

A Mathematical Model for the Effect of Magnetic, Body Acceleration and Time Dependence on Blood Flow in Stenosed Artery

¹Ahmad Reza Haghighi and ²Nasim Asghari

¹Department of Mathematics, Urmia University of Technology, Urmia, Iran.

²Department of Mathematics, Islamic Azad University, Central Tehran Branch, Tehran. Iran.

Abstract: Blood flow in stenosed tube has been modeled in the present investigation. The model is aimed at studying the effect of stenosis on flow rate and shear stress distribution on time dependent (pulsatile) flow of blood in the arteries. The model has been compared with the time independent model and shown that the time dependent has significant effect on the flow situation. The model accounts for the anomalies of blood flow such as; blunted velocity profile, and Fahraeus – Lindquist effect (FLE). The model also has been studied for various of parametric effects such as magnetic, body acceleration, size of Blood cells (Couple Stress) frequency, amplitude, phase difference, percentage stenosis (constriction), and length of stenosis. One of the most important aspect of the present model is that the model has been studied for different blood diseases and compared with the case of normal blood.

Key words: Stenosed artery, Flow rates of blood, Shear Stress.

INTRODUCTION

One of the most unpredictable parts of human life is getting affected by heart attack. It is learnt by statistical survey that one out every 1000 is getting affected by heart attack around the globe. By and large we know now causes associated with the heart attack though exact cause of the same is still far away from the known aspect. One of the causes attributed due to heart attack is thinning of the passage (constriction) of blood flow. The reason again for constriction is attributed due to accumulation of fatty substance (cholesterol) and blood clots on the interior part of tube (leuman). The experimental study suggest that up to 70% constriction the patient do not feel uncomfortable at all and he/she can cope up with the requirement of blood supply to various other parts. However for the constriction in the range of 80% and above, doctors generally resort to surgery for the patients. At present there are no guidelines when is the right time to go for the surgery.

Surgeons need to have guide line what effects constriction has on strength of the arteries and how resistive are the arteries with varied degree of constriction. In view of the importance of blood flow studies in Cardiovascular system the present studied have been oriented with its application to blood diseases. Also the model has been aimed at to include effects of time dependency (Srivastava, L.M. and Srivastava, V.P., 1984; Mandal, P.K. Mandal and Amin, N., 2007) (pulsatile nature) anomalies of blood flow [Fahraeus – Lindquist effect (FLE)] (Dintenfass, L., 1967), mild magnetic effects on blood flow (Barnothy, M.F., 1969; Vardanyan, V.A., 1973; Deshikachar, K.S. Ramchandra Rao, A. 1985), blood cells suspended in plasma (Bungliarello, G. and Sevilla, J., 1970), viscosity of blood (for different blood diseases) (Pralhad, R. N. and Schultz, D.H., 2004; Sankar, D.S. and K. Hemalatha, 2007) and frequency related effects (Haghighi, A.R. and R.N. Pralhad, 2008; Mustapha, N. and Mandal, P.K., 2010; Womersley, J.R., 1955) on the flow. The blood is assumed to be represented by a micro-continuum (couple stress) fluid proposed by Stokes (1966).

In this paper, the studies have been basically focusing onto accounting of pulsatile nature of blood, magnetic and body acceleration in the stenotic artery with a view to account it for different blood diseases. Mathematical treatment of the studies has been opted from Hankel transformation approach instead of conventional either numerical or empirical approach. Detailed aspect of flow parameters and its variations with the pulsatile, magnetic, body acceleration and blood cells has been explored in this paper.

Analysis:

It is assumed that the flow is pulsatile and laminar and turbulence effects in the body are neglected

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \mu \nabla^2 u - \eta \nabla^4 u \quad (1)$$

Equation (1) in cylindrical polar co-ordinates under the periodic body acceleration in the presence of magnetic field is given by

Corresponding Author: Ahmad Reza Haghighi, Department of Mathematics, Urmia University of Technology, Urmia, Iran.

E-mail: ah.haghighi@gmail.com

$$\rho \frac{\partial u}{\partial t} + \eta \nabla^2 (\nabla^2 u) - \mu \nabla^2 u + \sigma B_0^2 u = -\frac{\partial p}{\partial z} + \rho G \tag{2}$$

Where $u(r,t)$ is the velocity in the axial direction, ρ and μ are the density and viscosity of blood, η is the couple stress parameter, σ is the electrical conductivity, B_0 is the external magnetic field and r is the radial coordinate.

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \tag{3}$$

The flow geometry of blood flow has been shown in Figure 1.

Fig. 1: Blood flow in stenosed tube.

The relation between tube geometry and axis is given by the following relation (Ikbal, Md.A., S. Chakravarty S. and Mandal, P.K., 2009).

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\delta}{2R_0} \left\{ 1 + \cos \frac{2\pi}{L_0} \left(z - d - \frac{L_0}{2} \right) \right\} & d \leq z \leq d + L_0 \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

For the initial calculation of velocity, flow rate (Q), pressure gradient and body acceleration are assumed to be of the form (Tanveer, S., 2005).

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \cos(\omega t) \quad t \geq 0 \tag{5}$$

$$G = a_0 \cos(\omega_1 t + \phi) \quad t \geq 0 \tag{6}$$

Where A_0 the steady-state part of pressure gradient, A_1 is the amplitude of the oscillatory part, $\omega = 2\pi f$ and f is the heart pulse frequency, a_0 is the amplitude of body acceleration, $\omega_1 = 2\pi f_1$ and f_1 is body acceleration frequency, ϕ is the phase difference, z is the axial distance and t is time. Flow variables have been normalized by using following relations:

$$u^* = \frac{u}{\omega R}, r^* = \frac{r}{R}, A_0^* = \frac{R}{\mu \omega} A_0, A_1^* = \frac{R}{\mu \omega} A_1, a_0^* = \frac{\rho R}{\mu \omega} a_0, z^* = \frac{z}{R}, t^* = t \omega \tag{7}$$

Equation (2) simplifies to [after dropping stars]

$$\begin{aligned} \bar{\alpha}^2 a^* \frac{\partial u}{\partial t} = & \bar{\alpha}^2 A_0 + \bar{\alpha}^2 A_1 \cos t + \bar{\alpha}^2 a_0 \cos(bt + \phi) + \bar{\alpha}^2 \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right) \\ & - \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right) \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right) - \bar{\alpha}^2 H^2 u \end{aligned} \tag{8}$$

Where $\bar{\alpha}^2 = \bar{a}^{*2} \left(\frac{R}{R_0} \right) \frac{\mu}{\mu_1}$, couple stress parameter, $a^* = a \left(\frac{R}{R_0} \right)$ Womersley parameter, $H = H^* \left(\frac{R}{R_0} \right) \sqrt{\frac{\mu}{\mu_1}}$ is the Hartmann number, and R is the radius of the pipe.

$$\bar{a}^{*2} = \frac{R^2 \mu_1}{\eta}, H^* = B_0 R \sqrt{\frac{\sigma}{\mu_1}}, a = R \sqrt{\frac{\omega \rho}{\mu_1}}, b = \frac{\omega_1}{\omega} \tag{9}$$

The initial and boundary conditions for this problem are (Haghighi, A.R. and R.N. Pralhad,2008):

$$u(r,0) = 2 \sum_{n=1}^{\infty} \frac{J_0(r\lambda_n) \bar{\alpha}^{-2}}{\lambda_n J_1(\lambda_n)} \frac{[A_0 + A_1 + a_0 \cos \phi]}{[\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)]} \tag{10}$$

$$u \text{ and } \nabla^2 u \text{ are finite at } r=0 \tag{11}$$

$$u = 0, \nabla^2 u = 0 \text{ at } r=1. \tag{12}$$

Tanveer (2005) have analyzed the model for velocity computation and for the straight tube this expression for velocity has been taken for further investigations in the present studies with appropriate changes for the constricted (stenosed) tube.

$$\begin{aligned} u(r^*,t) = & 2 \sum_{n=1}^{\infty} \frac{J_0(r^* \lambda_n) \bar{\alpha}^{-2}}{\lambda_n J_1(\lambda_n)} \left\{ \frac{A_0}{[\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)]} + \frac{A_1 [\{\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)\} \cos t + m \sin t]}{[\{\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)\}^2 + m^2]} \right. \\ & + \frac{a_0 [\{\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)\} \cos(bt + \phi) + bm \sin(bt + \phi)]}{[\{\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)\}^2 + m^2 b^2]} \\ & - e^{-h_1 t} \left\{ \frac{A_0}{[\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)]} + \frac{A_1 [\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)]}{[\{\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)\}^2 + m^2]} \right. \\ & \left. \left. + \frac{a_0 [\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2) \cos \phi + bm \sin \phi]}{[\{\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)\}^2 + m^2 b^2]} - \frac{[A_0 + A_1 + a_0 \cos \phi]}{[\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)]} \right\} \end{aligned} \tag{13}$$

Where

$$h_1 = \frac{[\lambda_n^4 + \bar{\alpha}^2 (\lambda_n^2 + H^2)]}{m}, m = \bar{\alpha}^{-2} a^2 \tag{14}$$

And J_0 and J_1 are the Bessel functions of order zero and one respectively and λ_n are the roots of equation $J_0(r) = 0$. Used of transform techniques (Sneddon, N., 1980) have been made use in solving equation (8).

