin{bmatrix} 1 & a_1 & 0 \\ a_1 & b_2 & b_1 \\ a_2 & 0 & b_2 \end{bmatrix} \begin{bmatrix} f_1 \\ g_0 \\ g_1 \end{bmatrix} = \begin{bmatrix} t_1 - a_1 \\ t_2 - a_2 \\ 0 \end{bmatrix} \tag{7}$$

3. The control parameters of F and G are sent to the controller where the control output is sent to piezo actuator via DAC to control the process to meet the desired response. (Fig. 2 part D).The control signal at an instant of time is given by:

$$u(t) = -f_1(t)u(t-1) - g_0(t)y(t) - g_1(t)y(t-1) \tag{8}$$

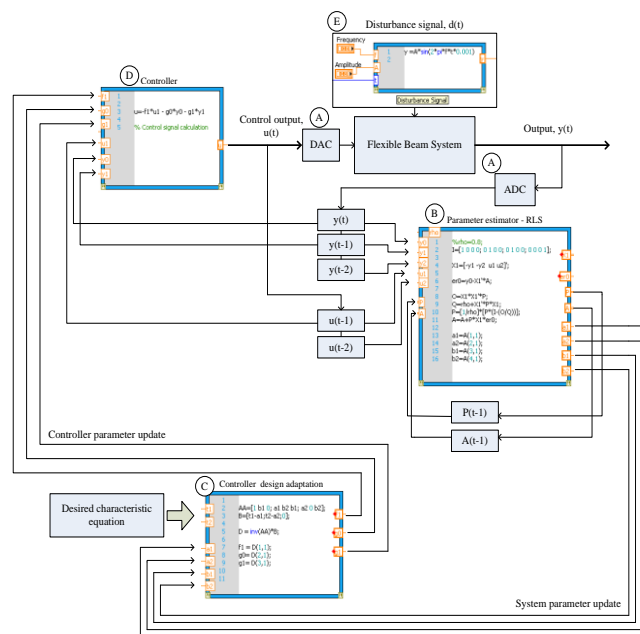


Fig. 2: Algorithms of self-tuning pole placement control

3. Experimental Setup:

In order to validate the developed model and verify the effectiveness proposed control schemes, experimental study was conducted based on the schematic diagram shown in Fig. 3. The Properties and dimensions of the aluminum beam and piezoceramic actuator are showed in Table 1. Two piezoceramic patches model P-876.A12 DuraAct were surface bonded on the beam to act as disturbance actuator and control actuator. Two linear power amplifiers were used. Both amplifiers have gains up to 10 and output voltage from -100 V to +100 V which are sufficient to amplify the input signals to actuate the piezoceramic patches. A Sunx laser displacement sensor model ANR-1250 (range: 50 ± 10 mm and resolution: $5 \mu\text{m}$) was used to measure beam displacement. This

experiment was conducted in collocated control where the locations of sensor and control actuator are at the same position. A real-time computer control system was implemented using personal computer (Intel core I3, 2.93 GHz). Online monitoring and control algorithm were developed with LabVIEW programming software. LabVIEW programming is suitable for on-line real time application where interfacing between input-output signal and computer system can be done easily using data acquisition card (National Instruments PCI-6259). National instruments (NI) data acquisition card was used to acquire analog input from the laser displacement sensor, computed for actuating signals by the controller, and sends the actuating signal to piezo amplifier.

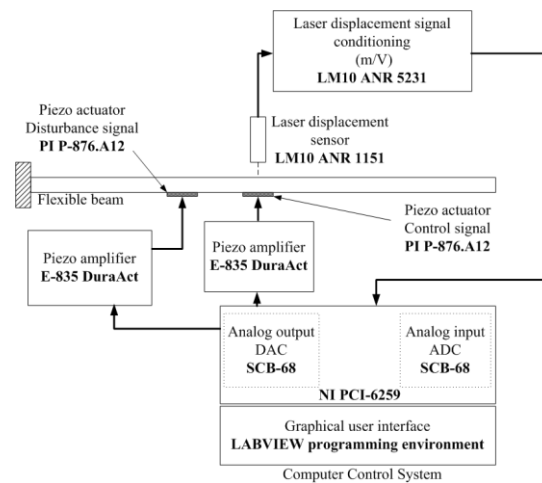


Fig. 3: Schematic blok diagram of experimental rig for flexible beam

Table 1: Properties and dimensions of the aluminum beam and piezoceramic actuator

Parameter	Numerical Values
Length (mm)	500
Width (mm)	50
Thickness (mm)	1.1
Young's modulus (GPa)	71
Density (Kg/m ³)	2700
Dimensions (mm)	61×35×0.5
Mass (g)	3.5
Active area (cm ²)	15
Supply voltage (V)	-100 to 400
Blocking force (N)	265
Young's modulus (GPa)	23.3

