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Evaluation of three constitutive models to characterize Granular Base for Pavement Analysis using Finite Element Method

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Abstract: Resilient modulus is an important input for computation of the structural response of pavements in Mechanistic -Empirical Methods. It has a significant effect on computed pavement responses and predicted pavement performance. The main objective of the research study that is introduced here is to develop a material model subroutine applicable to Finite Element program to simulate the nonlinear behavior of unbound granular materials, and evaluate the influence of using different constitutive models for characterization of unbound granular materials on critical responses of Flexible pavements. These constitutive models include Linear Elastic, Uzan and Universal models. For this purpose, three different pavement sections are assumed and FE models for these sections were verified by comparing elastic responses with KENLAYER Program. A granular base with known material constants for these three models was selected and sections were analyzed using different constitutive models. The results indicate that the selection of a proper nonlinear model has a significant effect on prediction of pavement responses and so on predicted performance of pavement.

Key words: Granular base**,** Universal Model, Pavement Nonlinear Analysis, FEM.

INTRODUCTION

Existing pavement design procedures are principally based on either empirical or mechanistic-empirical methods. An empirical approach mainly is based on observed performance, without consideration of theoretical behavior. From other side, a mechanistic-empirical design approach ties together the theoretical behavior of a pavement with observed performance. Resilient modulus is a required input for computation of the structural response of pavements in Mechanistic -Empirical Methods. It has a significant effect on computed pavement responses and predicted pavement performance. Resilient modulus can be measured directly from the laboratory or obtained through the use of correlations with other material strength properties such as CBR. M-E Design Guide 2002 has proposed different levels of inputs for resilient modulus for new, reconstruction, and rehabilitation design. M-E Design Guide suggests using a Nonlinear constitutive model for determining of resilient modulus values for unbound granular materials, Subgrade, and bedrock in level 1 of inputs(NCHRP, 2004).

Several computer programs have been developed for nonlinear analysis of flexible pavements. Based on Bermister's layered theory, KENLAYER (Huang, 2004) was developed to account for stress-dependent characteristics by assigning various moduli to different layers. KENLAYER incorporates two nonlinear models one for granular material and the other for fine grained soils. ILLI-PAVE (Thompson and Elliot 1985) and MICH-PAVE (Harichandran *et al*. 1989) are two well-known axisymmetry Finite-Element programs for pavement analysis taking nonlinear behaviors of materials into account. Same as KENLAYER, both of these programs used only the K-θ model for granular materials and the bilinear model for fine-grained subgrade soils. Efficiency in computer resources and computation time is the main advantage of the axisymmetrical Finite element programs, whereas the limitation of single loading restricts their applicability. Using general-purposed FE programs, such as ABAQUS and ANSYS, has become popular among researchers during recent years. (Hjelmstad and Tacirglu, 2000; Kim *et al*, 2009). Zaghloul and White (1993) achieved a good agreement between measured and computed surface deflections for a two layered pavement using the Drucker-Prager material model in a 3D ABAQUS model. Tutumluer and Barksdale (1995) used an Axisymmetric FEM program GT-PAVE with a cross-anisotropic and Uzan resilient modulus model to model a flexible pavement. The predicted strains and stresses in the subgrade and granular layers were comparable with measured values.