Shear Stress:

Shear stress τ_{rz} which is one of the physiological importance parameters has been computed by using following relation

$$\tau_{rz} = -\mu \frac{\partial u(r)}{\partial r} \tag{15}$$

Using Equation (13), Shear stress simplifies to

$$\begin{aligned} \tau^* = 2 \frac{\mu}{\mu_1} \sum_{n=1}^{\infty} \frac{J_1(r^* \lambda_n) \bar{\alpha}^{-2}}{\lambda_n J_1(\lambda_n)} & \left[\left\{ \frac{A_0}{[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]} + \frac{A_1[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\} \cos t + m \sin t]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2]} \right. \right. \\ & \left. \left. + \frac{a_0[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\} \cos(bt + \phi) + b m \sin(bt + \phi)]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2 b^2]} \right\} \right. \\ & \left. - e^{-ht} \left\{ \frac{A_0}{[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]} + \frac{A_1[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2]} \right. \right. \\ & \left. \left. + \frac{a_0[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2) \cos \phi + b m \sin \phi]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2 b^2]} - \frac{[A_0 + A_1 + a_0 \cos \phi]}{[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]} \right\} \right] \end{aligned} \quad (16)$$

Where μ_1 is the viscosity of plasma which is taken as 1.2 cPin the present model and $\tau^* = \frac{\tau_{rz}}{\mu_1 \omega}$ (non-dimensional shear stress).

Flow Rate:

Flow Rate (Q) has been computed by using following relation

$$Q = 2\pi \int_0^1 r u(r^*, t) dr \quad (17)$$

$$\begin{aligned} Q(r^*, t) = 4\pi \sum_{n=1}^{\infty} \frac{\bar{\alpha}^{-2}}{\lambda_n^2} & \left[\left\{ \frac{A_0}{[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]} + \frac{A_1[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\} \cos t + m \sin t]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2]} \right. \right. \\ & \left. \left. + \frac{a_0[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\} \cos(bt + \phi) + b m \sin(bt + \phi)]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2 b^2]} \right\} - \right. \\ & \left. e^{-ht} \left\{ \frac{A_0}{[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]} + \frac{A_1[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2]} \right. \right. \\ & \left. \left. + \frac{a_0[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\} \cos \phi + b m \sin \phi]}{[\{\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)\}^2 + m^2 b^2]} - \frac{[A_0 + A_1 + a_0 \cos \phi]}{[\lambda_n^4 + \bar{\alpha}^{-2}(\lambda_n^2 + H^2)]} \right\} \right] \end{aligned} \quad (19)$$

RESULTS AND DISCUSSION

One of the main objectives of the model is to study exhaustively the effects of shear stress and resistance to flow on blood flow. Hence, computations are mainly focused onto shear stress and resistance to flow for various parametric effects.

Shear stress and resistance to flow have been computed for the case of Normal Blood and for the diseased blood (Polycythemia, Plasma Cell Dyscrasias, and for Hb.ss). The data required for the computation has been taken from [Tanveer,S.,2005;Pralhad, R. N. and D.H. Schultz,2004] and shown in Table 1.Use of Matlab and Maple software (Yang, W.Y., Cao, W., Chung, T.S. and Morris, J. 2004;Hanselman, C.D. 2002) has been made use while computing for shear stress and resistance to flow.

Table 1: Viscosity Data (Pralhad, R. N. and D.H. Schultz,2004).

Diseases	μ, cP	μ_1, cP
Normal Blood	3.81	1.2
Polycythemia	6.75	1.2
Plasma cell Dyscrasis	4.99	1.2
Hb.ss	3.29	1.2

Shear stress has been computed for the case of Normal Blood and other blood diseases [Polycythemia, Plasma cell Dyscrasis and Hbss]. Effect of parametric variation have also been observed in the analysis. The results have been shown in Figure (2-19).The observations are highlighted below.

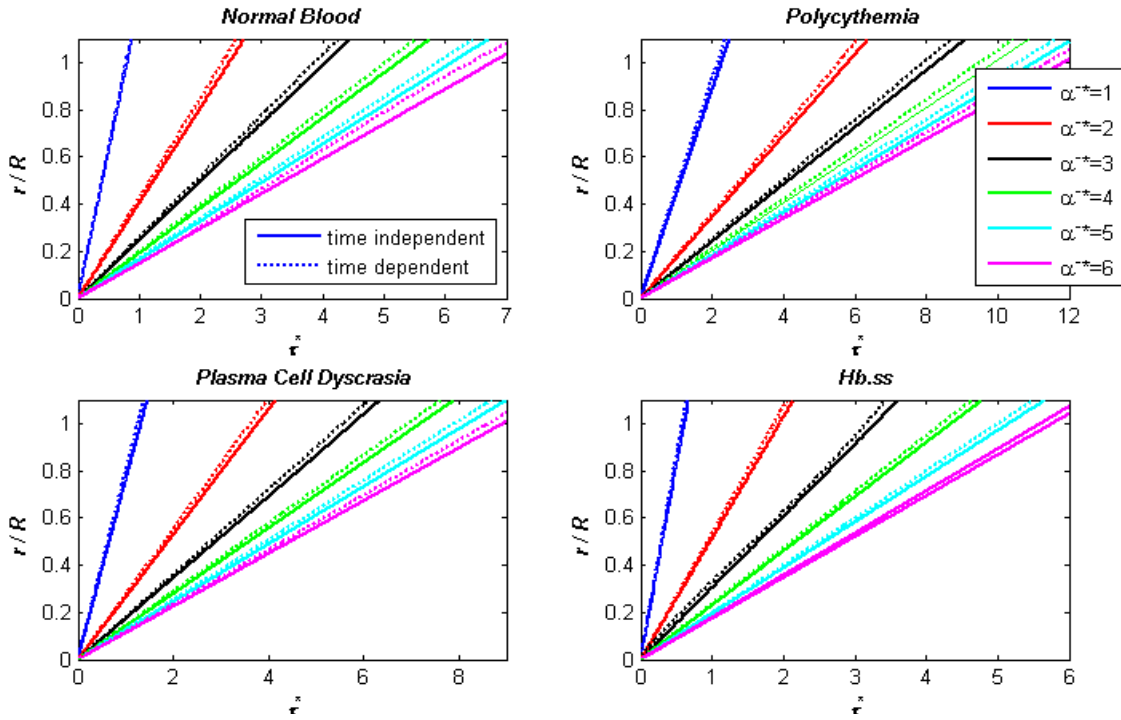


Fig. 2: Shear Stress for stenosed, different $\bar{\alpha}^*$'s [$H^*=2, A_0=2, A_1=4, a_0 = 3, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, t = 0.5, b = 0, a = 1$].

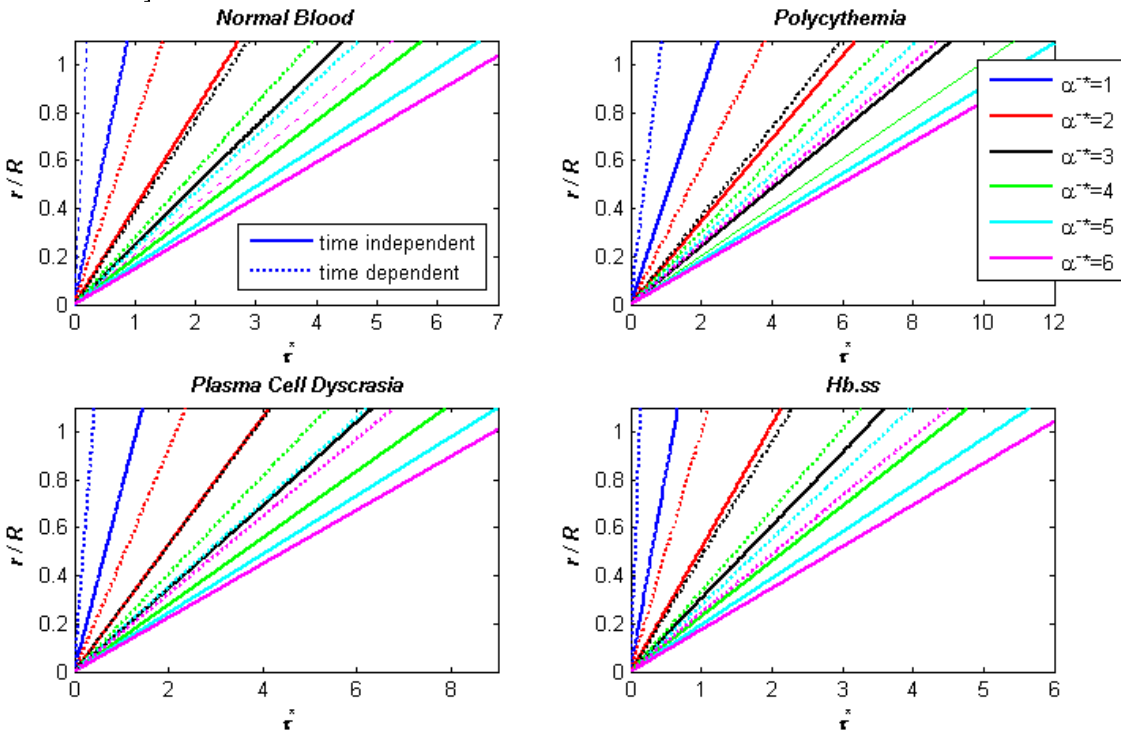


Fig. 3: Shear Stress for stenosed, different $\bar{\alpha}^*$'s [$H^*=2, A_0=2, A_1=4, a_0 = 3, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, t = 1.5, b = 1.5, a = 5$].

