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Marangoni Boundary Layer Flow in Micropolar Fluid

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ABSTRACT

Steady Marangoni boundary layer flow of a micropolar fluid, with the interface temperature assumed to be a quadratic function of the distance x along the interface, is considered in this paper. The governing partial differential equations are transformed to the ordinary differential equations using similarity variables, which have been then solved numerically by using Keller-box technique along with the shooting method. The numerical solutions for the velocity and temperature profiles, as well as results for the surface shear stresses and surface heat transfer are computed and presented in the form of tables and figures for different values of the governing parameters, namely material parameter, K, and Prandtl number, Pr. Apart from that, the solutions have been analyzed for two different values of n which is $n = 0$ and $n = 0.5$ that represent strong and weak concentration, respectively. The governing boundary layer equations of the problem were also solved analytically for the case of $n = 0.5$

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INTRODUCTION

The phenomenon that liquid flows along a gas-liquid or a liquid-liquid interface from areas having low surface tension to areas having higher surface tension is named the Marangoni effect, after an Italian physicist living in the nineteenth century. Solutal Marangoni convection is flow caused by surface tension gradients originating from concentration gradients, while thermocapillarity is flow caused by surface tension gradients originating from temperature gradients. The study of Marangoni convection has attracted the interest of many researchers in recent years. This is mainly because of its vast contributions in the industrial field especially in the art work of dyeing on the ground (see Kuroda, 2000), in the field of crystal growth (see Arafune and Hirata, 1998), etc. The existence of the dissipative layers or Marangoni boundary layers seem to have been first observed by Napolitano (1979). These layers may exist on both sides of a Marangoni interface, if the non-dimensional Reynolds, *Re*, and P´eclet, *Pe*, numbers are much larger than one. Since *Re* and *Pe* increase with the reference length (i.e. the extension of the interface), in microgravity environment conditions may easily exist to establish a boundary layer regime, contrary to what will happen on Earth (see Golia and Viviani (1986)). Problems of this type are of great importance due to their relevance in several fields of microgravity sciences and space processing. Studies of these problems are also motivated by their importance in terrestrial materials processing, and oceanography. Some of the relevant works are done by Christopher and Wang (2001) who studied the effects of Prandtl number on the Marangoni convection over a flat plate. They have also presented a similarity solution for Marangoni flow for both the momentum and the energy equations assuming a developing boundary layer along a surface. Moreover, Pop *et al*. (2001) analyzed the forced convection thermal and solute concentration Marangoni boundary layers. Al-Mudhaf and Chamka (2005) investigated MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption with fluid suction and injection. Besides, Chamkha *et al*. (2006) have dealt with a steady coupled dissipative layer, called Marangoni mixed convection boundary layer. The mixed convection boundary layer is generated when besides the Marangoni effects there are also buoyancy effects due to the gravitational and external pressure gradient effects. Hamid *et al*. (2011) extended the problem of the thermosolutal Marangoni forced convection boundary layer flow by Pop *et al*. when the wall is permeable. Very recently, Mat *et al*. (2013) discussed the radiation effects on

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the problem of Marangoni boundary layer with permeable surface. They showed that when radiation parameter increased, the heat transfer rate at the surface decreased and the imposition of suction was to decrease the surface temperature gradient, whereas injection showed the opposite effects. An extension of Marangoni boundary layer to the case of nanofluid based on a model proposed by Tiwari and Das (2007), in which the effects of nanoparticles are taken into account, was made by Arifin *et al*. (2011, 2013).

The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equations for Newtonian fluids, so that more complex fluid such as particle suspensions, liquid crystals, animal blood, lubrication, and turbulent shear flows can be described by this theory. The micropolar fluid theory which takes into account the inertial characteristics of the substructure particles which are allowed to undergo rotation has been proposed by Eringen (1966) and was further developed by Eringen (1972). This theory has generated much interest, and many classical flows are being re-examined to determine the effects of the fluid microstructure. The key points to note in the development of Eringen's microcontinuum mechanics are the introduction of new kinematic variables, e.g. the gyration tensor and microinertia. It seems that Gorla (1983) was the first who has obtained numerical results for the micropolar boundary layer flow at a stagnation point on a moving wall using fourth order Runge Kutta method. A study of combined heat and mass transfer by natural convection of micropolar flow near a continuously moving vertical permeable surface has been investigated by Damseh *et al*. (2009) and they solved numerically the corresponding ordinary differential equations using also the fourth order Runge Kutta method. It is reported that increasing the vortex viscosity parameter increased the coefficient of friction due to higher mixing of fluid layers, while the effect of increasing the micro-gyration vector caused a decrease in the coefficient of friction and also, increasing the heat generation or absorption parameter and the chemical reaction parameter enhanced the respective heat and mass transfer coefficients or the Nusselt and Sherwood numbers and decreased the coefficient of friction.

