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Mathematical Analyses Of Some Stochastic Sets In Plan And Their Applications

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Abstract: In this paper we study some stochastic models for random sets in plan, particularly we consider a random process in plan including infinite but countable number of points distributed randomly in two-dimensional space. This processes are known as the main part of stochastic geometry (Cahoy, D., V. ovhaikin, 2010; Esmaeili, H., C. Kiuppeldberg, 2010). We divide this random set into two disjoint sets and assume that for each point from one of the set there is a point in another one with a special distance and assume that all these distances are equal then for each point we define a function from one set to another one according to the distances of the set to our reference point. These distances are random variables so the defined function is again a random function (Balakrishnan, N., T. Kozubowski, 2008). Then according to this function we define another random variable that this random variable is very important speciallyin the application of random variables. In some specified case the probability and cumulative density functions of this random variable exist so using them we study the random variable. In the rest we extend the random variable to a more general and complicated form as the product of another random variable and by using the estimations we study this random variable same as the another one. However in this case our result are not as exact as the our previous results. Our tools are very simples but the results are very interesting.

Key words: Stochastic models,random sets, point processes, random variables

INTRODUCTION

However the study of random sets and random variables goes back to many years ago but recently the applications of point processes in other field of science has made great strides. Specially the spatial point process. A spatial point process is a random pattern of points in d-dimensional space where usually d=2 or d=3 in application (Vtale, R., Y. Zang, 2008; Stoyan, D., et al., 1995; Werner, W., 2010; Kingman, J.F.C., 1993). In our work we focus on d=2 because in applications of point processes for examples, trees in a forest or bird nests we assign to elements their locations in plan that means we assign a 2-dimensional point process to 3-dimensional objects. Spatial point process are useful as statistical models in the analysis of observed patterns of points where the points represent the locations of some object of study of point process play a special role as the building blocks of more complicated random set models and as instructive simple example of random sets. A point process in one dimensional that is usually the time , is a useful for the sequence of random times when a particular events occurs but a spatial point process is a useful model for a random pattern of [points in d-dimensional space where $d \geq 2$. for example if we make a map of the locations of all the people who called emergency service during a particular day, this map constitutes a random patterns of points in two dimensions. There will be a random number of such points and their locations are also random. Spatial point process in plan can be used directly to model and analyze data which take the form of a point pattern such as locations of users in a network. Spatial point process occur frequently in a wide variety of scientific disciplines including seismology, geography,…. The classical spatial point process papers and textbooks usually deal with relatively small point patterns, where the assumption of stationary and homogeneity is central and non-parametric methods based on summary statistics play a major role. In recent years fast computers and advances in computational statistics, particularly Markove chains Monte Carlo methods, have had a major impact on the development of statistics for spatial Poisson point processes. The focus has now changed or likelihood-based inference for flexible parametric models often depending on covariant and librated from restrictive assumptions of stationary.

If it is known on which region the point process is defined or if the process extends over a very large region or if certain invariance assumption such as stationary are imposed then it is maybe appropriate to consider an infinite point process in R^2. In fact a spatial point process X on R^2 as a locally finite random subset of R^2 that means for every subset of R^2 the number of point process in that subset is a random variable whenever that subset is a bounded region.

Random sets:

We consider a random set including a countable number of point in two dimensional plan. We call this set random because the points are distributed randomly in the R^2this random set is called also a point process. We divide this set into two subsets, A and B. we assume that for each point in A there is a point in B that distance

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between this two points is a constant real number l and also assume that the number of points belong to Ain a unit area is m. in fact m is volume density of process and is a real positive number. For each point x in B we define a function. Assume that the assigned point for x is y_1

$f_x: A \rightarrow R^+$

When $y \neq y_1$

$$f_x(y)= \sum_{y \in A-y_1} d^{-a}(x,y)$$

And $f_x(y_1) = l^{-a}$.