Variation of shear stress with $\bar{\alpha}^*, A_0, A_1, a_0$ [Figure (2-9)] and α Figure (19) shows that, the values of shear stress increases with the respective increase in parametric values. However with H^*, ϕ , Figure (9-13) and time t Figure (6-17), it is observed that, shear stress decrease with increase in parametric values. These observations are found to be in agreement with the physics of flow and deformation. For instance increase in $\bar{\alpha}^*$ means decrease in blood cells counts which in term should increase velocity gradient and hence in shearing stress value which is been observed by the findings of the present analysis. Similarly increases in magnetic effect, body

acceleration and time observation should decrease the shear stress since magnetic effect and body acceleration enhance body functioning, and time will subside all initial perturbations. It is also noted that distinct behavior of time dependency with that of time independency has been observed if we observe the shear stress value at $t=0.5$ and at $t=1.5$ [Figure (2-15)]. One of the most significant observation in stress distribution found is when computed with the variation of percentage increase of stenosis height(constriction) [Figure (14-15)].

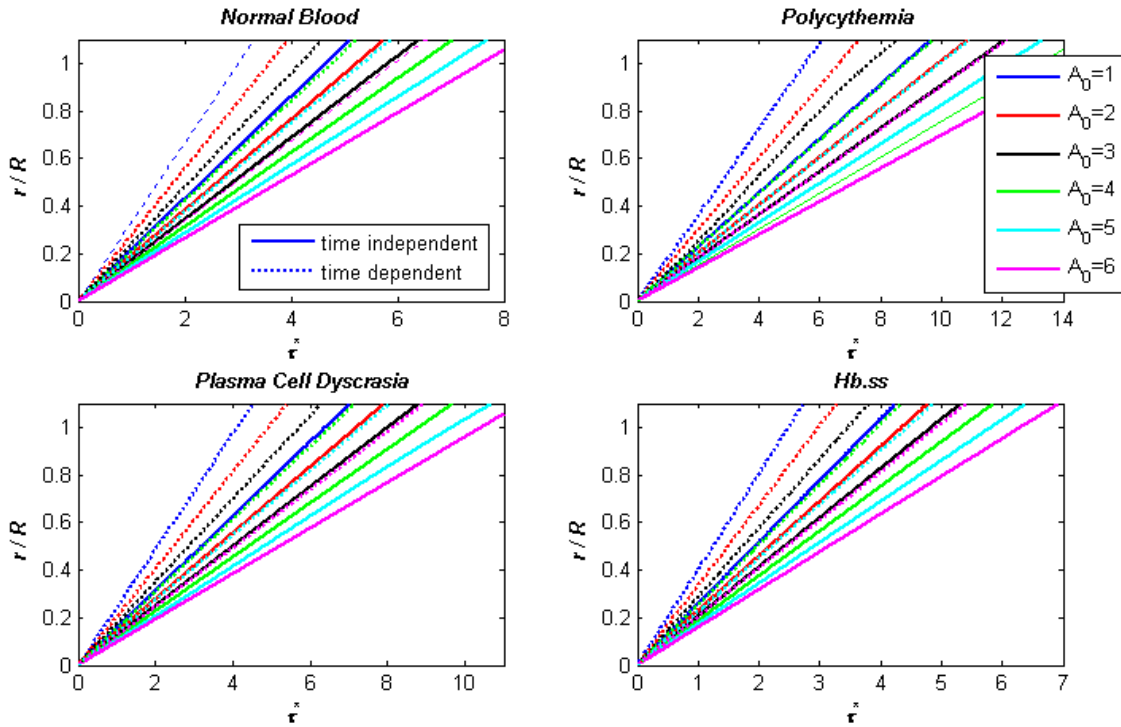


Fig. 4: Shear Stress for stenosed, different A_0 's [$H^*=2, \bar{\alpha}^*=4, A_1=4, a_0=3, \phi=15^\circ, \frac{\delta}{R_0}=0.6, t=0.5, b=0, a=1$].

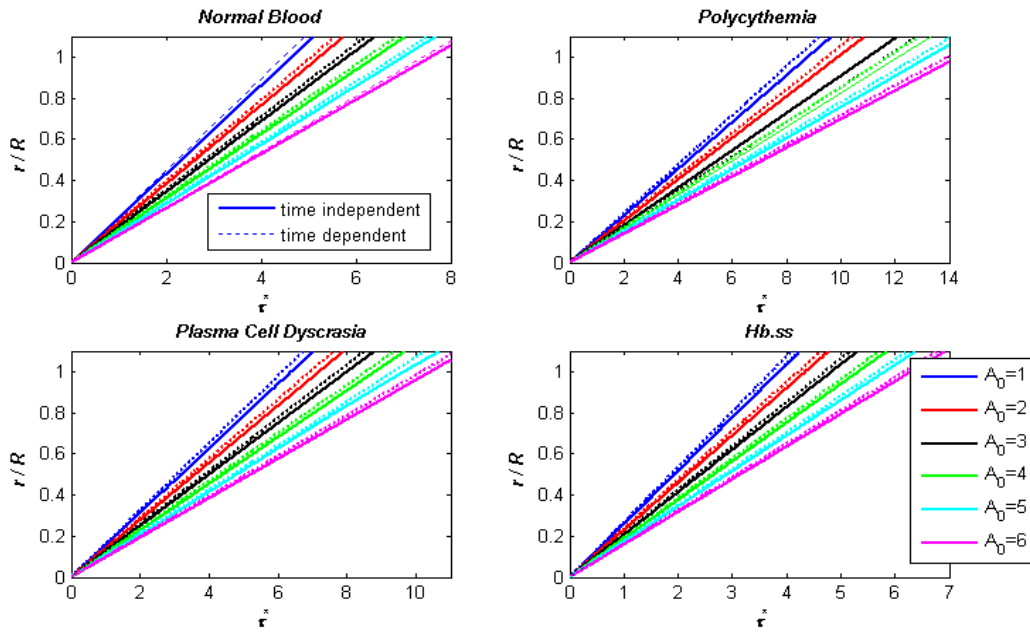


Fig. 5: Shear Stress for stenosed, different A_0 's [$H^*=2, \bar{\alpha}^*=4, A_1=4, a_0=3, \phi=15^\circ, \frac{\delta}{R_0}=0.6, t=1.5, b=1.5, a=5$].

It is observed that shear stress increases with increases of stenosis height from 0 to 0.6 however when computed for 0.8 we found that the stress values suddenly drops off. This strange behavior can be attributed due to sudden choking of the leuman and at this stage the reverse flow may start of and that could be the reason for sudden decreasing in stress value. This observation is in agreement with that of Young [1979] whose experimental observation found that, the flow behaves abnormally after 70-80% of constriction.