Stagnation-point flow of a micropolar fluid has been treated by McNitt (1970), who considered both the plane and axisymmetric-stagnation point flows, using boundary layer theory, with the condition that the spin should vanish on the solid boundary. Later, John (1972) applied the micropolar model to turbulent shear flow, and used a boundary condition analogous to the vanishing of the eddy viscosity, namely, that the spin should be equal to the velocity gradient. The boundary layer flow of a micropolar fluid over a semi-infinite flat plate was studied by Goodarz (1976). Guram and Smith (1980) investigated the stagnation flows of micropolar fluids with strong and weak interactions. They obtained numerical results using a fourth order Runge-Kutta method. Further, Jena and Mathur (1981) obtained the similarity solutions for laminar free convection flow of thermomicropolar fluid and they reported that microelement are unable to rotate. Besides, two types of boundary conditions are prescribed for the microrotation on the wall.

Based on the above results or findings on similarity solutions as well as boundary conditions, many studies have been conducted recently. Lok *et al* (2003) studied the unsteady problem near the stagnation point of plane surface and they found that the presesence of microelements thoroughly influences the characteristic features on unsteady flow. Whilst, Nazar *et al*. (2004) has been investigated the problem of micropolar fluid towards a stretching sheet. Further, Lok *et al*. (2006) extends her research by injecting mixed convection effects and it is shown that with the increase of the Prandtl number and a fixed value of the material parameter, the reduced steady-state skin friction decreases when the flow is assisting and it increases when the flow is opposing. Ishak *et al* (2006) produced an excellent paper by considering a problem on micropolar fluid with different kind of surfaces, which are fixed or moving, namely a wedge and a flat plate. The results indicate that the effect of the material parameter on the skin friction and heat transfer depends on the velocity ratio of the plate and the fluid. Attia (2008) applied the conditions of strong and weak interactions in his study on stagnation point flow and heat transfer of a micropolar fluid with uniform suction or blowing. Finally, we mention to this end that an incompressible micropolar fluid on a vertical flat plate with prescribed surface heat flux has been investigated by Yakob *et al*. (2010) and it is found that there exist dual solution, which have been obtained.

Due to the complexity effects of Marangoni convection for micropolar fluids, none of the above references attempted to obtain numerical solutions for such flows. The aim of the present paper is, therefore, to study the forced convection Marangoni boundary layer flow of a micropolar fluid. Detailed numerical solutions of the transformed ordinary differential equations have been obtained for some values of the material parameter K and for the boundary condition for microrotation $n = 0$ (strong concentration of microelements) and $n = 0.5$ (weak concentration of microelements), respectively, using the Keller-box technique along with the shooting method. These solutions refer to the velocity and microrotation fields as well as to the skin friction coefficient in the region up to the separation point of the boundary layer. To our best knowledge this problem has not been studied before.

Problem Formulation:

Consider the steady laminar Marangoni boundary layer flow of a micropolar fluid, where the interface temperature is assumed to be a quadratic function of the distance x along the interface. This assumption guarantees the existence of similarity solutions of the governing equations. The governing equations for this investigation are based on the balance laws of mass, momentum, microrotation and energy. Taking into consideration the boundary layer assumption, these equations can be written in dimensional form as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial N}{\partial y} \tag{2}$$

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\gamma\frac{\partial^2 N}{\partial y^2}-\kappa\left(2N+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where the surface tension is assumed to depend on the temperature linearly,

$$\sigma=\sigma_0\left[1-\gamma_T\left(T-T_\infty\right)\right] \tag{5}$$

and γ_T denotes the temperature coefficient of the surface tension and is given by

$$\gamma_T=-\frac{1}{\sigma_0}\left(\frac{\partial\sigma}{\partial T}\right). \tag{6}$$

The boundary conditions of this problem are

$$\begin{gathered} v(x,0)=0,\ \mu\left(\frac{\partial u}{\partial y}\right)=-\left(\frac{\partial\sigma}{\partial x}\right)=\sigma_0\gamma_T\left(\frac{\partial T}{\partial x}\right), \\ N=-n\frac{\partial u}{\partial y}, T(x,0)=T_\infty+T_0X^2, \\ X=\frac{x}{L}, u(x,\infty)=0,\ N(x,\infty)=0,\ T(x,\infty)=T_\infty. \end{gathered} \tag{7}$$