In which a is a real number. For each $(x,y) \in (A,B)$, $d^{-a}(x,y)$ is a random variable. According to the defined function, we define the image of f as the random variable:

$$Y = \sum_{y \in A-y_1} d^{-a}(x,y)$$

The probabilistic behavior of Y has very important role in mathematical analyses of random sets. The pdfof Y:

$$f_Y(x)= \frac{\pi m}{2x^{3/2}} e^{-\frac{\pi^3 m^2}{4x}}$$

Consequently for a small real number $c \in [0,1]$:

$$f(Y>c)= \int_c^{\infty} f_y(x)dx$$

$$= \frac{\pi m}{2} \int_c^{\infty} x^{-3/2} e^{-\frac{\pi^3 m^2}{4x}} dx$$

Estimations:

The above integration is a incomplete gamma function and its calculation is impossible for $c > 0$ to estimate that we use the simple inequality:

$$e^{-\frac{\pi^3 m^2}{4x}} > 1 - \frac{\pi^3 m^2}{4x}$$

So:

$$\rho(Y>c) > \frac{\pi m}{2} \int_c^{\infty} m^{-3/2} (1 - \frac{\pi^3 m^2}{4x}) dx$$

$$= \frac{\pi m}{2} (2c^{-1/2} - \frac{\pi^3 m^2}{4x} . \frac{2}{3} c^{-3/2})$$

If $n \in [0,1]$ then:

$\rho(Y>c)=n$

ignoring the term $c^{-3/2}$ will result in the interesting equation:

$$\frac{\pi m}{\sqrt{c}} = n$$

And from that:

$$m = \frac{n\sqrt{c}}{\pi}$$

Now we define another function as the extension of f to be:

$$g= \sum \psi_y d^{-\delta}(x,y)$$

in which ψ_y are independent random variables. Now define

$$Y = \sum \psi_y d^{-\delta}(x,y)$$

If we assume that ψ_y have the Rayligh distribution then :

$$f_Y(x) = \frac{m}{4} (\pi/x)^{3/2} e^{-\frac{\pi^4 m^2}{16x}}$$

And from that:

$$F_y(x) = 1 - \mathrm{erf}(\frac{\pi^2 m}{4\sqrt{x}})$$

So:

$$\rho(y > c) = 1 - F_Y(c)$$

$$= \mathrm{erf}\,(\frac{\pi^2 m}{4\sqrt{c}})$$

On the other side:

$$\mathrm{erf}(x) \sim 1 - \frac{e^{-x^2}}{\sqrt{\pi}}\,(1/x - \frac{1}{2x^3} + \ \ldots$$

$$\sim 1 - \ 1/x \frac{e^{-x^2}}{\sqrt{\pi}}$$

$$\sim 1 - \ 1/x\,(\ 1/\sqrt{\pi}\ (1 - x^2 + \cdots$$

$$\sim 1 - \frac{1}{x\sqrt{\pi}}$$

Consequently:

$$\rho(Y > c) \approx 1 - \frac{1}{\sqrt{\pi}} \frac{4\sqrt{c}}{\pi^2 m}$$

And then:

$$\rho(Y > c) = n$$

Will result:

$$1 - \frac{1}{\sqrt{\pi}} \frac{4\sqrt{c}}{\pi^2 m} = c$$

And:

$$m = \frac{4\sqrt{c}}{\pi^{3/2}(1-n)}$$

On the other hand we also have:

$$\rho(Y > c) = \frac{m}{4} \int_c^{\infty} (\pi/x)^{3/2}\, e^{-\frac{\pi^6 m^2}{16x}}\ dx$$

$$= \frac{m\pi^{3/2}}{4}(2c^{1\,1/2} - \frac{\pi^4 m^2}{16}.\,2/3\, c^{-3/2}\,)$$

$$\approx \frac{m\pi^{3/2}}{2\sqrt{c}}$$

And again solving the equation:

$$\frac{m\pi^{3/2}}{2\sqrt{c}} = n$$

Give us:

$$m = \frac{2\sqrt{c}\ \ n}{\pi^{3/2}}$$

If we calculate the first and second moment:

$$E[y] = \pi m/2 \int_0^{\infty} x.\,1/x^{3/2}\, e^{-\frac{\pi^3 m^2}{4x}} dx$$

$$= 2\pi^{5/2} m^2\ \Gamma\ (-1/2) \rightarrow\ \infty$$

and

$$E[y^2] = \frac{\pi m}{2}\int_0^{\infty} x^2 . \frac{1}{x^{3/2}} e^{-\frac{\pi^3 m^2}{4x}} dx \to \infty$$

So these moments give us the trivial inequality.

Conclusions:

The above calculations show that working with CDF and pdf of random sets in plan however has its benefits but there are many difficulties such as integrations that can not be calculated directly as well. But maybe using these estimations give us a lot of useful results that can be used in describing random and even stochastic phenomena.

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