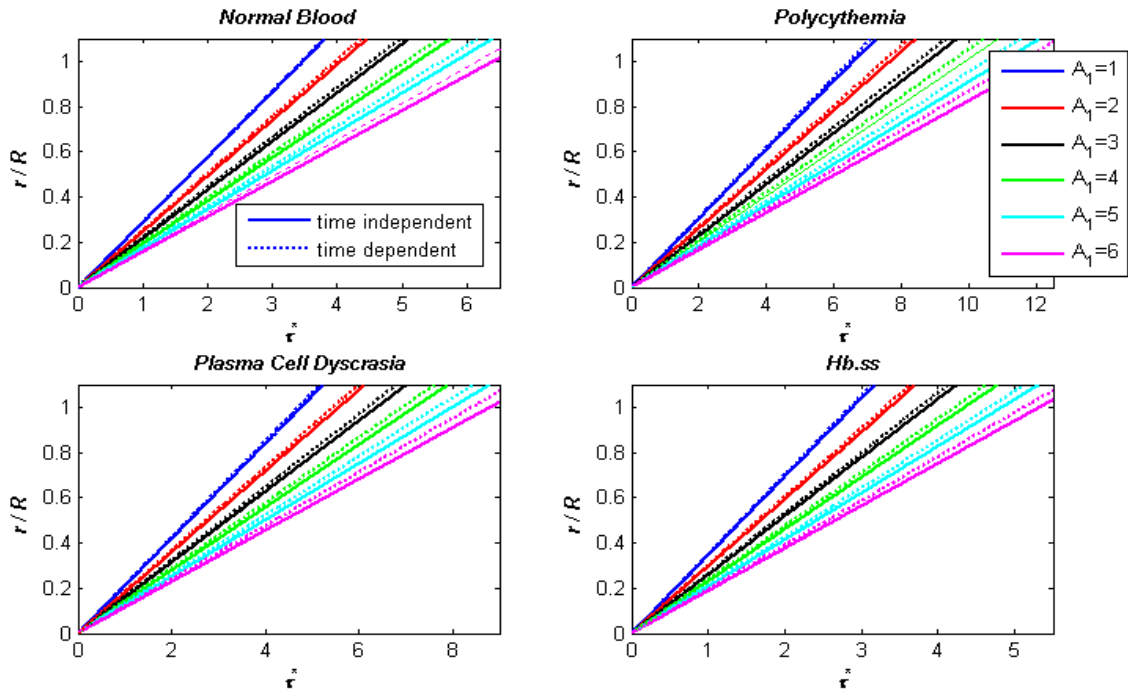


Fig. 6: Shear Stress for stenosed, different A_1 's [$H^*=2, \bar{\alpha}^* = 4, A_0=2, a_0 = 3, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, t = 0.5, b = 0, a = 1$].

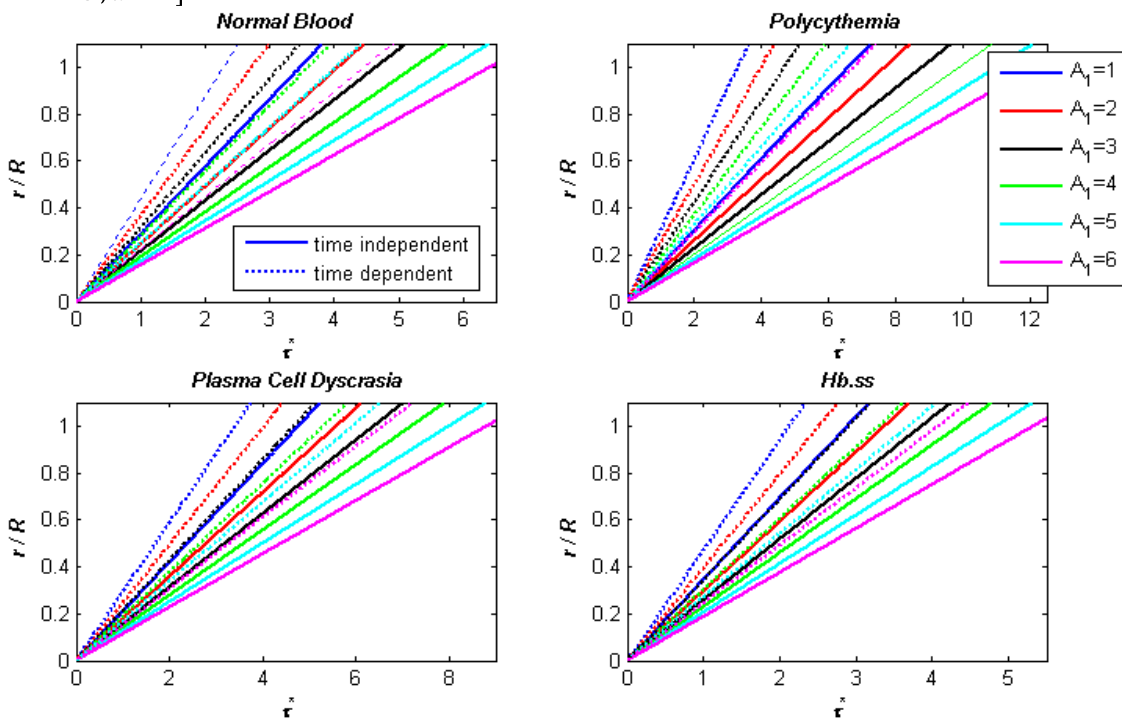


Fig. 7: Shear Stress for stenosed, different A_1 's [$H^*=2, \bar{\alpha}^* = 4, A_0=2, a_0 = 3, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, t = 1.5, b = 1.5, a = 5$].

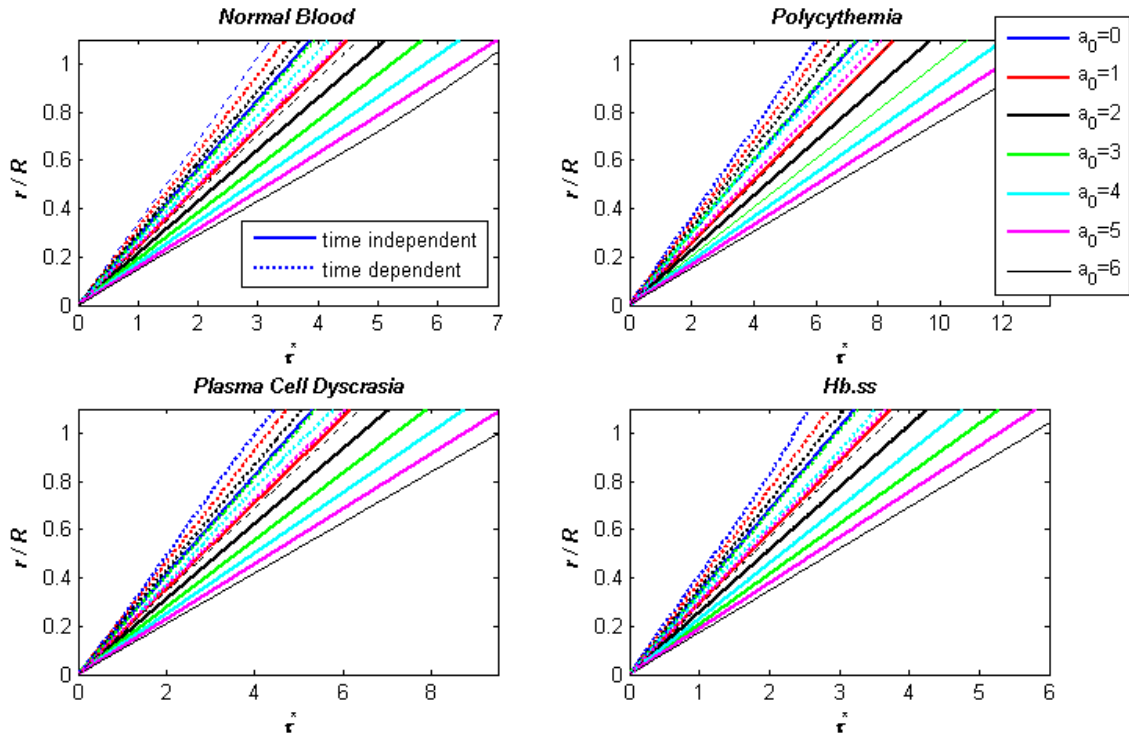


Fig. 8: Shear Stress for stenosed, different a_0 's [$H^* = 2, \bar{\alpha}^* = 4, A_0 = 2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, t = 0.5, b = 0, a = 1$].

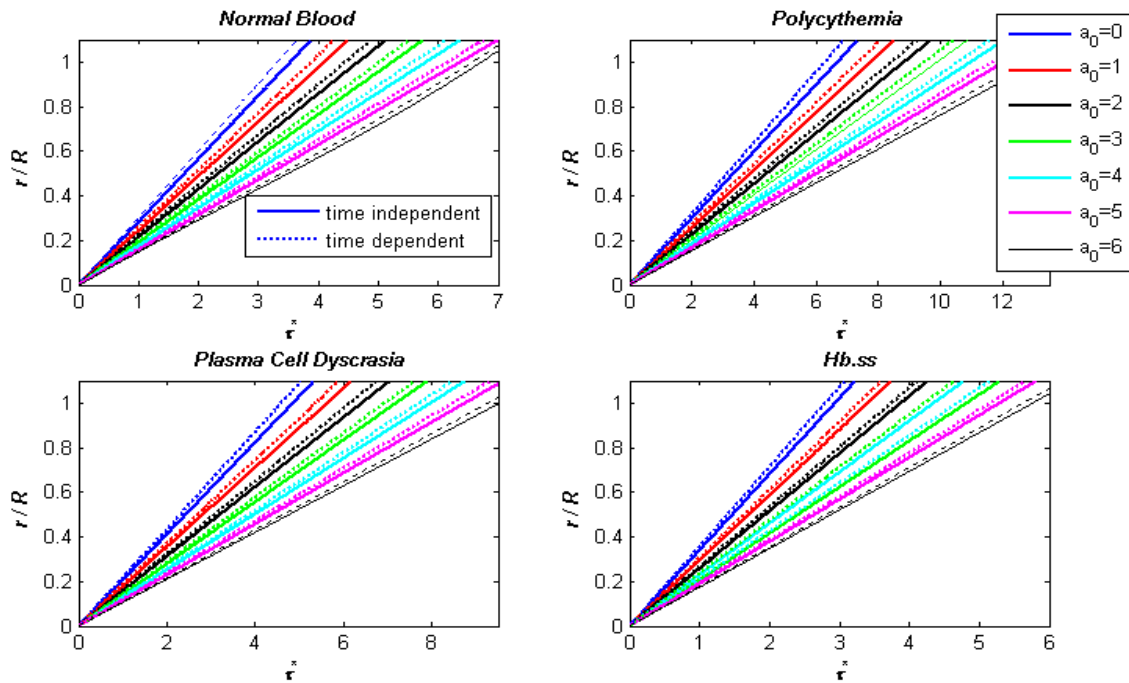


Fig. 9: Shear Stress for stenosed, different a_0 's [$H^* = 2, \bar{\alpha}^* = 4, A_0 = 2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, t = 1.5, b = 1.5, a = 5$].