Here u and v are the velocity components along x and y axes, T is the fluid temperature, is N the microrotation (or angular velocity), T_0 is a positive or negative dimensional constant, L is reference length (which will be specified below), μ, κ, ρ, j, γ and α are the dynamic viscosity, vortex viscosity (or the microrotation viscosity), fluid density, spin gradient viscosity, thermal and thermal diffusivity, respectively. Further, n is a constant which varies in the range of $0 \le n \le 1$. The case $n=0$, is called strong concentration by Guram and Smith (1980) and indicates $N=0$ near the wall, represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate. The case $n=0.5$ indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentrations. The case $n=1$, as suggested by Goodarz (1976), is used for the modeling of turbulent boundary layer flows. Further, we follow the work of many recent authors by assuming that

$$\gamma=\left(\mu+\frac{\kappa}{2}\right)j=\mu\left(1+\frac{K}{2}\right)j,$$

where $K=\kappa/\mu$ is the micropolar or material parameter. This assumption is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity (Damseh *et al*., 2009)

We will introduce now the similarity variables defined as

$$\begin{gathered} \psi(x,y)=\nu Xf(\eta), \quad N(x,y)=\left(\frac{\nu}{L^2}\right)Xg(\eta), \\ T(x,y)=T_\infty+T_0X^2\theta(\eta), \quad \eta=\frac{y}{L}. \end{gathered} \tag{8}$$

Substituting (8) into Eqs. **Error! Reference source not found.**- (4), we get the following system of ordinary differential equations

$$(1+K)f'''+ff''-f'^2+Kg'=0 \tag{9}$$

$$(1+K/2)g''+fg'-f'g-K(2g+f'')=0 \tag{10}$$

$$\frac{1}{\Pr}\theta''+f\theta'-2f'\theta=0 \tag{11}$$

subject to the boundary conditions

$$f(0)=0,\ f''(0)=-2,\quad g(0)=-nf''(0)=2n,$$
$$\theta(0)=1, \tag{12}$$
$$f'(\infty)=0,\quad g(\infty)=0,\quad \theta(\infty)=0.$$

where prime denote differentiation with respect to η and $\Pr$ is the Prandtl number. We notice that the reference length L has been chosen as

$$L=-\frac{\mu\nu}{\sigma_0\gamma_T T_0} \tag{13}$$

It should be, however, mentioned that having in view that with increasing temperature the surface tension σ in general decreases, its temperature gradient γ_T given by Eq. (6) is positive. Thus, the reference length L chosen according to Eq. (13) is positive only if T_0 is negative (see Christopher and Wang, 2001).

2.1 Exact solution for n = 0.5:

In this case, it is easily seen that we have

$$\left(1+\frac{K}{2}\right)f'''+ff''-f'^2=0 \tag{14}$$

$$g=-\frac{1}{2}f'' \tag{15}$$

The function

$$f(\eta)=f_\infty\left(1-e^{-a\eta}\right), \tag{16}$$

is an exact solution of Eq. (14) when for the constants a and f_∞ are connected by the relationship

$$f_\infty=\left(1+\frac{K}{2}\right)a\,. \tag{17}$$

On the other hand, we notice that the boundary condition $f(0)=0$ is satisfied automatically, the asymptotic condition $f'(\infty)=0$ requires $a>0$, and thus $f_\infty=f(\infty)$. Moreover, the interface condition $f''(0)=-2$ gives

$$a^2 f_\infty=2\,. \tag{18}$$

Thus, Eqs. (16) and (17) gives for a the expression

$$a=\sqrt[3]{\frac{4}{2+K}}, \tag{19}$$

which determines the value of a for specified values of the micropolar parameter K. The similar surface velocity results as

$$f'(0)=af_\infty=\sqrt[3]{2(2+K)}, \tag{20}$$

and the similar velocity $f'(\eta)$ is according to Eqs. (16) and (18) a simple exponential function of η,

$$f'(\eta)=f'(0)e^{-a\eta} \tag{21}$$

Further, by using Eq. (21) , the approximate expression (15) admit the exact solution

$$g'(\eta)=\frac{a}{2}f'(\eta). \tag{22}$$

Using the solution (16) of Eq. (14) it can be obtained, following Magyari and Chamka (2008), an analytic solution also of Eqs. (11) for the temperature field.