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White *et al*. (1997) used the Drucker-Prager elasto-plastic model in a dynamic 3D ABAQUS to model an aircraft wheel for a pavement structure with a 50 mm thick asphalt surfacing overlying a 150 mm thick granular base. They achieved good agreement with MDD measurements and surface deflections. Hjelmstad and Taciroglu (2000) studied eight possible implementations of the algorithms for nonlinear analysis of granular material based on Universal model and implemented these algorithms using UMAT capability of ABAQUS. They showed that some conventional algorithms are destined to fail at higher load levels and offered two competitive methods for global analysis using Universal nonlinear model. Perkins *et al*. (2004) made a proper numerical implementation of the nonlinear elastic analysis with the tension cutoff model using the ABAQUS program. A good agreement was seen between the ABAQUS numerical and theoretical solutions for a specific set of load steps. Sukumaran (2004) presented a three-dimensional analysis model of airport flexible pavements using ABAQUS. In nonlinear material analysis, granular materials used the Mohr-Coulomb failure criterion and medium strength subgrade and Dupont clay were modeled using the von-Mises failure criterion. The model was also compared with the available failure data from the National Airport Pavement Test Facility (NAPTF) of the Federal Aviation Administration (FAA). To properly characterize the resilient response of coarse-grained unbound aggregates and fine-grained subgrade soils, Kim *et al*. (2009) developed a user-defined nonlinear material model using UMAT in the ABAQUS program. They used a direct secant stiffness approach to converge smoothly in each loading for both ABAQUS axisymmetric and 3D flexible pavement response analysis. Nonlinear FE mechanistic model predictions showed generally good agreement with the measured responses of the NAPTF MFC test section. The predicted values of subgrade vertical stress, surface displacement, and subgrade displacement compared reasonably well with measured responses in the MFC section.

The main objective of this study is to evaluate the effect of using different constitutive models for simulation of granular base behavior on critical responses of flexible pavements and so on predicted performance of pavements. For this purpose, Universal Nonlinear model is implemented in Finite Element code for nonlinear analysis of pavement structure based on Axisymmetry FEM. After validation of FE models for three different pavement sections using linear elastic characterization of all layers, three different constitutive models were used for modeling of granular base and then the critical responses of each section using these models were calculated and compared to each other.

Constitutive Models for Unbound Granular Materials:

The response of a granular soil sample under repeated loading during construction phase and initial trafficking tends to shake down to elastic response. The amount of plastic deformations decrease with increase in load repetitions until the response is essentially elastic. These observations have led researchers in the pavement community to simulate the behavior of granular materials as elastic or resilient materials(Hjelmstad and Taciroglu, 2000).

The concept of a resilient modulus of a material was originally introduced by Seed *et al*. (1962). Seed *et al*. defined resilient modulus, M_r , as the ratio of applied dynamic deviatoric stress, σ_d , to the resilient or recovered strain, ε_r , under a transient dynamic pulse load given by $M_r = \sigma_d / \varepsilon_r$. Repeated load triaxial test is commonly employed to quantify the resilient modulus of granular materials and cohesive soils. The resilient response of granular materials and fine-grained soil is stress dependent (resilient modulus is not constant, but depends on the repeated stress state). Several Models have been developed over the years that combine applied stresses and material characteristics to describe the nonlinear behavior of granular materials under traffic loading. The K-θ model has been the most famous for characterizing the resilient response of the granular bases and subbase materials (Hicks and Monismith, 1971). The resilient modulus (M_R) is given as follows:

$$M_R = K_1 \theta^{K_2} \tag{1}$$

Where

θ = First invariant of stress tensor = $|\sigma_1 + \sigma_2 + \sigma_3|$ /3

σ_1= Major principal stress.

σ_2= Intermediate principal stress

σ_3 = Minor principal stress/confining pressure

K_1, K_2 = Regression analysis constants obtained from experimental data.

Uzan (1985) observed that the K-θ model did not summarize measured data well when shear stresses were significant, and proposed a three parameter model. This model is given as

$$M_R = K_1 \theta^{K_2} \sigma_d^{K_3} \tag{2}$$

Where
θ = First invariant of stress tensor = $|\sigma_1 + \sigma_2 + \sigma_3| /3$

$\sigma_d = |\sigma_1 - \sigma_3|$ = The deviatoric stress in a triaxial test configuration

K_1, K_2, and K_3 = material constants

Witczak and Uzan (1988) proposed a modification to the Uzan model by replacing the deviator stress term in Equation (2) by an octahedral shear stress term. This octahedral shear stress model also considers the dilation effect that takes place when a pavement element is subjected to a large principal stress ratio σ_1 / σ_3. This model is called Universal Model and is given as follows:

$$M_R = K_1 \theta^{K_2} \tau_{oct}^{K_3} \tag{3}$$

Where
θ = First invariant of stress tensor = $|\sigma_1 + \sigma_2 + \sigma_3|/3$

τ_{oct} = Octahedral shear stress= $1/3\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$

K_1, K_2, and K_3 = Multiple regression constants evaluated from resilient modulus test data.

