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Chaos in Wireless Communication Channels

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ABSTRACT

Chaos has been previously observed in physical systems, electrical circuits, optoelectronics, biological-systems, physiology, economics etc. In this paper we report, for the first time, the presence of chaotic behavior in the fluctuations of amplitude and phase of the radio signal received through a multipath wireless channel. The time series analysis indicates the existence of a positive value of the dominant Lyapunov exponent. This fact is corroborated by exhaustive simulations and analysis of measurement-data obtained from an indoor wireless-channel. We have also detected the presence of chaos in the received signal when two closely-spaced receiver antennas are gradually moved away from the transmitter. Thus, both temporal and spatial chaos has been observed in these wireless channels. These interesting findings open up a new area of research where multipath wireless channels can be used to generate chaos, with potential applications in chaos-based secure-communications for the next-generation wireless communications devices.

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INTRODUCTION

There has been a phenomenal growth in the domain of wireless communications in the recent years. A wireless propagation channel is the medium linking the transmitter and the receiver. Wireless channels differ from wired channels by multipath propagation, i.e., there exists a multitude of paths from the transmitter to the receiver. The signal from the transmitter can be reflected, diffracted or scattered along its way from the transmitter to the receiver (T.S. Rappaport, 2002: D. Tse and P. 2005: W.C.Y. Lee, 1997) It is known that the received power, on an average, decreases with increase in the separation between the transmitter and receiver. However, there are fluctuations around the mean which can be modeled stochastically.

Small scale fading is used to describe the rapid fluctuations of the amplitude and phases of a radio signal over short periods of time or travel distance. Fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times (J.G. Proakis, 2001: W.C. Jakes: A. Goldsmith, 1974). The objective of this paper is to report the presence of chaotic behavior in the fluctuations of amplitude and phase of the radio signal received through a multipath wireless channel. To the best of knowledge of the author, a systematic investigation of the chaotic behavior of multipath wireless channels has not been reported earlier. The findings are corroborated both by exhaustive simulations as well as actual measurements carried out on wireless channels.

2. Fading Channels:

In the case when there is no Line of Sight (NLOS) between the transmitter and receiver, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelop of a fading signal (A.F. Molisch, 2005). The probability density function (pdf) of the Rayleigh distribution is given by

$$p_{NLOS}(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) & 0 \le r \le \infty \\ 0 & r < 0 \end{cases} \tag{1}$$

where is the root mean square (rms) value of the received voltage. The pdf of the phase of the received signal is uniformly distributed and can be expressed as

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$$p_{NLOS}(\phi)=\frac{1}{2\pi} \tag{2}$$

However, if there is a Line of Sight (LOS) between transmitter and receiver, there exists a dominant non-fading signal component along with the multipath components. In this scenario, the pdf of the received envelop is given by the Rice distribution (A.F. Molisch, 2005)

$$p_{LOS}(r)=\begin{cases}\frac{r}{\sigma^2}\exp\left(-\frac{r^2+A^2}{2\sigma^2}\right)I_0\left(\frac{Ar}{\sigma^2}\right) & 0\le r\le\infty, A\ge 0\,,\\ 0 & r<0\end{cases} \tag{3}$$

where A is the amplitude of the dominant component and $I_0(x)$ is the modified Bessel function of the first kind and zero order. The Rice factor, $K=\left(\frac{A^2}{2\sigma^2}\right)$, represents the ratio of the power of the LOS component and the diffuse components. The pdf of the received phase for the LOS scenario is given by (A.F. Molisch, 2005)

$$p_{LOS}(\phi)=\frac{1}{2\pi}\exp\left(-\frac{A^2}{2\sigma^2}\right)\times\left[1+\sqrt{\frac{\pi}{2}}\frac{A\cos(\phi)}{\sigma}\exp\left(\frac{A^2\cos(\phi)^2}{2\sigma^2}\right)1+erf\left(\frac{A\cos(\phi)}{\sigma\sqrt{2}}\right)\right]. \tag{4}$$

In this paper we report, for the first time, the presence of both temporal and spatial chaos in a multipath wireless channel.