RESULTS AND DISCUSSION

The nonlinear ordinary differential, Eqs. (9) to (11) subject to the boundary conditions (12) were solved numerically using the Keller-box technique along with the shooting method. The values of the surface velocity

$f'(0)$ and the heat flux $-\theta'(0)$ as well as the velocity $f'(\eta)$, angular velocity $g(\eta)$ and temperature profiles $\theta(\eta)$ are obtained for various values of n, material parameter, K and Prandtl number, $\Pr$. In this study, we are considering two values of n which is $n=0$ (strong concentration) and $n=0.5$ (weak concentration). Values of $f'(0)$, $g'(0)$ and $-\theta'(0)$ for different values of the parameter K are given in Table 1 for the case of $n=0$ (strong concentration of microelements) and $n=0.5$ (weak concentration of microelements). It is worth mentioning that $K=0$ corresponds to a Newtonian fluid and $K\neq 0$ corresponds to a micropolar fluid, which is the main issue of this study. The values obtained by Al-Mudhaf and Chamka (2005) are also included in this table. We can see that the agreement between the present results and those by Al-Mudhaf and Chamka (2005) are very good.

Table 1. Values of $f'(0)$, $g'(0)$ and $-\theta'(0)$ for different values of K

n	Pr	K	A-Mudhaf and Chamka (2005)		Present Results		
			$f'(0)$	$-\theta'(0)$	$f'(0)$	$g'(0)$	$-\theta'(0)$
0	0.78	0	1.587671	1.442203	1.58737	0	1.442203
		1			2.029266	0.579422	1.745330
		2			2.355857	0.862850	1.935095
		4			2.845289	1.171865	2.186710
		12			4.040406	1.614195	2.697840
		24			5.122329	1.835214	3.083303
0.5	0.78	0			1.587320	-1.259984	1.442356
		1			1.816998	-1.100499	1.612409
		2			1.999898	-0.999898	1.734423
		4			2.288751	-0.873065	1.908605
		12			3.036566	-0.658623	2.290299
		24			3.731605	-0.535572	2.588825

Figs. 1 to 3 show the velocity $f'(\eta)$, angular velocity $g(\eta)$ and temperature $\theta(\eta)$ profiles for different values of n. One can see that the fluid velocity $f'(\eta)$ decreases as n increase. Differ from the velocity profile, as n increase, the angular velocity and temperature profiles increase. Figs. 4 to 6 present the effects of material parameter K in each profiles. As K increase, fluid velocity and angular velocity also increase but the temperature profile decreases. Figs. 7 to 8 presents the effects of the Prandtl number $\Pr$ on the temperature profiles $\theta(\eta)$. It is seen that these profiles become steeper and steeper along with increment of the Prandtl number. As expected, the thermal boundary layer thickness decreases with an increase in $\Pr$ for both cases of $n=0$ (strong concentration) and $n=0.5$ (weak concentration). Therefore, we can conclude that a higher-Prandtl-number fluid results in a thinner boundary layer thickness and hence a higher heat transfer rate at the surface.

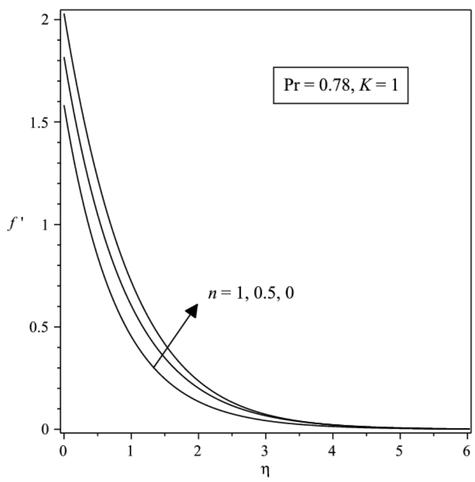

Fig. 1: The effects of *n* on velocity profiles $f'(\eta)$ when *K* = 1 and Pr = 0.78.

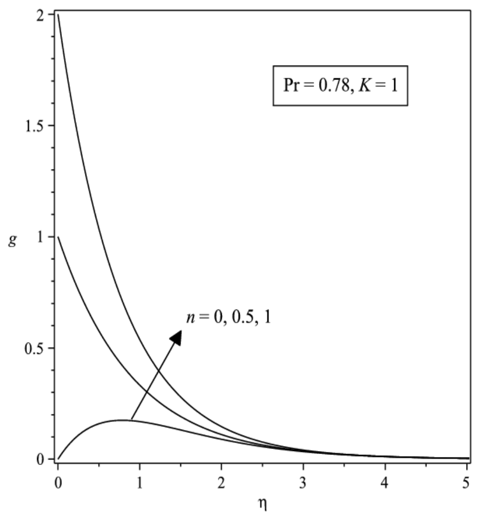

Fig. 2: The effects of *n* on angular profiles $g(\eta)$ when *K* = 1 and Pr = 0.78.