In 2002 M-E Guide, resilient modulus is estimated using a generalized constitutive model for Level 1 analysis for the nonlinear stress-dependent modeling of both the unbound aggregates and fine-grained soils (NCHRP, 2004). The difference in material behavior predicted by Universal and 2002 M-E Guide were only found in the regression variables and both of them give same values for resilient modulus (Kim *et al.*, 2009). The 2002 M-E Guide model used in design procedure is as follows:

$$M_R = K_1 p_a \left(\frac{\theta}{p_a} \right)^{K_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k3} \tag{4}$$

Where
θ = First invariant of stress tensor = $|\sigma_1 + \sigma_2 + \sigma_3|/3$
τ_{oct}= Octahedral shear stress
p_a = Atmospheric pressure
K_1, K_2, and K_3 = multiple regression constants evaluated from resilient modulus test data.

Validation of Finite Element Models:

To study the influence of using different characterization model of aggregate base layer on critical responses and predicted performance of pavement, three different pavement sections including low thickness, medium thickness and thick section were selected. These sections have been called 1st, 2nd and 3rd section, respectively.

For each of these sections a FE model was created and a fine mesh was generated to get more accurate results. For validation of FE models, three pavement sections were analyzed using both KENLAYER and Finite Element models. All pavement layers assumed to be linear elastic. Pavement structure and material properties for this set of analysis is given in table (1). Pavement sections are subjected to a circular load which has radius of 9.5cm and uniform pressure of 690KPa. FE mesh domain has the size of 140-times R in the vertical direction and 20-times R in the horizontal directions, where R is the radius of load footprint. This mesh domain gives comparable results with analytical solutions (Kim *et al*., 2009). The element type for all analysis was selected as 8-node biquadratic axisymmetry quadrilateral. Generated mesh near applied load is presented in fig. (1) for 1st section.

Table 1: Pavement Structure and Material Properties for validation of Axisymmetric FE analysis.

Pavement Layer	E or M_r (MPa)	Poisson ratio	Thickness(cm)		
			1st Section	2nd Section	3rd Section
Asphalt concrete	3500	0.35	6	12	20
Granular Base	240	0.34	15	30	60
Subgrade soil	50	0.45	1309	1288	1250

Note: Subgrade thickness just is used for FE analysis.

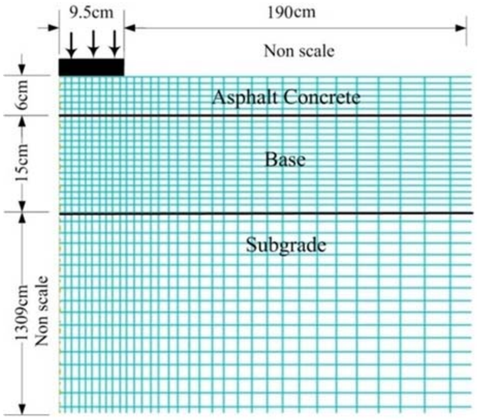


Fig. 1: Generated Mesh for 1st pavement section.

Table (2) shows the critical responses calculated using Finite Element and KENLAYER. As it can be seen, the good agreement can be proved between responses computed from KENLAYER and Finite Element method.

Table 2: Predicted Responses by FEM and KENLAYER.