3. Lyapunov Index for Wireless Channels:

The dominant Lyapunov exponent is one of the commonly used indicators to describe the qualitative behavior in a dynamical system using the analysis of uni-dimensional time series. The presence of chaos in a dynamical system can be detected by measuring the largest Lyapunov exponent (R.C. Hilborn, 1994 : H.G. Schuster, 1984). Lyapunov exponents quantify the exponential divergence of initially close state-space trajectories and estimate the amount of chaos in a system (S.H.Strogatz, 1994). The Lyapunov exponent may be defined as

$$\lambda=\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\ln\left|\frac{d_{n+1}}{d_n}\right|, \tag{5}$$

where d_n is the separation after n discrete steps and N is the evolution length. Taking the expectation on both sides of (5) we get

$$E[\lambda]=\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}E\left[\ln\left|\frac{d_{n+1}}{d_n}\right|\right]\ . \tag{6}$$

Now from Jenson's Inequality (T.M. Cover and J.A. Thomas), for a concave function $f(X)$, $E[f(X)]\le f(E[X])$. Thus, (6) can be rewritten as

$$E[\lambda]\le\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\ln\left(E\left[\left|\frac{d_{n+1}}{d_n}\right|\right]\right)$$

$$=\ln\left(E\left[\left|\frac{d_{n+1}}{d_n}\right|\right]\right)=\lambda_{UL}, \tag{7}$$

where λ_{UL} represents the upper limit on the possible value of the Lyapunov exponent. Extensive simulations were carried out to determine the value of λ_{UL} for the LOS and NLOS scenarios and the results are given in Table 1. For the simulations, 10^6 data points were taken in order to calculate the value of λ_{UL}.

Table 1: Values of λ_{UL} obtained by simulations.

	Amplitude Process	Phase Process
Line of Sight (Rice factor $K = 2$)	1.6952	1.7039
Non Line of Sight	1.7560	1.7354

We also estimated the value of the largest Lyapunov exponent for a time series by the method called the length of evolution (A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastano, 1991 : X. Zeng, R. Eykholt, and R. A. Pielke, Phys. Rev. 1991 : J. Wright, Phys. 1984). The largest value of the Lyapunov exponent is given by

$$\lambda_1(i) = \frac{1}{i\Delta t}\frac{1}{(M-i)}\sum_{j=1}^{M-i}\ln\frac{d_j(i)}{d_j(0)}, \tag{8}$$

where, Δt is the sampling period of the time series and $d_j(i)$ is the distance between j^{th} pair of nearest neighbors after i discrete-time steps, i.e., $i\ \Delta t$ seconds. In (8), $M = N - (m - 1)J$ where J is the lag or reconstruction delay, and m is the embedding dimension (S. Sato, M. Sano, and Y. Sawada, Prog. Theor. 1987). Simulations were carried out for $3 \le m \le 7$ and $1 \le J \le 7$. The average value of the largest Lyapunov exponent for the NLOS amplitude process is found to be 0.3071 while for the NLOS phase process it is found to be 0.3132. The Monte-Carlo simulations have been run 10^6 times. It is interesting to note that the estimate of the largest Lyapunov exponent is fairly consistent across the entire range of embedding dimension and reconstruction delay, as shown in Fig. 1. Fig. 1(a) depicts the $\lambda_1$computation for the NLOS amplitude process while Fig. 1(b) depicts the computation for the NLOS phase process. In both the cases, the standard deviation for the different m and J values is less than 0.03. The existence of a positive value of the Lyapunov exponent shows the presence of chaos in both amplitude and phase processes in NLOS wireless channels.