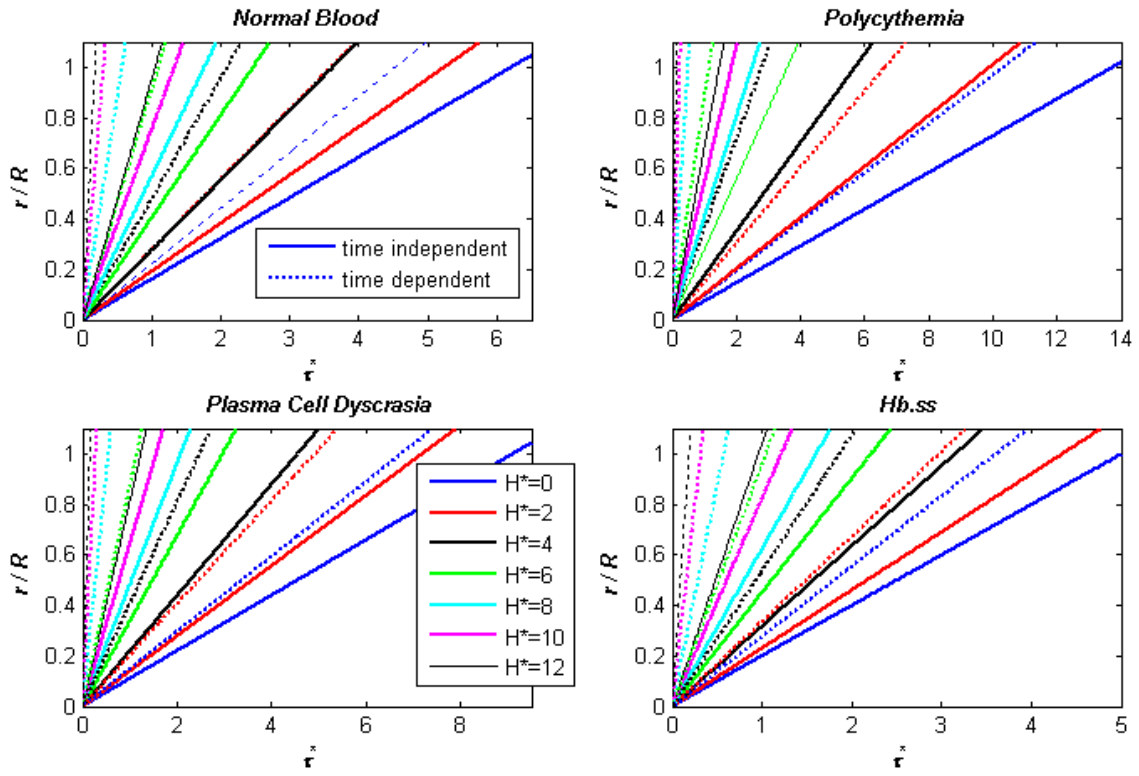


Fig. 10: Shear Stress for stenosed, different H^* 's [$a_0 = 3, \bar{\alpha}^* = 4, A_0=2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0,6, t = 0.5, b = 0, a = 1$].

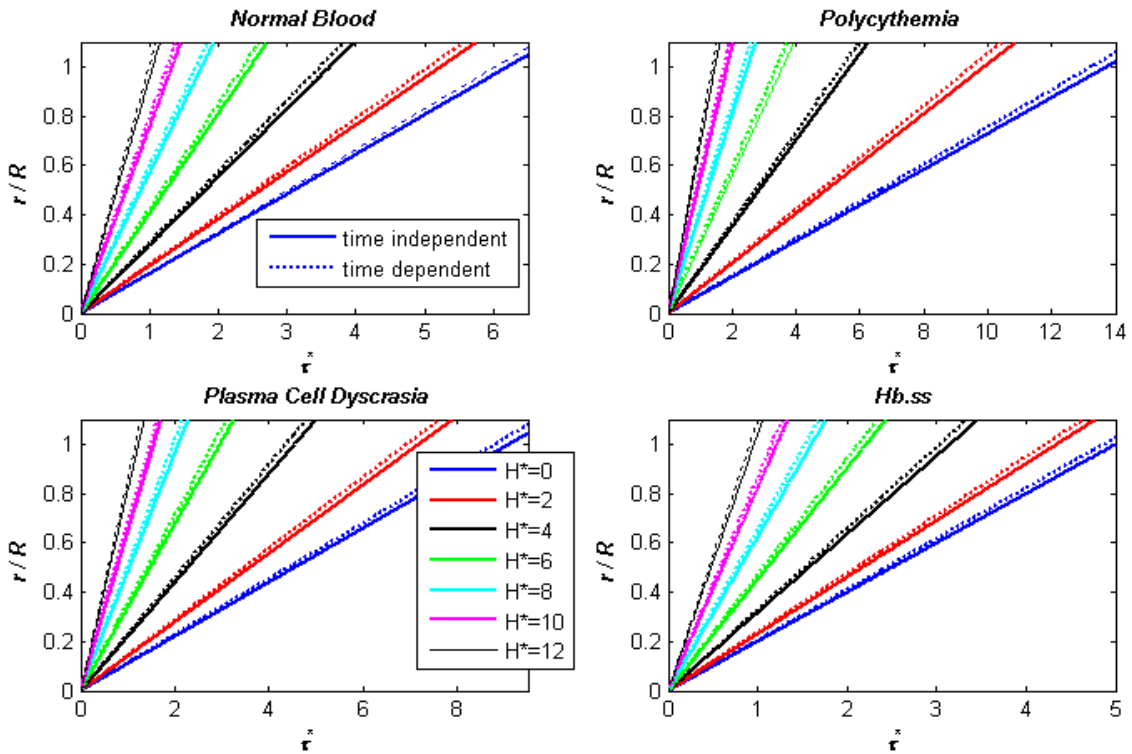


Fig. 11: Shear Stress for stenosed, different H^* 's [$a_0 = 3, \bar{\alpha}^* = 4, A_0=2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0,6, t = 1.5, b = 1.5, a = 5$].