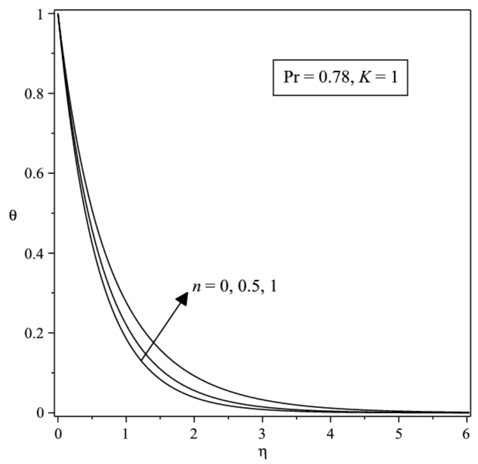

Fig. 3: The effects of *n* on temperature profiles $\theta(\eta)$ when *K* = 1 and Pr = 0.78.

Furthermore, our computations show that the velocity profiles $f'(\eta)$ and angular velocity profiles $g(\eta)$ do not vary with the Prandtl number Pr. This is expected since the Prandtl number only appear in the energy equation, i.e. in Eq. 11. Since we also find the exact values for $n = 0.5$, we have compared it to our numerical results and it is also shown in Figs. 9 to 12. The comparison of the results shows a very good agreement and, therefore, we are confident that the present results are accurate. Fig. 13 shows the effects of the parameters n and K on the interface velocity when $\mathrm{Pr} = 0.78$. It illustrated that the interface velocity increase with K but decrease when n increase. Finally, Figs. 14 and 15 illustrate the effects of Pr on the heat flux for both values of $n = 0$ and $n = 0.5$. The figure shows that as Pr increases, the surface heat transfer also increases. This is because the higher values of the Prandtl number fluid has a lower thermal conductivity which results in thinner thermal boundary layer and hence a higher heat transfer rate at the surface.

Conclusion:

In this paper, we have considered the steady laminar Marangoni boundary layer flow of a micropolar fluid, where the interface temperature is assumed to be a quadratic function of the distance *x* along the interface. The similarity equations Equations are solved numerically for some values of the micropolar parameter *K* and the Prandtl number Pr when $n = 0$ (strong concentration) and $n = 0.5$ (weak concentration). The development of the surface velocity, and the heat flux, as well as the velocity, angular velocity, and temperature profiles has been illustrated in tables and graphs. It was found, as expected, that a higher-Prandtl-number fluid has a thinner thermal boundary layer, which increases the gradient of temperature. Consequently, the surface heat transfer is increased as Pr increases. The increase of the material parameter *K* will lead to an increase of the velocity and angular velocity profiles, which also increase the surface velocity.

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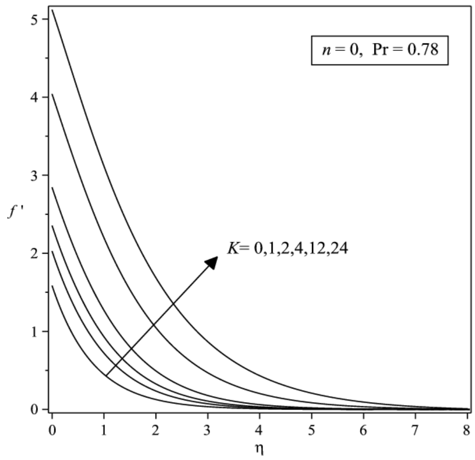

Fig. 4: The effects of the parameter *K* on velocity profiles $f'(\eta)$ when n = 0 (strong concentration) and Pr = 0.78.

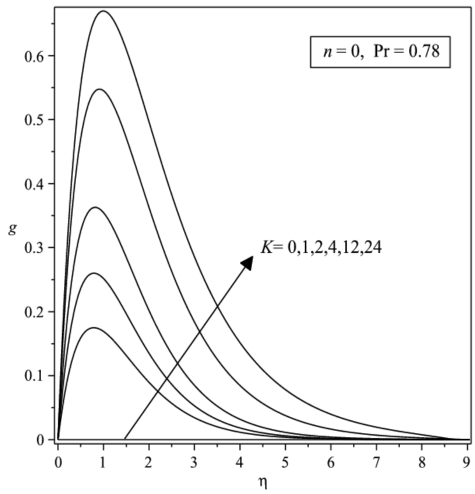

Fig. 5: The effects of the parameter *K* on angular velocity profiles $g(\eta)$ when n = 0 (strong concentration) and Pr = 0.78.