Pavement responses	1st Section		2nd Section		3rd Section	
	FEM	KENLAYER	FEM	KENLAYER	FEM	KENLAYER
$\delta_{surface}$ (mm)	0.536	0.576	0.336	0.321	0.023	0.020
$\sigma_{h\ botton\ of\ AC}$ (Mpa)	1.745	1.741	0.881	0.881	0. 408	0.414
$\varepsilon_{h\ botton\ of\ AC}$ ($\mu\varepsilon$)	352.22	351.90	175.780	175.500	81.215	82.310
$\sigma_{v\ top\ of\ subgrade}$ (Mpa)	-0.0560	-0.0562	-0.0152	-0.0152	-0.0049	-0.0046
$\varepsilon_{v\ top\ of\ subgrade}$ ($\mu\varepsilon$)	-1056.91	-1057.00	-282.49	-282.30	-86.986	-88.500

Material Model Subroutine for Nonlinear Analysis:

The material model subroutine is called at every material integration point for any iteration. Finite Element program passes in stresses, strains, and state variables at the beginning of each time increment along with the current strain increment. The user defined material model subroutine then updates the stresses and state variables to the values at the end of the time increment and provides the material stiffness matrix. Material model subroutine for simulation of nonlinear behavior of granular base materials was developed based on algorithm which is proposed by Hjelmstad and Tacirglu (2000). The nonlinear constitutive model of resilient behavior can be written as:

$$S = \frac{M_r(\theta, \tau_{oct})}{1+\nu}(\alpha \varepsilon I + E) = \frac{M_r(\rho, \gamma_{oct})}{1+\nu}[1 + \alpha I \otimes I]E \tag{5}$$

Where:
S=Stress tensor
E=Strain tensor

$M_r(\theta, \tau_{oct}) = M_r(\rho, \gamma_{oct})$ =Resilient Modulus

ε= First invariant of strain tensor ($\varepsilon_{11}+ \varepsilon_{22}+\varepsilon_{33}$)
ρ= | ε|
γ_{oct}= Octahedral shear strain
I= Identity tensor

$\alpha = \nu/(1-2\nu)$

ν=Poisson ratio

For Universal Model, Hjelmstad and Tacirglu showed that the material tangent stiffness ($C = \partial\sigma/\partial\varepsilon$) can be computed directly using the following equation:

$$C = \frac{(K_1.\bar{\alpha}^{K_2}.\rho^{K_2}.\gamma_{oct}{}^{K_3})^{\mu}}{(1+\nu)^{1+\mu}}[1+(\mu.K_2.\bar{\alpha}+\alpha)I \otimes I + \frac{\mu.K_3}{3}N \otimes N$$
$$+\frac{\mu.\bar{\alpha}.\varepsilon.K_3}{3\gamma}I \otimes N + \frac{\mu.\gamma.K_2}{\varepsilon}N \otimes I] \tag{6}$$

where:
K_1,K_2 and K_3= Material constants for Universal model

$\alpha = \frac{\nu}{1-2\nu}$

$\bar{\alpha} = (1+\nu)/3(1-2\nu)$

$\mu = 1/(1-K_2-K_3)$

N=$\bar{E}/\gamma$

$\bar{E}$ =Deviatoric strain tensor ($E - \frac{1}{3}\varepsilon I$)

The analysis starts with an arbitrary stiffness matrix and the first increment of loading. The computed strains are used to adjust the stiffness matrix with Equation (6). An equilibrium check is then conducted by comparing the differences between the internal force and the external loading. If differences exceed the specified accuracy, the pavement responses are computed again with the adjusted stiffness matrix. The analysis of the specific loading increment is thus iterated until the equilibrium accuracy is achieved. The flow diagram of implementation of material model subroutine is shown in fig. 2.