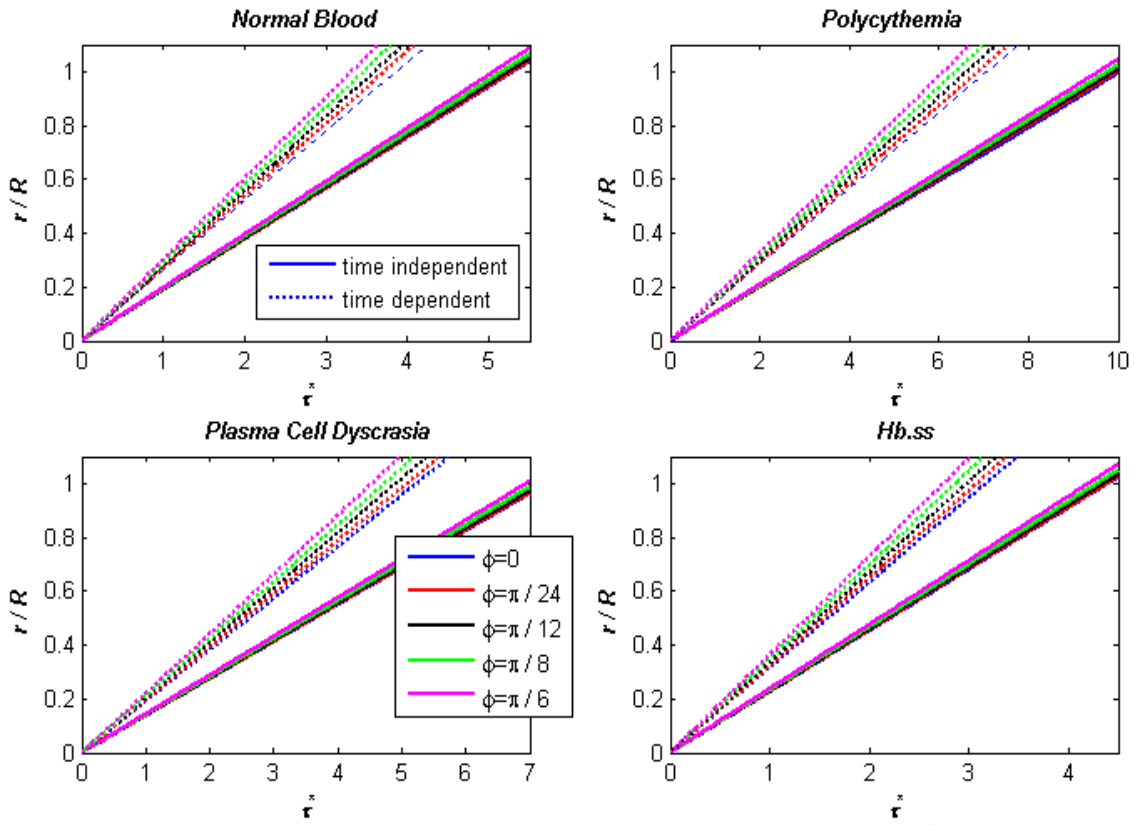


Fig. 12: Shear Stress for stenosed, different ϕ 's [$a_0 = 3, \bar{\alpha}^* = 4, H^* = 2, A_0 = 2, A_1 = 4, \frac{\delta}{R_0} = 0.6, t = 0.5, b = 0, a = 1$].

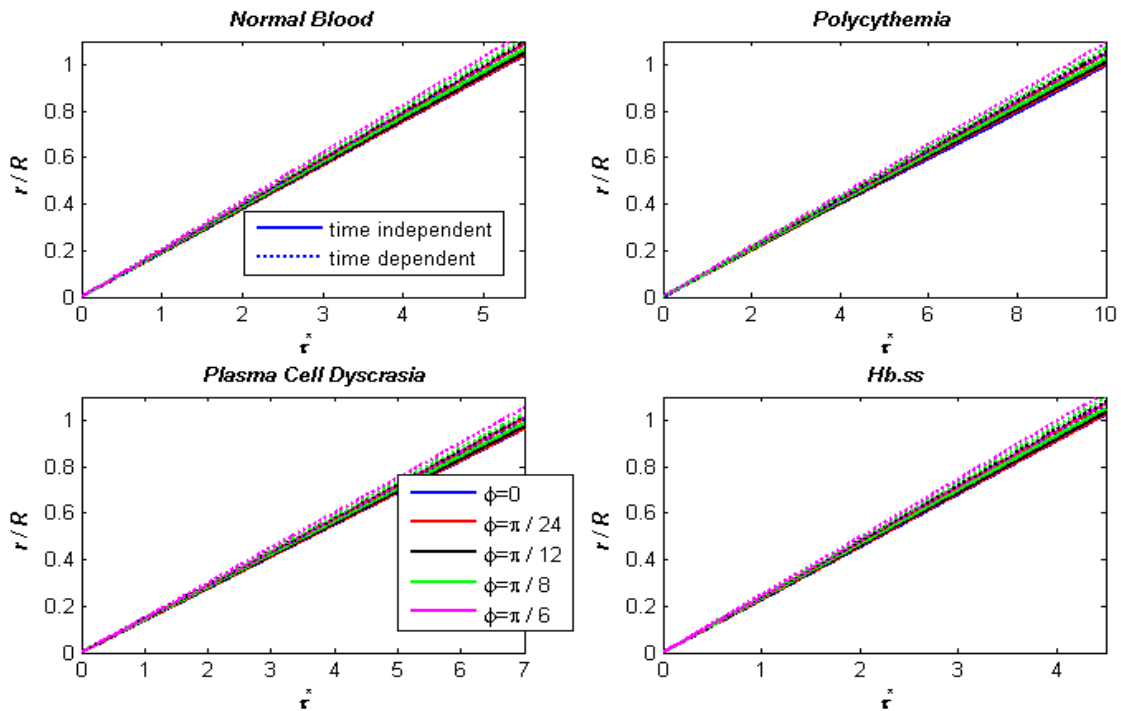


Fig. 13: Shear Stress for stenosed, different ϕ 's [$a_0 = 3, \bar{\alpha}^* = 4, H^* = 2, A_0 = 2, A_1 = 4, \frac{\delta}{R_0} = 0.6, t = 1.5, b = 1.5, a = 5$].

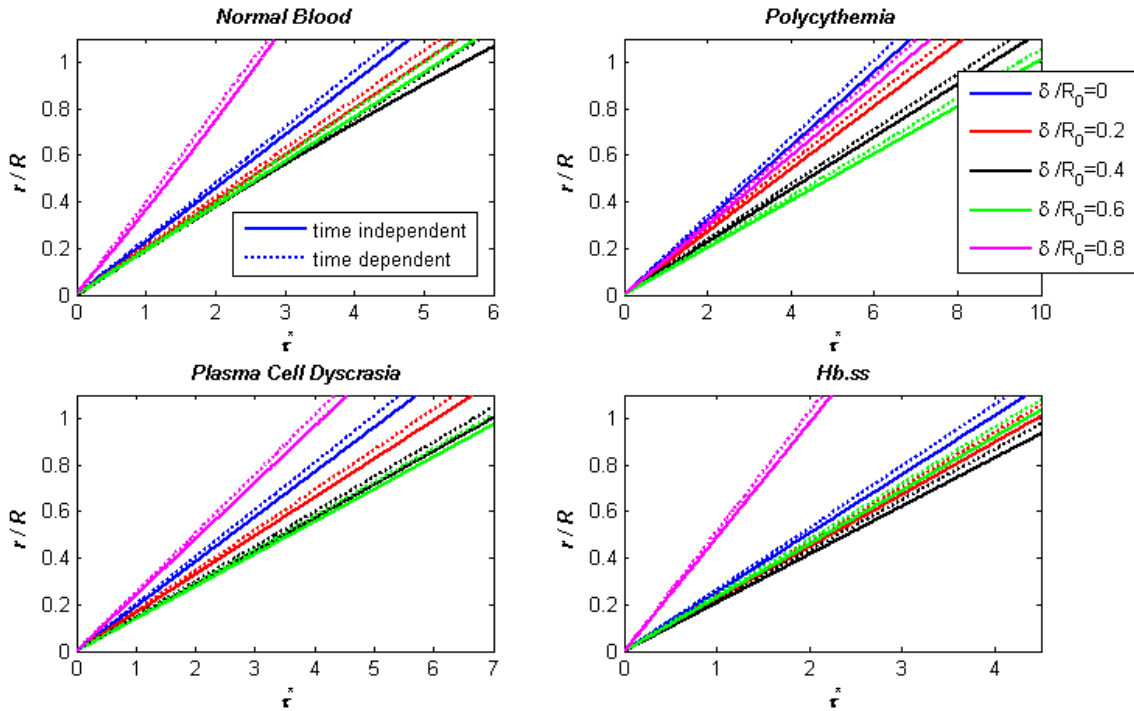


Fig. 14: Shear Stress for stenosed, different $\frac{\delta}{R_0}$'s [$a_0 = 3, \bar{a}^* = 4, A_0 = 2, H^* = 2, A_1 = 4, \phi = 15^\circ, t = 0.5, b = 0, a = 1$].