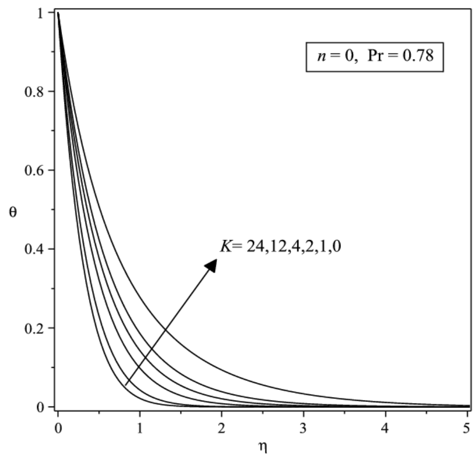

Fig. 6: The effects of the parameter *K* on temperature profiles $\theta(\eta)$ when n = 0 (strong concentration) and Pr = 0.78.

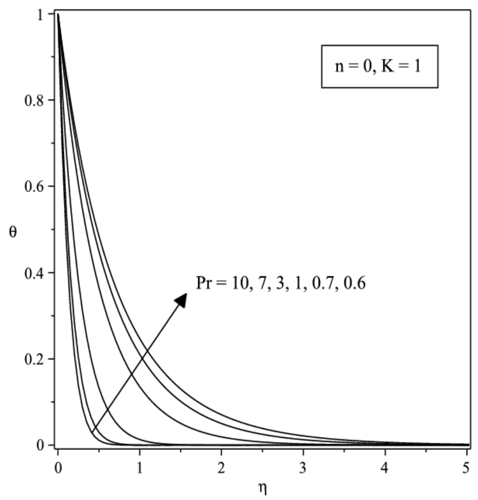

Fig. 7: The effects of Pr on the temperature profiles $\theta(\eta)$ for n = 0 (strong concentration) when K = 1.

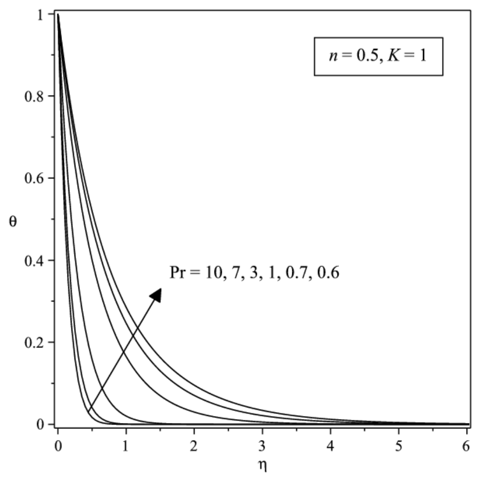

Fig. 8: The effects of Pr on the temperature profiles $\theta(\eta)$ for n = 0.5 (weak concentration) when K = 1.

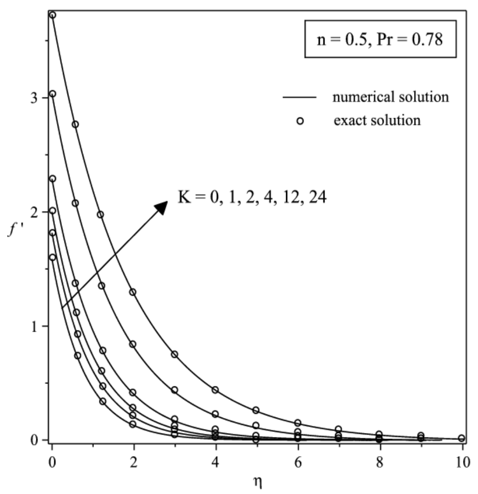

Fig. 9: The effects of the parameter K on velocity profiles $f'(\eta)$ when n = 0.5 (weak concentration) and Pr = 0.78.

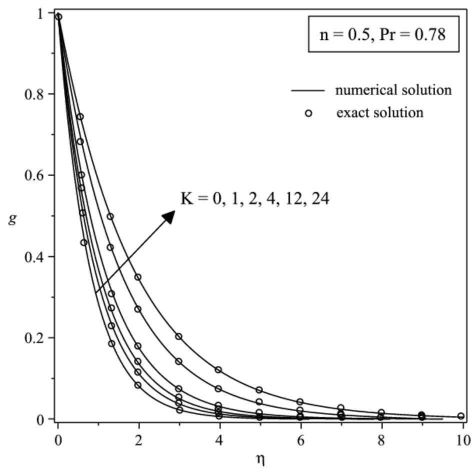

Fig. 10: The effects of the parameter K on angular velocity profiles $g(\eta)$ when $n = 0.5$ (weak concentration) and Pr = 0.78.