The convergence properties of the mentioned method can be improved with the introduction of a damped fixed-point iteration in which a modulus is formed from the current state and the previous state (Brown and Pappin 1981; Tutumluer 1995; Kim et. al 2009). This can be written as:

$$M_r^i = (1-\beta)M_r^{i-1} + \beta\left(M_r^i\right)_{\mathrm{mod}\,el} \tag{7}$$

where

M_r^i =new M_r at the end of iteration number i

M_r^{i-1} = M_r used at the end of iteration number i−1; M_r model

$\left(M_r^i\right)_{\mathrm{mod}\,el}$ = M_r computed from the model at the end of iteration number i

$\beta =$ damping parameter [0, 1].

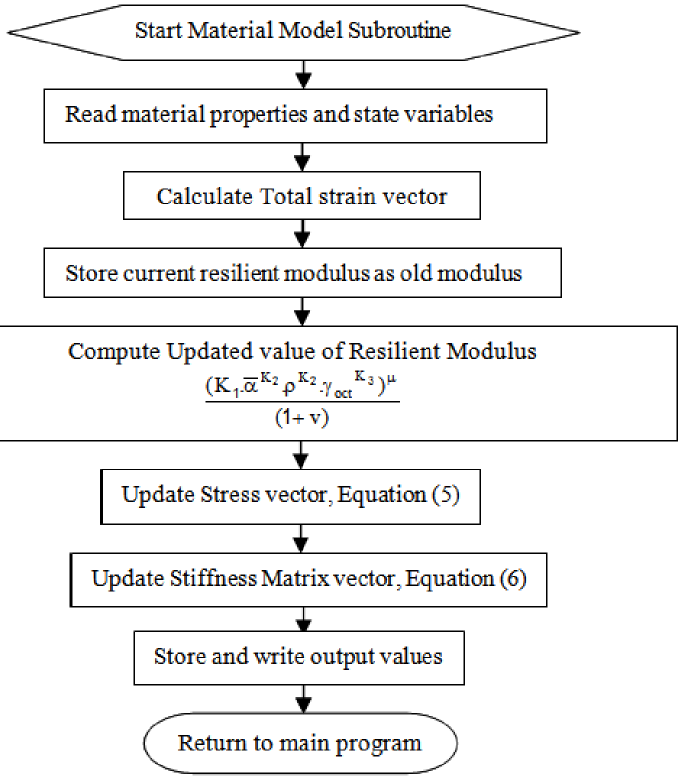


Fig. 2: Flow diagram of material model subroutine for implementation of nonlinear model.

For analysis of each pavement, Load includes both gravity and wheel load that are imposed on model in two different stages. Both gravity and wheel load are applied incrementally in which the load increment is increased or decreased according to the various convergence characteristics throughout the analysis. For validation of developed subroutine, critical responses (including bulk stress and octahedral shear stress) for computation of resilient modulus and computed resilient modulus were compared to each other.

A typical value of critical responses and computed resilient modulus is represented in Fig. 3 for 1st section.

Comparison of Different Constitutive Models for Modeling of Aggregate Base:

The triaxial test data was used for determining material constants for different models including Linear Elastic, K-θ Nonlinear Elastic and Universal Nonlinear Elastic. This triaxial test data was obtained by Allen and Thompson at the University of Illinois (Allen, 1973). The samples were first subjected to several cycles of certain stress paths to achieve resilient behavior. The values of the material constants for the characterization of base layer were established by fitting the given model, using a weighted nonlinear least-squares curve fitting technique, to the laboratory test data by Hjelmstad and Tacirglu (2000). Material constants for modeling granular base layer using each model are given in table (3). For evaluation of different constitutive models to simulate the behavior of base layer and comparison of computed critical responses of pavement using each of these models, three different sections were considered and each of these sections was analyzed with assuming different model for granular base layer. Pavement sections in this study are given in table (3).