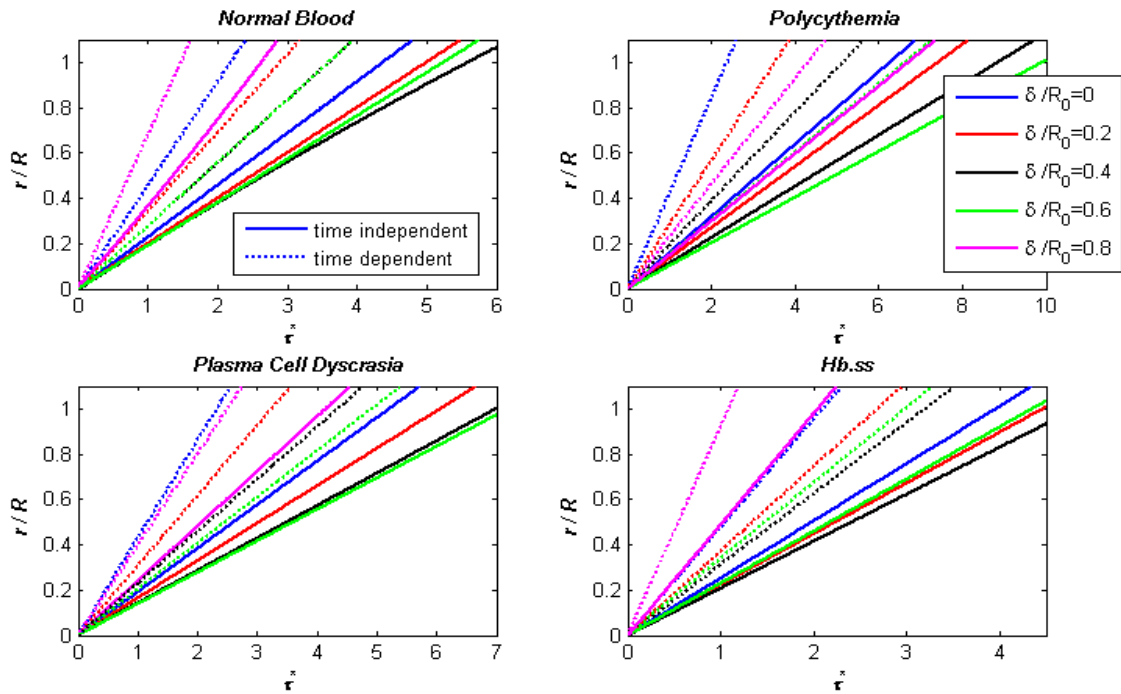


Fig. 15: Shear Stress for stenosed, different $\frac{\delta}{R_0}$'s [$a_0 = 3, \bar{a}^* = 4, A_0 = 2, H^* = 2, A_1 = 4, \phi = 15^\circ, t = 1.5, b = 1.5, a = 5$].

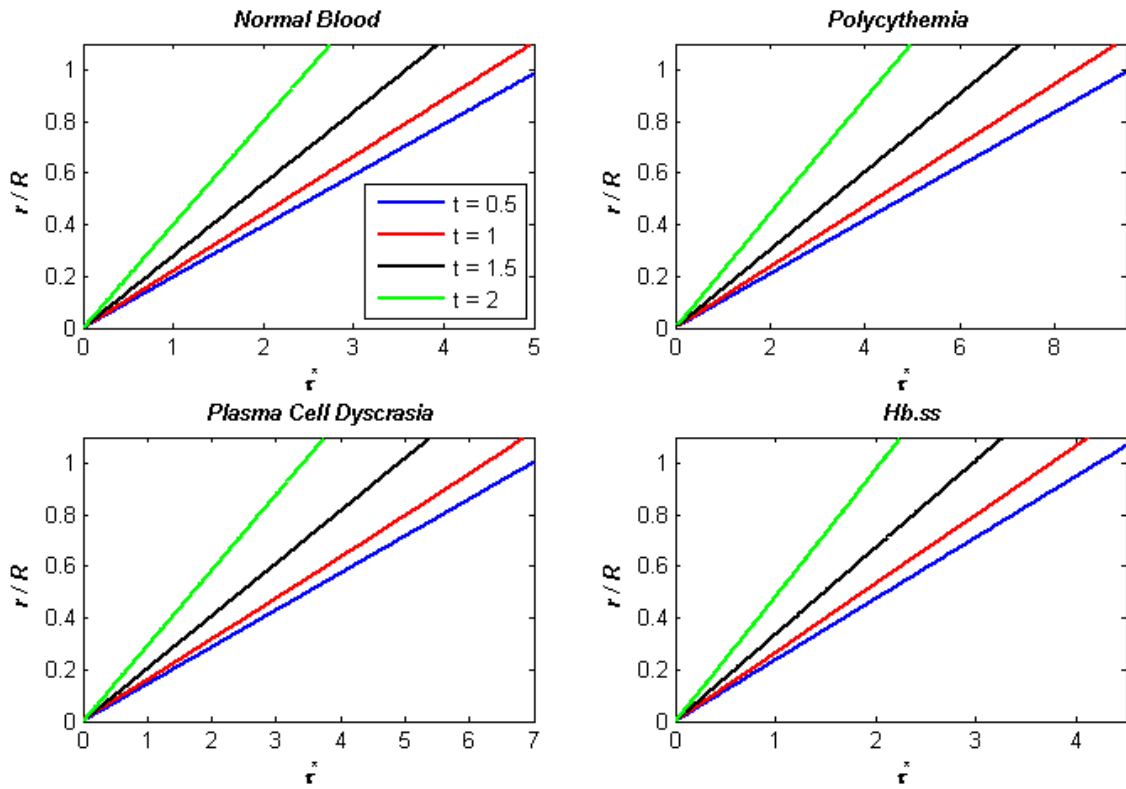


Fig. 16: Shear Stress for stenosed, different t 's [$a_0 = 3, a^- = 4, A_0 = 2, H^* = 2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, b = 0, a = 1$].

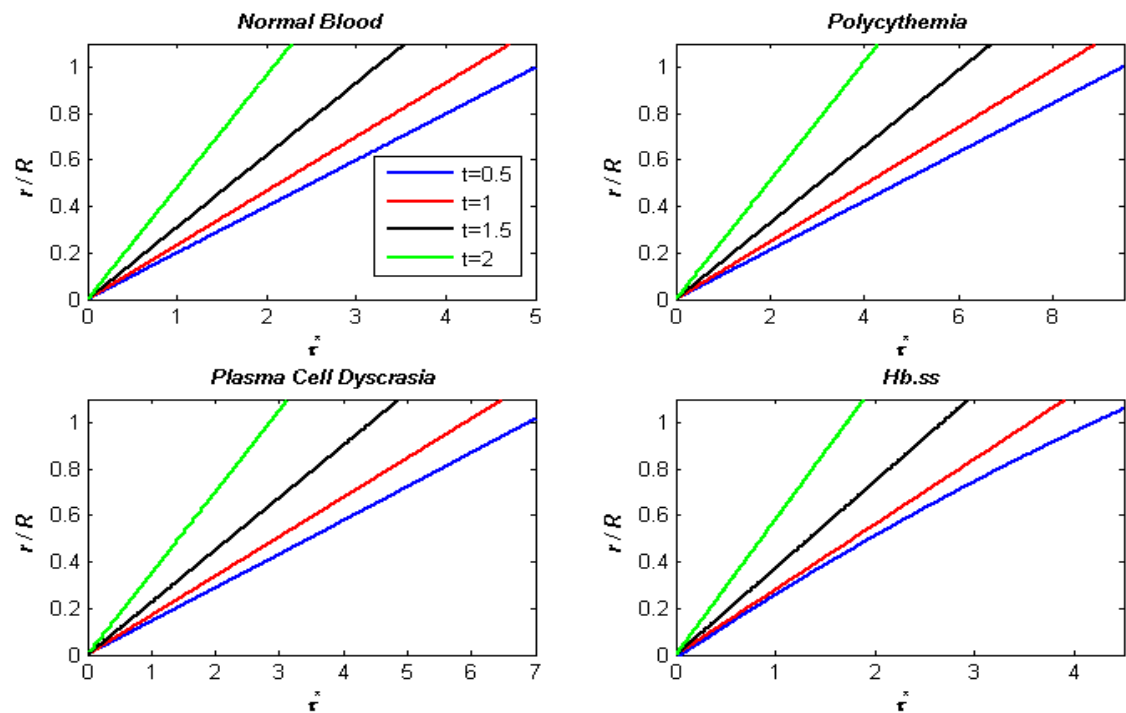


Fig. 17: Shear Stress for stenosed, different t 's [$a_0 = 3, \bar{a}^* = 4, A_0 = 2, H^* = 2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, b = 1.5, a = 5$].