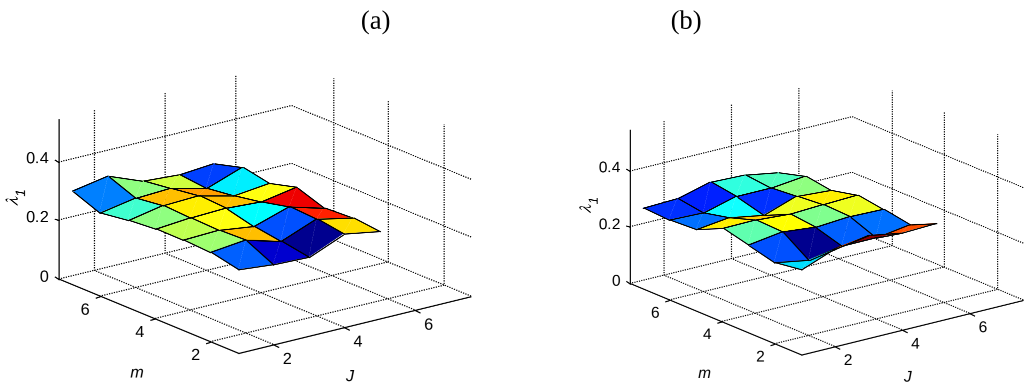

Fig. 1: The estimate of the largest Lyapunov exponent, λ_1, for the case of NLOS multipath wireless channel for (a) the amplitude process and (b) the phase process.

Simulation Results:

In order to corroborate the simulation results, time series analysis was carried out on actual wireless-channel measurement-data. The measurements were carried out indoors in a laboratory environment at 2.4 GHz. Dipole antennas were used both at the transmitter and receiver. The transmitter was kept stationary while the receiver was moved with respect to the transmitter. The received power was measured using a Rhodes and Schwartz spectrum analyzer. The measurement data obtained at each receiver location was averaged 100 times.

The experimental set-up is shown in Fig 2(a). A typical profile of the received signal is shown in Fig. 2(b). From a set of 100 measurement data, the average value of the largest Lyapunov exponent, λ_1 is calculated to be 0.3315. The existence of a positive value of the Lyapunov exponent shows the presence of *temporal* chaos in the amplitude process of the indoor wireless channel. The complex phasor of the received signal is plotted in Fig 3(a). In a multipath wireless channel, yet another variable is the angle of arrival of the signal at the receiver antenna. This is because the signal can be reflected, diffracted or scattered along its way from the transmitter to the receiver. For an omni-directional antenna, a typical 3-D phasor plot taking into consideration the angle of arrival of the received signal is shown in Fig. 3 (b).

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(a) (b)

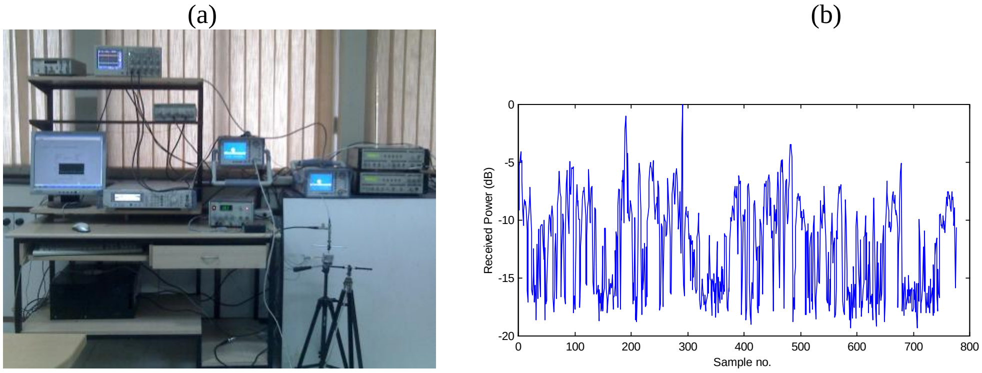

Fig. 2: (a) The experimental set-up. (b) A typical profile of the received signal.

We have also observed spatial chaos for multipath wireless channels. We investigated the signals obtained simultaneously at two closely-spaced receiver antennas as they move away from the transmitter. The experiment was conducted indoors in the laboratory environment at 2.4 GHz with the spacing between the receiver antenna pair equal to 0.25Λ, where Λ represents the wavelength of the radio signal.