One of the scopes of this research is to develop a visual graphical user interface (GUI) for online monitoring and control the unwanted vibration signal occurring on a flexible beam system. The GUI was developed using LabVIEW programming referred as virtual instrument (VI), where VI source code is created using the graphical programming language, *G*, in a window called the block diagram. LabVIEW programming software is used to develop the proposed control algorithms and provide online monitoring such as graphs and numerical indicators incorporated with personal computer based data acquisition system (PC based DAQ). Online monitoring and control provides user to change the control parameter, on/off controller, change controller mode, change disturbance amplitude, etc. while the LabVIEW programming is running. With these important features user has the ability to observe the performance of active vibration control of a flexible beam system in real-time environment.

The corresponding GUI is shown in Fig. 4. The GUI is splitted into 8 areas. The DAQ configuration (area (1)) enables user to select the respective input/output channel, save data, stop program and input/output range setting. The timing setting area (2) allows user to set input/output rate and sampling time. The sampling period must be adapted to the dynamics of the physical system. If the sampling period is too large, the original signal's information may not be completely recoverable from the sampled signal, hence it will cause detrimental effect on the controllability and stability of closed loop system (Perdikaris, 1991). The amplitude and frequency for the disturbance and actuator signals can be adjusted in area (3) via a numerical control box. It also enables user to control manually the actuator signal (i.e. open loop control). User is allowed to select open loop control (system without pole placement controller) in area (4) which can be activated by clicking on the respective control button. In area (5) user is allowed to set the desired pole locations, adjust the controller gain, and monitor the desired polynomial equation. The online chart display panel (area (6)) displays real time data on a PC screen which helps user to monitor the behavior of the system. The display is updated at each sample interval. Area (7) displays the open loop and closed loop pole locations on the *z*-plane. Finally, the model parameter estimate convergence

profiles and model predictive error can be observed in area (8). With this GUI, it really helps user to determine suitable control parameter settings to which that the unwanted vibration can be suppressed effectively.

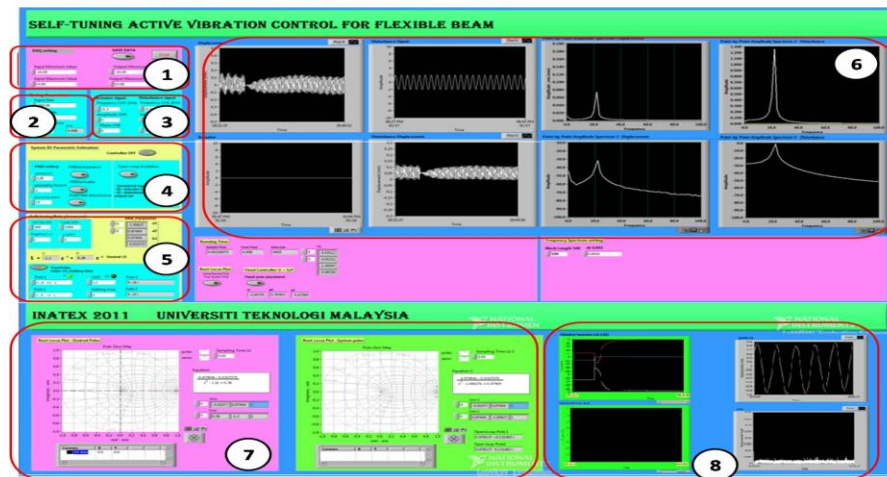


Fig. 4: Graphical user interface of active vibration control for flexible beam

Experimental Result:

The experiment was developed to investigate the performance of self-tuning pole placement control in suppressing the unwanted vibration of a flexible beam when subjected to the external disturbance. The beam is subjected to the unwanted disturbance of 1 V for about 26 s, and then the unexpected disturbance at magnitude of 5 V is applied. The response of the self-tuning STPPC is compared with the responses of the fixed controller.

Fixed Controller:

The overall time response of vibration displacement for the fixed controller when the disturbance amplitude is changed from 1 to 5 V can be seen in Fig. 5. It can be seen clearly that when the controller is ON, the beam's deflection is decreased to 0.009-(-0.013) = 0.022 mm peak to peak deflection at disturbance of 1 V. Then, when the disturbance is changed to 5 V, beam's deflection increased to settle at its steady state oscillations at about 0.046-(-0.048) = 0.094 mm peak to peak deflection. The control voltage has increased from 2.29 to 6.90 V in order to cope with the change of disturbance signal.