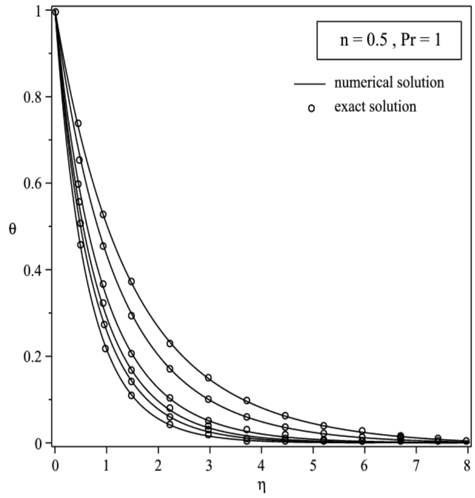

Fig. 11: Temperature profile $\theta(\eta)$ when $n = 0.5$ (weak concentration) for Pr = 1.

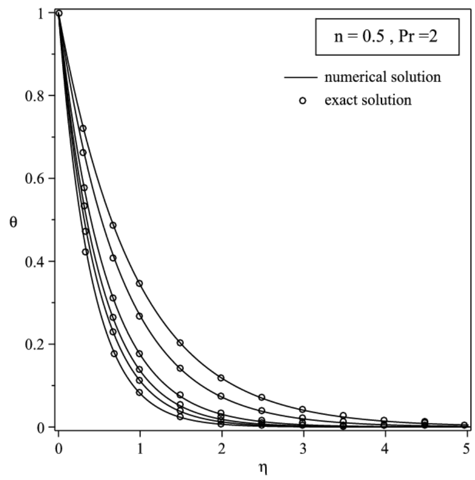

Fig. 12: Temperature profile $\theta(\eta)$ when $n = 0.5$ (weak concentration) for Pr = 2.

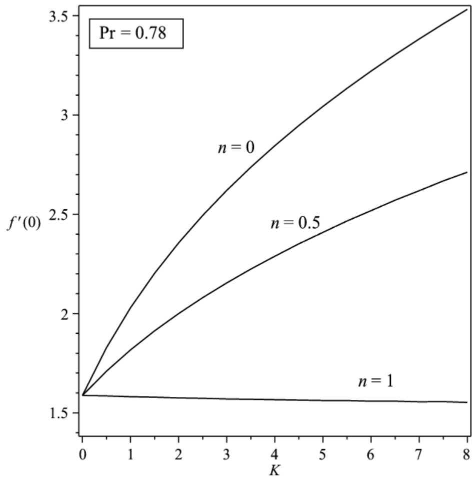

Fig. 13: Variations of $f'(0)$ with *K* for several values of *n* when Pr = 0.78.

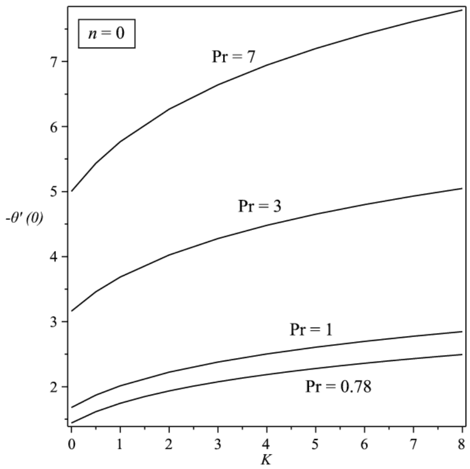

Fig. 14: Variations of $-\theta'(0)$ with *K* for several values of Pr when *n* = 0.

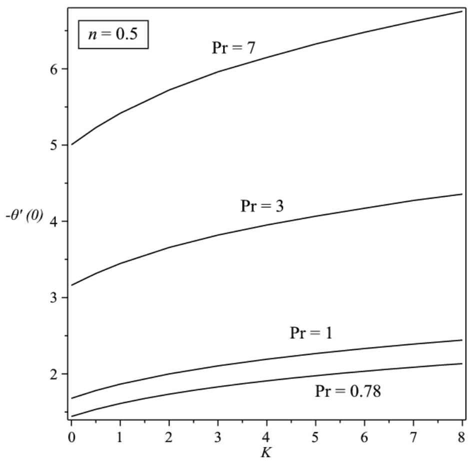

Fig. 15: Variations of $-\theta'(0)$ with *K* for sev.

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REFERENCES

Al-Mudhaf, A. and A.J. Chamkha, 2005. Similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects, Heat and Mass Transfer, 42(2): 112-121.

Attia, H.A., 2008. Stagnation Point Flow and Heat Transfer of a Micropolar Fluid with Uniform Suction or Blowing, Brazil Society of Mechanical Science and Engineering, 5(1): 51-55.

Arafune, K. and A. Hirata, 1998. Interactive Solutal and Thermal Marangoni Convection in a Rectangular Open Boat, Numerical Heat Transfer, Part A: Applications, 34(4): 421-429.