Asphalt surface and subgrade soil was assumed to be linear elastic. Each Pavement section is subjected to a circular load which has radius of 9.5cm and uniform pressure of 690KPa. Distribution of computed resilient Modulus in Base layer has been shown in fig. (3) for two different radial distances(R=0cm and R=9.5cm). The variation of resilient modulus in different depth is very Severe for 1st section which has a low bearing capacity but for 3rd section, with high bearing capacity, resilient modulus varies smoothly when depth increases.

Critical responses include surface deflection, radial stress and strain at the bottom of Asphalt layer and Vertical stress and strain at top of subgrade were computed using different constitutive model for granular base. Computed responses are presented in table (4).

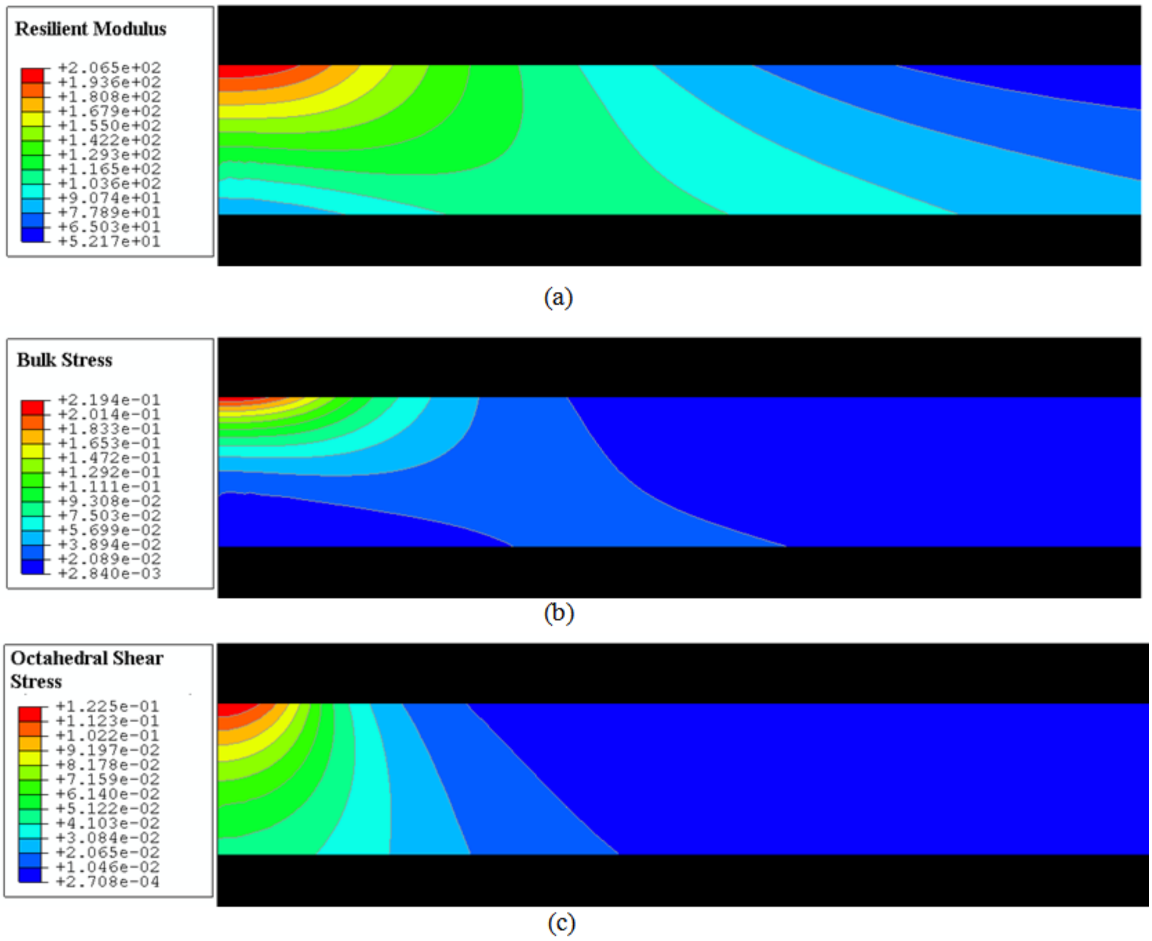
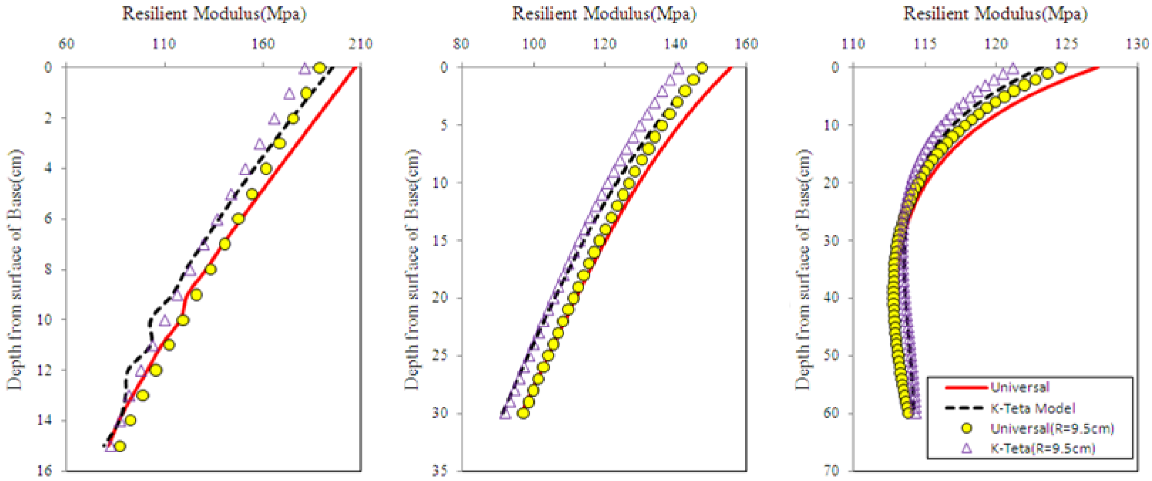