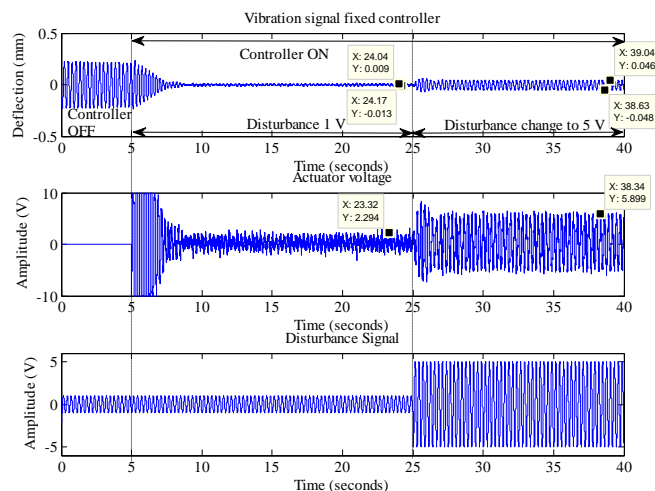


Fig. 5: Fixed controller time response

Self-tuning Pole Placement Control:

Fig. 6 shows the overall performance for the STPPC with the disturbance amplitude is changed from 1 to 5 V. It is observed that after controller ON at 5 s, the peak to peak deflection amplitude is reduced to 0.011-(-0.012) = 0.022 mm. Then soon after the disturbance changed from 1 to 5 V, the peak to peak deflection amplitude of a flexible beam is increased to 0.030-(-0.038) = 0.068 mm. The vibration attenuation obtained from STPPC has shown a better performance compared with the fixed controller. It can be seen that the control voltage during 5 V disturbance produced by STPPC is a bit higher than the fixed controller which is about 6.16 V. The controller

parameter continuously update with the changed of system parameter. It is observed that, the changed of controller parameters is very small. This due to the behavior of STPPC itself which means that the controller parameters will significantly changed when there is a change in its physical system.

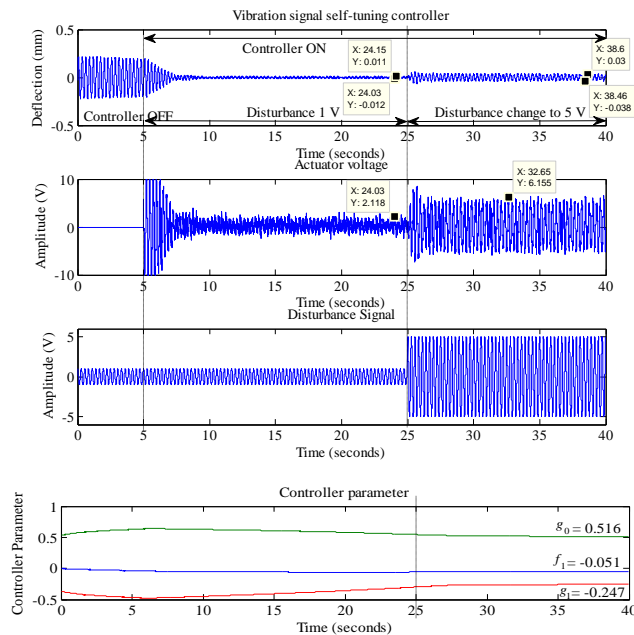


Fig. 6: STPPC time response

Figs. 7 and 8 demonstrate the robustness of all the control schemes when the amplitude of the disturbance signal varies from 1 to 5 V. It is observed that fixed controller and STPPC produce almost the same settling time which is at 2.10 and 1.94 s respectively. Moreover, it is also noted that the maximum peak amplitude for the fixed controller and STPPC has produced almost the same value. This is due to the behavior of STPPC which significantly vary its controller parameters to the change of physical system.

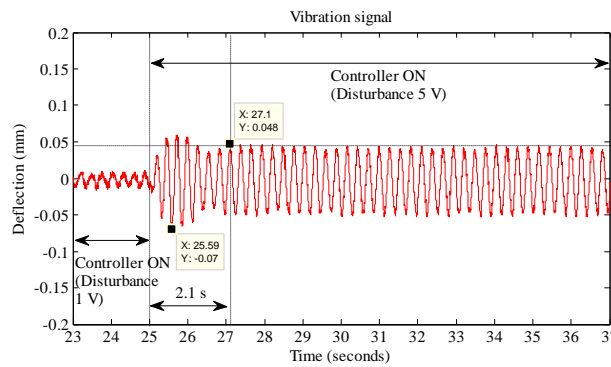


Fig. 7: Time response during disturbance change from 1 to 5 V for fixed controller