Arifin, N.M., R. Nazar and I. Pop, 2011. Non-isobaric Marangoni boundary layer flow for Cu, Al2O3 and TiO2 nanoparticles in a water based fluid, Meccanica, 46(4): 833-843.

Arifin, N.M., R. Nazar and I. Pop, 2013. Similarity Solution of Marangoni Convection Boundary Layer Flow over a Flat Surface in a Nanofluid, Journal of Applied Mathematics, Article ID 634746, 8 pages.

Chamkha, A.J., I. Pop and H. Takhar, 2006. Marangoni Mixed Convection Boundary Layer Flow, Meccanica, 41(2): 219-232.

Christopher, D.M. and B. Wang, 2001. Prandtl number effects for Marangoni convection over a flat surface, International Journal of Thermal Sciences, 40(6): 564-570.

Damseh, R.A., M.Q. Al-Odat, A.J. Chamkha and B.A. Shannak, 2009. Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface, International Journal of Thermal Sciences, 48(8): 1658-1663.

Eringen, A.C., 1966. Theory of Micropolar Fluids, Journal of Math Mech., 16: 1-18.

Eringen, A.C., 1972. Theory of Thermomicropolar Fluids, Journal Math Anal Appl., 38: 480-496.

Golia, C. and A. Viviani, 1986. Non isobaric boundary layers related to Marangoni flows, Meccanica, 21(4): 200-204.

Goodarz, A., 1976. Self-similar solution of imcompressible micropolar boundary layer flow over a semi-infinite plate, International Journal of Engineering Science, 14(7): 639-646.

Gorla, R.S., 1983. Micropolar boundary layer flow at a stagnation point on a moving wall, International Journal of Engineering Science, 21(1): 25-33.

Guram, G.S. and A.C. Smith, 1980. Stagnation flows of micropolar fluids with strong and weak interactions, Computers & Mathematics with Applications, 6(2): 213-233.

Hamid, R.A., N.M. Arifin, R. Nazar, F.M. Ali and I. Pop, 2011. Dual Solutions on thermosolutal Marangoni forced convectionboundary layer with suction and injection, Mathematical Problems in Engineering, Article ID 875754, 19 pages.

Ishak, A., R. Nazar and I. Pop, 2006. Moving wedge and flat plate in a micropolar fluid, International Journal of Engineering Science, 44(18-19): 1225-1236.

John, J.P., 1972. An application of the micropolar fluid model to the calculation of a turbulent shear flow, International Journal of Engineering Science, 10(1): 23-32.

Jena, S.K. and M.N. Mathur, 1981. Similarity solutions for laminar free convection flow of a thermomicropolar fluid past a non-isothermal vertical flat plate, International Journal of Engineering Science, 19(11): 1431-1439.

Kuroda, T., 2000. The Marangoni Effect and Its Artistic Application, Forma, 15(3): 203-204.

Lok, Y.Y., N. Amin and I. Pop, 2003. Unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface, International Journal of Thermal Sciences, 42(11): 995-1001.

Lok, Y.Y., N. Amin, I. Pop, 2006. Unsteady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface, International Journal of Thermal Sciences, 45(12): 1149-1157.

Mat, N., N.M. Arifin, R. Nazar, F. Ismail and I. Pop, 2013. Radiation effects on Marangoni convection boundary layer over a permeable surface, Meccanica, 48: 83-89.

Magyari, E. and A.J. Chamkha, 2008. Exact analytical results for the thermosolutal MHD Marangoni boundary layers, International Journal of Thermal Sciences, 47(7): 848-857.

McNitt, R. P. 1970. Boundary-layer theory for a micropolar fluid, Recent Advances in Engineering Science. Gordon and Breach.

Napolitano, L.G., 1979. Marangoni boundary layers, Proc 3rd European Symposium on Material Science in Space, Grenoble, ESA SP-142.

Nazar, R., N. Amin, D. Filip and I. Pop, 2004. Stagnation point flow of a micropolar fluid towards a stretching sheet, International Journal of Non-Linear Mechanics, 39(7): 1227-1235.

Pop, I., A. Postelnicu and T. Groşan, 2001. Thermosolutal Marangoni Forced Convection Boundary Layers, Meccanica, 36(5): 555-571.

Tiwari, R.K. and M.K. Das, 2007. Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids, International Journal of Heat and Mass Transfer, 50(9-10): 2002-2018.

Yacob, N.A., A. Ishak and I. Pop, 2010. Non newtonian micropolar fluid flow towards a vertical plate with Prescribed Wall Heat Flux, Australian Journal of Basic and Applied Sciences, 4(8): 2267-2273.