Fig. 2: Responses for Base layer of 1st section ($M_R = 425\theta^{0.22}\tau_{oct}^{0.07}$); (a) Resilient Modulus (MPa), (b) Bulk Stress(3θ) (MPa), (c) Octahedral Shear Stress (MPa).

Table 3: Pavement Structure and Material Properties for validation of Axisymmetric FE analysis.

Pavement Layer	E or M_r (Mpa)	Poisson ratio	Thickness (cm)	Density (kgf /cm^3)	Material Properties
AC	3500	0.35	6 -12-20	0.0029	Linear Elastic
Base	240	0.34	15-30-60	0.0022	Linear Elastic
		0.33			M_r = 240MPa
					Nonlinear: K-θ model(Eq. 1)
					K_1=420Mpa K_2=0.29
					Nonlinear: Universal model(Eq. 2)
		0.32			K_1=425Mpa K_2=0.22 K_3=0.07
Subgrade	50	0.45	1309	0.0017	Linear Elastic

As it can be seen, a good agreement exists among responses that are computer using K-θ and Universal model. The agreement would be higher if the structural capacity of pavement increases. Responses that are computed using Linear Elastic behavior of Base layer have significant difference from which are computed assuming nonlinear behavior of base layer. Radial strain at the bottom of Asphalt layer and Vertical strain at the top of Subgrade are two critical responses which are used to control fatigue and rutting of flexible pavements respectively. Error in computation of these two responses results in erroneous predicting of pavement performance. Allowable Load repetitions for fatigue and rutting life are given in table (5). These values were calculated one time using responses obtained assuming linear behavior of base and the other time using responses obtained assuming nonlinear behavior (Universal Model) of base layer. Transfer functions for predicting fatigue and rutting life are accepted from Asphalt Institute (AI, 1981; AI, 1982). Load repitations calculated based on the assumption of nonlinear behaviour of base are about one half of what calculated from linear bahavior of base layer and this means if a pavement is designed based on linear elastic theory, it just can carry one half of design load repitations during its design life. This approves the necessity of using proper contitutive model for charactrization of granular materials for precise prediction of pavement performance.