(a) (b)

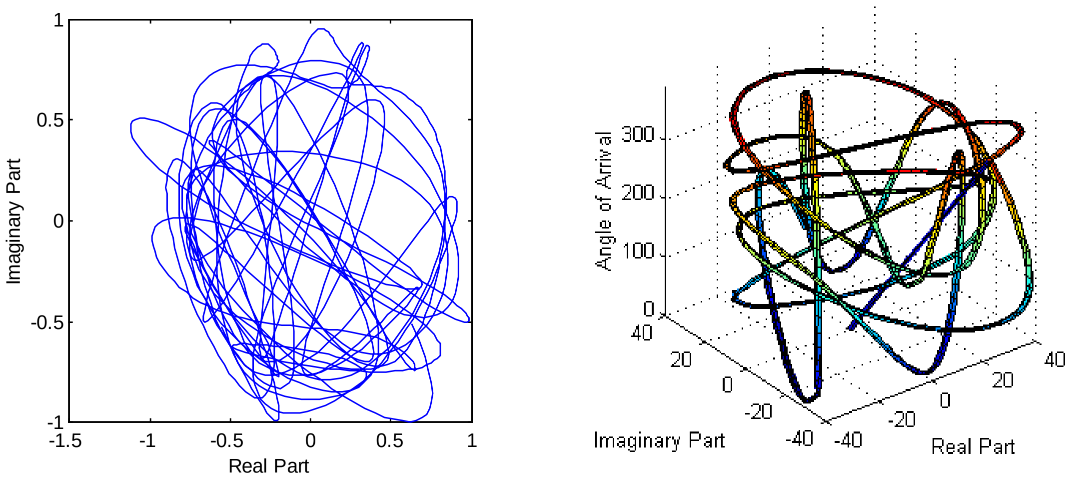

Fig. 3: (a) A typical phasor plot of the (normalized) complex received signal obtained by measurement for an indoor environment. (b) A typical 3-D phasor plot taking into consideration the angle of arrival of the received signal.

The received power profiles obtained at the two closely-spaced antennas are plotted in Fig. 4. The objective here is to investigate the rate of divergence of the two trajectories. The starting point is chosen such that the received signal in both antennas is approximately equal (difference of about 10^{-3}). As we move the receiver antenna pair away from the transmitter, the received signal trajectories diverge. Experiments were repeated 100 times at different locations of the laboratory in order to obtain different wireless channel realizations. The average value of the Lyapunov exponent was found to be 0.2941, indicating the presence of *spatial* chaos in wireless channels.

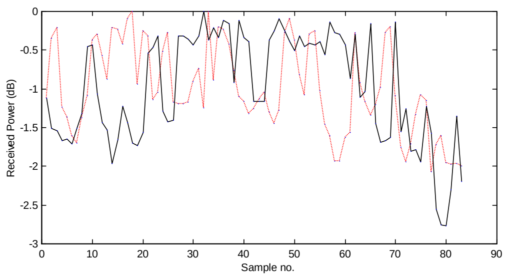

Fig. 4: Measured power at two closely-spaced antennas at the receiver as they move away from the transmitter.

Conclusion:

This paper reports, for the first time, the presence of chaotic behavior in wireless channels. We have shown, by exhaustive simulations, the existence of a positive Lyapunov exponent, both in the case of amplitude fluctuations as well as phase fluctuations for the received radio signal in a multipath wireless channel. Experimental data obtained from an indoor wireless channel measurement set-up also corroborates the existence of chaotic behavior. The presence of chaos was also detected in the received signal when a closely-spaced antenna pair at the receiver was gradually moved away from the transmitter. Thus, both temporal and spatial chaos in multipath wireless channels has been observed. These findings open up an entirely new gamut of research where multipath wireless channels can be used to generate chaos. A potential application would be chaos-based secure-communications for the next-generation wireless communications devices. Considering the phenomenal projected growth rate in the domain of wireless communications, these research findings would have far-reaching consequences.

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