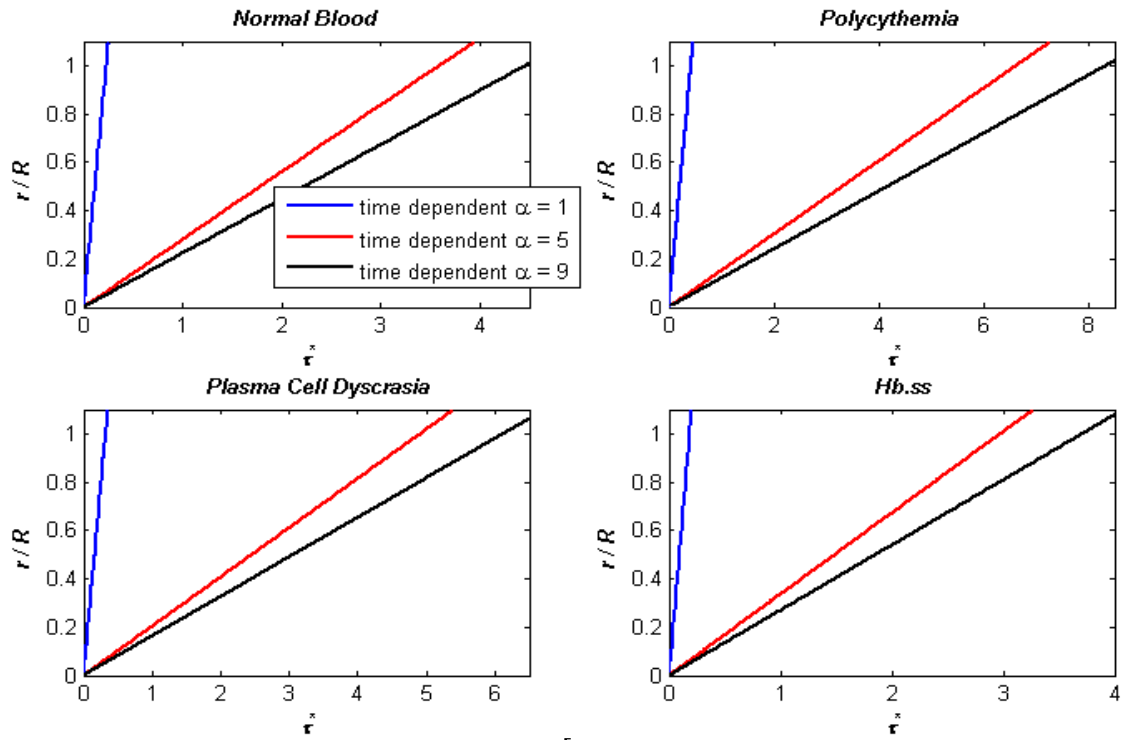


Fig. 18: Shear Stress for stenosed, different b 's [$a_0 = 3, \bar{\alpha}^* = 4, A_0 = 2, H^* = 2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, t = 0.5, a = 1$].

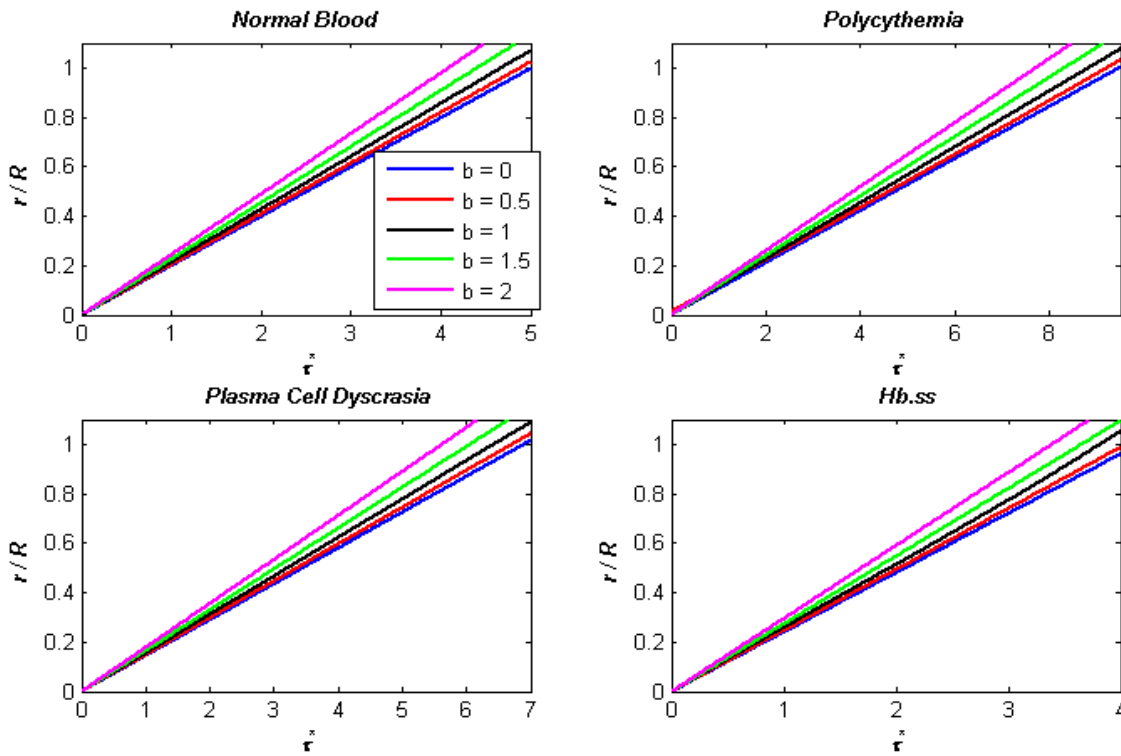


Fig. 19: Shear Stress for stenosed, different α 's [$a_0 = 3, \bar{\alpha}^* = 4, A_0 = 2, H^* = 2, A_1 = 4, \phi = 15^\circ, \frac{\delta}{R_0} = 0.6, b = 1.5, t = 1.5$].

Conclusions:

Blood flow in a stenosed (constriction) tube has been studied in the present investigation due to its importance in Cardio-vascular and biomedical science [Fung, Y.C., 1984, 1986]. The flow of blood is assumed to be represented by a couple stress fluid. Effect of magnetic, Body acceleration together with time dependency (pulsatile nature of blood flow) has been considered in the model. The study also accounts for different blood diseases [Polycythemia, Plasma Cell dyscrasia, and Hb ss] and compared with the case of normal blood.

The results indicate that, the effects of time dependency on flow parameters such as shearing stress have significant effect on the flow in comparison to steady aspect. Also, Effect of magnetic, body acceleration has significant effects with time dependency. The size effects have found to aggravate flow situation in Normal blood and worsen in case of diseased case due to the presence of more blood cells. One of the most significant aspects of the present investigation is that the model confirms with the findings of Young [1979] beyond constriction of 80%, the flow becomes critical and exhibit quite abnormal behavior.

REFERENCES

- Srivastava, L.M. and V.P. Srivastava, 1984. Peristaltic transport of blood: Casson model II. *J. Biomechanics*, 17(1): 821-829.
- Barnothy, M.F., 1964, 1969. Biological effects of magnetic fields: Vol.1 and 2 Plenum Press, New York.
- Bunigliarello, G. and J. Sevilla, 1970. Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes, *Biorheology*, 7(1): 85-107.
- Deshikachar, K.S. and A. Ramchandra Rao, 1985. Effect of magnetic field on the flow and blood oxygenation in channels of variable cross-section, *Int. J. Engg. Sci.*, 23(10): 1121-1139.
- Dintenfass, L., 1967. Inversion of the Fahraeus-Lindqvist phenomenon in blood flow through capillaries of diminishing radius, *Nature*, 215(5105): 1099.
- Fung, Y.C., 1984. *Bio-mechanics*, Springer-Verlag.
- Fung, Y.C., 1986. *Bio-dynamics*, Springer-Verlag.
- Haghighi, A.R. and R.N. Pralhad, 2008. Effect of Low Pressures on the Physiological aspect of human body for varied meteorological and metabolic conditions, *International Journal of Mathematical Sciences and Engineering Applications (IJMSEA)*, 3(2): 205-219.
- Hanselman, C.D., 2002. *Mastering Matlab7*, London.
- Ikkal, Md.A., S. Chakravarty, K.L. Kelvin Wong, J. Mazumdar and P.K. Mandal, 2009. Unsteady response of non-Newtonian blood flow through a stenosed artery in magnetic field, *Journal of computational and applied Mathematics*, 230: 243-259.
- Mandal, P.K., S. Chakravarty, A. Mandal and N. Amin, 2007. Effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosed artery, *Appl. Math. Comput.*, 189: 766-779.
- Mustapha, N., P.K. Mandal, P.R. Johnston and N. Amin, 2010. A numerical simulation of unsteady blood flow through multi-irregular arterial stenoses, *Applied Mathematical Modelling*, 34: 1559-1573.
- Pralhad, R.N. and D.H. Schultz, 2004. Modeling of arterial stenosis and its applications to blood diseases. *J. Mathematical Biosciences*, 203-220.
- Sankar, D.S., K. Hemalatha, 2007. A non-Newtonian fluid flow model for blood flow through a catheterized artery-Steady flow, *Applied Mathematical Modeling*, 31: 1847-1864.
- Sneddon, N., 1980. *Special function of mathematical physics and chemistry*, Longman.
- Stokes, V.K., 1966. Couple stresses in fluid, *The Phy. Of Fluids*, 9: 1709-1715.
- Tanveer, S., 2005. Blood flow through narrow tubes with periodic body acceleration in presence of magnetic field and its application to cardiovascular diseases, Ph.D, Thesis, Gulbarga University, India.
- Vardanyan, V.A., 1973. Effect of magnetic field on blood flow, *Biofizika*, 18(3): 491-496.
- Yang, W.Y., W. Cao, T.S. Chung and J. Morris, 2004. *Applied Numerical Methods Using Matlab*, Wiley-Interscience, New Jersey.
- Young, D.F., 1979. Fluid mechanics of arterial stenoses, *J. of Biomechanical Eng. (Trans ASME)*, 101: 157-175.
- Womersley, J.R., 1955. Method For The Calculation of Velocity, Rate of Flow and Viscous drag in Arteries when the Pressure Gradient is Known, *J. Physiol.*, 553-563.