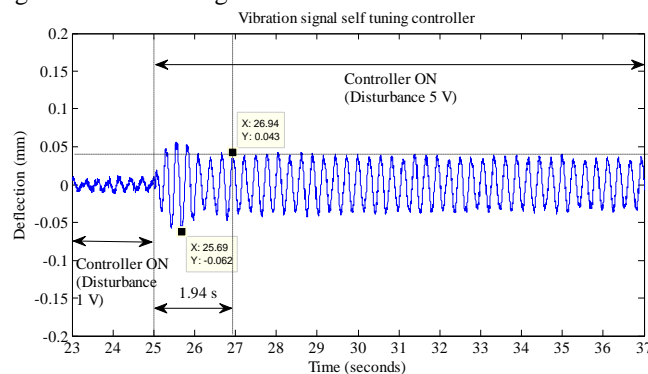


Fig. 8: Time response during disturbance change from 1 to 5 V for STPPC

Frequency Response:

A better representation of controller performance at 1 and 5 V disturbances can be observed using frequency response plot as shown in Figs. 9 and 10 respectively. From Fig. 9, it can be observed that both control schemes have shown good vibration attenuation. It is noted that the fixed controller and STPPC offer almost similar performance with the attenuation achieved at 28.09 and 28.76 dB respectively.

While there is a disturbance changed from 1 to 5 V at 26 s, vibration attenuation produced by the fixed controller and STPPC has increased to 13.10 and 16.11 dB respectively (Fig. 10). The performance degradation before and after disturbance for the fixed controller and STPPC are increased about 14.99 and 12.66 dB. The overall results are summarized in Table 2. From the comparative study, it can be concluded that the performance of STPPC with the change of disturbance is not really significant, since the system parameters do not really affected by the disturbance signal.

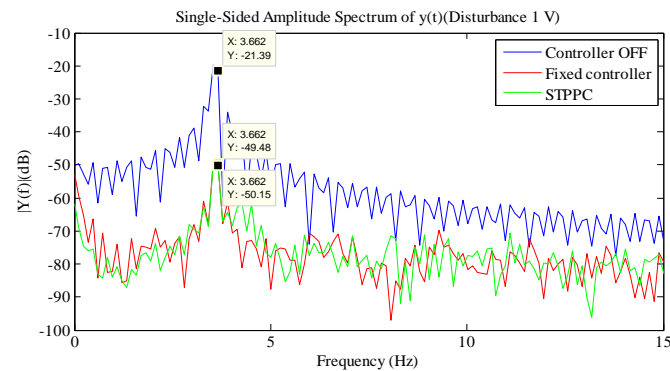


Fig. 9: Frequency response at disturbance 1 V

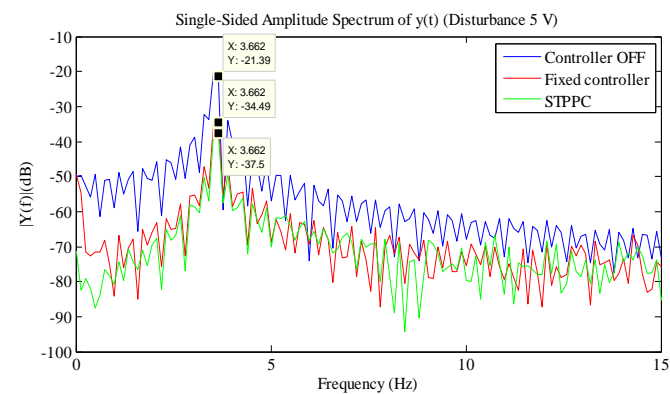


Fig. 10: Frequency response at disturbance 5 V

Table 2: Comparison of controller performance

Control schemes	Settling time disturbance changed from 1 to 5 V (s)	Peak amplitude at disturbance changed (mm)	Vibration attenuations at 1 V disturbance (dB)	Vibration attenuations 5 V at disturbance (dB)
Fixed controller	2.10	0.07	28.09	13.10
STPPC	1.94	0.06	28.76	16.11

Conclusion:

A self-tuning pole placement control scheme which is implemented in PC based control has been developed to control the vibration of a flexible aluminum beam using piezoelectric actuators. The controller is designed using system identification and pole placement to control the vibration. A graphical user interface has been developed using LabVIEW software that helps user to easily change the controller setting and provide online monitoring of the vibration of the flexible beam. The experimental results show that piezoelectric actuators bonded on the beam has effectively suppressed the vibration of flexible beam. STPPC offers better performance over a fixed controller when tested with disturbances of different magnitudes. In view of the above practical results, it is shown that self-tuning control scheme offer a viable solution for real-time vibration problems.

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