(a) (b) (c)

Fig. 3: Computed Resilient Modulus for two different radial distances; (a)1st section, (b)2nd section, (c) 3rd section

Table 4: Critical responses of each section using different constitutive model for base layer.

Section	Base Layer Model	$\delta_{surface}$ (mm)	$\sigma_{h\ botton\ of\ AC}$ (Mpa)	$\varepsilon_{h\ botton\ of\ AC}$ ($\mu\varepsilon$)	$\sigma_{v\ top\ of\ subgrade}$ (Mpa)	$\varepsilon_{v\ top\ of\ subgrade}$ ($\mu\varepsilon$)
1st section	Linear Elastic	8.445	1.746	352.63	-0.0607	-1076.73
	Nonlinear (K-θ)	8.557(1.3%)	2.290(31.2%)	448.87(27.3%)	-0.0706(16.3%)	-1258.82(16.9%)
	Nonlinear (Universal)	8.547(1.2%)	2.215(26.9%)	435.60(23.5%)	-0.074(21.9%)	-1255.30(16.6%)
2nd section	Linear Elastic	8.266	0.869	173.519	-0.0244	-331.54
	Nonlinear (K-θ)	8.344(0.9%)	1.112(28.0%)	214.42(23.6%)	-0.0277(13.5%)	-383.76(15.8%)
	Nonlinear (Universal)	8.343(0.9%)	1.106(27.3%)	214.65(23.7%)	-0.0278(13.9%)	-384.23(15.9%)
3rd section	Linear Elastic	8.828	0.406	81.215	-0.0225	-170.078
	Nonlinear (K-θ)	8.343(-5.5%)	0.514(26.6%)	99.876(23.0%)	-0.0236(4.9%)	-187.476(10.2%)
	Nonlinear (Universal)	8.348(-5.4%)	0.511(25.9%)	99.62(22.7%)	-0.0237(5.3%)	-188.974(11.1)

Note: all responses include self weighing.

Table 5: Number of allowable repetitions of load for Fatigue and Rutting life based on the assumption of different constitutive model for base layer

Constitutive Model	1st Section		2nd Section		3rd Section	
	Fatigue	Rutting	Fatigue	Rutting	Fatigue	Rutting
Linear Elastic	246,280	21,545	2,540,768	4,057,975	30,906,028	78,966,042
Nonlinear (K-θ)	111,308	10,755	1,266,083	2,117,479	15,646,942	51,208,637
Nonlinear (Universal)	122,862	10,889	1,261,623	2,105,985	15,779,659	49,427,963

Conclusion:

Developed Finite Element Code for simulation of Nonlinear behavior of granular material based on axisymmetry Finite-Element Method can be used effectively for nonlinear analysis of flexible pavement.

According to the test results for characterization of granular base, three common constitutive models for modeling granular base were used for analysis of three different pavement sections and critical responses of these three sections were computed using each of these constitutive models. A good agreement can be indicated among responses that are computed using K-θ and Universal model. The agreement would be higher for thicker sections than low thickness section. This means that for accurate analysis of low thickness pavements, selected nonlinear model can affect more severely on critical responses and so on predicted performance of pavement.

Responses which are computed using Linear Elastic behavior of Base layer have significant difference from that are computed based on the assumption of nonlinear behavior of base layer. Computed fatigue and rutting life using linear and nonlinear analysis show that pavements which are analyzed with the assumption of linear elastic behavior of base just can carry one half of design load repetitions. This truth can approve the importance of nonlinear characterization of granular base for accurate pavement performance prediction.

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